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NOTE ON STRONG SUPPORT VERTEX COVERING OF FUZZY GRAPH $G(\sigma, \mu)$ BY USING STRONG ARC

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Abstract

In this paper, definition of strong degree and strong full degree, strong support and strong support regular of a vertex of fuzzy graph are newly introduced by using strong arc. Further, definition of strong support covering and anti-strong support covering of the fuzzy graphs are newly introduced by using strong arc.

Moreover, some standard theorems on strong full degree and strong support regular for the fuzzy graph are proved with example. Finally, results on strong support covering number and anti-strong support covering number for the standard fuzzy graph like fuzzy paths FP_n, fuzzy cycle FC_n are found.

1. Introduction

In 1965, L. A. Zadeh introduced the concept of fuzzy subset of a set as a way for representing uncertainty [19]. Zadeh's ideas stirred the interest of researchers worldwide. J. N. Monderson, S. N. Premchand, discussed fuzzy graph theory fuzzy hypergraph [4]. Fuzzy graph is the generalization of the ordinary graph. The formal mathematical definition of domination was given by O. Ore. in 1962[10]. In 1975, A. Rosenfeld introduced the notion of fuzzy graph and several analogs of theoretic concepts such as path, cycle and connectedness [11]. A. Somasundaram and S. Somasundaram discussed the domination in fuzzy graph using effective arc [12]. A. Nagoorgani and V. T.

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Chandrasekarn discussed the strong arc in fuzzy graph [8, 9]. K. R. Bhutani and A. Rosenfeld have introduced the concept of Strong arcs in fuzzy graph [1, 2]. Several works on fuzzy graph are also done by Mathumangal pal and Hossein Rashmanlou [6], Mathumangal pal [5] S. Methew and M. S. sunitha [7], C. Y. Ponnappan, P. Surilinathan and S. Basheer Ahamed [13, 17, 18], C. Y. Ponnappan and V. Senthilkumar [14, 15, 16]. Before discuss the study of strong Support cover of the Fuzzy graphs, we are placed few preliminary.

2. Preliminaries [14, 15, 16, 17]

Definition 2.1. Fuzzy graph $G(\sigma, \mu)$ is pair of function $V \rightarrow [0, 1]$ and $\mu : V\times \rightarrow [0, 1]$, where for all u, v in V, we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$.

Definition 2.2. The fuzzy graph $H(\tau, \rho)$ is called a fuzzy subgraph of $G(\sigma, \mu)$ if $\tau(u) \leq \sigma(u)$ for all u in V and $\rho(u, v) \leq \mu(u, v)$ for all u, v in V.

Definition 2.3. A fuzzy subgraph $H(\tau, \rho)$ is said to be a spanning sub graph of $G(\sigma, \mu)$ if $\tau(u) = \sigma(u)$ for all u in V. In this case the two graphs have the same fuzzy node set, they differ only in the arc weights.

Definition 2.4. Let $G(\sigma, \mu)$ be a fuzzy graph and τ be fuzzy subset of that is, $\tau(u) \leq \sigma(u)$ for all u in V. Then the fuzzy subgrpah of $G(\sigma, \mu)$ induced by τ is the maximal fuzzy subgraph of $G(\sigma, \mu)$ that has fuzzy node set τ. Evidently, this is just the fuzzy graph $H(\tau, \rho)$ where $\rho(u, v) = \tau(u) \wedge \tau(v) \wedge \mu(u, v)$ for all u, v in V.

Definition 2.5. The underlying crisp graph of a fuzzy graph $G(\sigma, \mu)$ is denoted by $G^* = (\sigma^*, \mu^*)$, where $\sigma^* = \{u \in C \mid \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \in V \times V \mid \mu(u, v) > 0\}$.

Definition 2.6. A fuzzy graph $G(\sigma, \mu)$ is a strong fuzzy graph if $\mu(u, v) = \mu^{\infty}(u, v)$ for all $u, v \in \mu^*$ and is a complete fuzzy graph if $\mu(u, v) > 0$ for all u, v in σ^*. Two nodes u and v are said to be neighbors if $\mu(u, v) > 0$.

Definition 2.7. A fuzzy graph $G(\sigma, \mu)$ is said to be Bipartite if the node set V can be Partitioned into two non empty sets V_1 and V_2 such that $\mu(v_1, v_2) = 0$ if $v_1, v_2 \in V_1$ or $v_1, v_2 \in V_2$. Further if $\mu(v_1, v_2) > 0$ for all $v_1 \in V_1$ and $v_2 \in V_2$ then G is called complete bipartite graph and it is denoted by K_{σ_1, σ_2} where σ_1 and σ_2 are respectively the restriction of σ to V_1 and V_2.

Definition 2.8. The complement of a fuzzy graph $G(\sigma, \mu)$ is a subgraph $\overline{G} = (\overline{\sigma}, \overline{\mu})$ where $\overline{\sigma} = \sigma$ and $\overline{\mu}(u, v) = \sigma(u) \wedge \sigma(v) - \mu(u, v)$ for all u, v in V. A fuzzy graph is self complementary if $G = \overline{G}$.

Definition 2.9. The order p and size q of a fuzzy graph $G(\sigma, \mu)$ is defined as $p = \sum_{u \in V} \sigma(u)$ and $q = \sum_{(u, v) \in E} \mu(u, v)$.

Definition 2.10. The degree of the vertex u is defined as the sum of weight of arc incident at u, and is denoted by $d(u)$.

Definition 2.11. A Path ρ of a fuzzy graph $G(\sigma, \mu)$ is a sequence of distinct nodes $v_1, v_2, v_3, \ldots, v_n$ such that $\mu(v_{i-1}, v_i) > 0$ where $1 \leq i \leq n$. A path is called a cycle if $u_0 = u_n$ and $n \geq 3$.

Definition 2.12. Let u, v be two nodes in $G(\sigma, \mu)$. If they are connected by means of a path ρ then strength of that path is $\Lambda_{i=i}^{n} \mu(u_{i-1}, v_1)$.

Definition 2.13. Two nodes that are joined by a path are said to be connected. The relation connected is reflexive, symmetric and transitive. If u and v are connected by means of length k, then $\mu^k(u, v) = \sup \{\mu(u, v_1) \wedge \mu(v_1, v_2) \ldots \wedge \mu(v_{k-1}, v_k) \mid u, v_1, v_2, \ldots v$ in such path $\rho\}$.

Definition 2.14. A Strongest path joining any two nodes u, v is a path corresponding to maximum strength between u and v. The strength of the strongest path is denoted by $\mu^{\infty}(u, v)$.

$$\mu^{\infty}(u, v) = \sup \{\mu^k(u, \rho) \mid k = 1, 2, 3 \ldots\}.$$

Example 1.

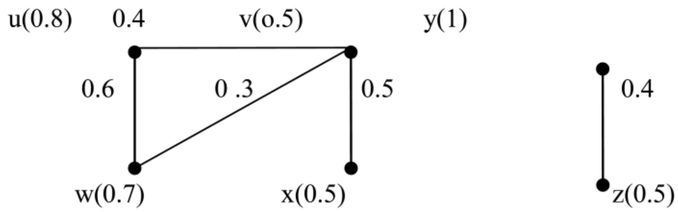


Figure (i)

In this fuzzy graph, figure (i) (a), $u = w, v, x$ is a $w - x$ path of length 2 and strength is 0.3.

Another path of $w - x$ is w, u, v, x of length 3 and strength is 0.4.

But strength of the strongest path joining w and x is $\mu^{\infty}(w, x) = \sup\{0.3, 0.4\} = 0.4$.

Definition 2.14. Let $G(\sigma, \mu)$ be fuzzy graph. Let x, y be two distinct nodes and G' be the fuzzy subgraph obtained by deleting the arc (x, y) that is $G'(\sigma, \mu')$ where $\mu'(x, y) = 0$ and $\mu' = \mu$ for all other pairs. Then (x, y) is said to be fuzzy bridge in G if $\mu'^{\infty}(\sigma, \mu) < \mu^{\infty}(u, v)$ for some u, v in V.

Definition 2.15. A node is a fuzzy cut node of $G(\sigma, \mu)$ if removal of it reduces the strength of the connectedness between some other pair of nodes. That is, w is a fuzzy cut node of $G(\sigma, \mu)$ iff there exist u, v such that w is on every strongest path from u to v.

Definition 2.16. An arc (u, v) of the fuzzy graph $G(\sigma, \mu)$ is called an effective edge if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ and effective edge neighborhood of $u \in V$ is $N_e(u) = \{v \in V : edge(u, v)$ is effective$\}$. $N_e[u] = N_e(u) \cup \{u\}$ is the closed neighborhood of u.

The minimum cardinality of effective neighborhood $\delta_e(G) = \min\{| N_e(u) | u \in V(G)\}$.

Maximum cardinality of effective neighborhood $\Delta_e(G) = \max\{| N_e(u) | u \in V(G)\}$.

3. Main Results

Strong support covering for the fuzzy graphs by strong arc. Here the Strong support vertex cover and anti-strong support vertex cover are introduced.

Definition 3.1 (Strong degree of a vertex)**.** The strong degree of a vertex u of a fuzzy graph $G(\sigma, \mu)$ is defined as the sum of strong neighborhood of u, and is denoted by Strong $\text{Deg}(u)$.

Definition 3.2 (Full Strong degree of a vertex)**.** Let u be a vertex in $G(\sigma, \mu)$. It is said to be full strong degree vertex if all other vertex of $G(\sigma, \mu)$ are strong neighborhood of u.

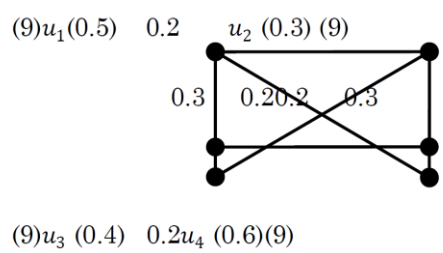
In other words,

Let $G(\sigma, \mu)$ be a simple fuzzy graph with n vertices.

Let u be a vertex of fuzzy graph $G(\sigma, \mu)$. Vertex u in $G(\sigma, \mu)$, is said to be full strong degree vertex if number of strong neighborhood of u is $n-1$.

Remark 1. Full strong degree vertex must be in one of the minimal strong support dominating set, but it is need not be in minimum strong support dominating set.

Counterexample 1.



In this fuzzy complete graph FK_4, all the arcs are strong.

Hence all vertices have full strong degree.

$D_1 = \{u_1\}$, $D_2 = \{u_2\}$, $D_3 = \{u_3\}$, $D_4 = \{u_4\}$ are all minimal strong support dominating set, but $D_2 = \{u_3\}$ is the only minimum strong support dominating set.

For strong support domination number,

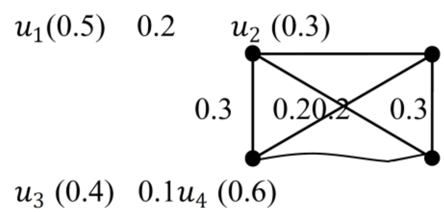
$$\gamma_{s\,\text{supp}}(G) = \text{Min}\,\{\Sigma\{\text{strong support of } u + \sigma(u)\},\ \text{where } u \in D\}$$

$$= \text{Min}\,\{9.5,\ 9.3,\ 9.4,\ 9.6\}$$

$$= 9.3.$$

The corresponding minimum strong support of dominating set is $D_2 = \{u_2\}$.

Counterexample 2.



In this fuzzy complete graph FK_4, arc (u_3, u_4) is a non- strong arc.

Hence, vertices u_1 and u_2 only have full strong degree and others are not. There are two minimal strong support dominating set for this fuzzy graph are followed as $D_1 = \{u_1\}.$, $D_2 = \{u_2\}$. In which $D_2 = \{u_2\}$ is only the minimum strong support dominating set.

Hence, full strong degree vertex must be in one of the minimal strong support dominating set, but it is need not be in minimum strong support dominating set.

Definition 3.3. The (Open) Strong support neighborhood of $u \in V$ is $N_{s\,\text{supp}}(u) = \{v \in V : arc\,(u, v)$ is strong supp $(u) \geq$ strong supp $(v)\}$. $N_{s\,\text{supp}}[u] = N_{s\,\text{supp}}(u) \cup \{u\}$ is the (Closed) strong support neighborhood of u.

Definition 3.4. A vertex u is strong support if strong supp $(u) \geq$ strong supp (v) where $v \in N_s(u)$. A subset S of V is called strong support set if for every vertex u in S is strong support.

Definition 3.5. A vertex u is anti-strong support if strong supp $(u) \geq$ strong supp (v), where $v \in N_s(u)$. A subset S of V is called anti-strong support set if for every vertex u in S is anti-strong support.

Definition 3.6 (Strong support regular)**.** A vertex v in $G(\sigma, \mu)$ is strong support regular if strong supp $(u) \geq$ strong supp (v) for all $v \in N_s(u)$. A subset S of $V(G)$ is called strong support regular if for every vertex u in S are strong support regular. A fuzzy graph $G(\sigma, \mu)$ is said to be strong support regular if strong supp (u) is constant for all u in $V(G)$.

Definition 3.7 (Vertex cover of a fuzzy graph)**.** A vertex and an arc (edge) are said to be cover each other if they are incident. A subset S of $V(G)$ is called vertex cover of a fuzzy graph $G(\sigma, \mu)$ if every arc $(e = uv)$ in $E(G)$ incident with at least one vertex of S.

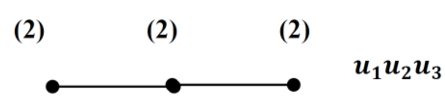
The minimum cardinality of vertex cover of a fuzzy graph G is denoted by $\alpha_0(G)$.

Definition 3.8 (Strong support vertex cover)**.** A subset S of $V(G)$ is strong support vertex cover of a fuzzy graph G if every arc u, v in $E(G)$ incident with a vertex of S such that strong supp $(u) \geq$ strong supp (v), where $v \in N_s(u) \cap (V - S)$.

The minimum cardinality of strong support vertex cover of a fuzzy graph G (Strong support vertex covering number) is denoted by $\alpha_{0-\text{supp}}(G)$.

Definition 3.9 (Anti-Strong support vertex cover). A subset S of $V(G)$ is a anti-strong support vertex cover of a fuzzy graph G if every arc (u, v) in $E(G)$ incident with a vertex of S such that strong supp $(u) \geq$ strong supp (v), where $v \in N_s(u) \cap (V - S)$. The minimum cardinality of anti-strong support vertex cover of a fuzzy graph G is denoted by $\alpha_{0-a-\text{supp}}$.

Example 1.



The upper strong support vertex cover is $S = \{u_2\}$ and lower strong support vertex cover is $\{u_2\}$

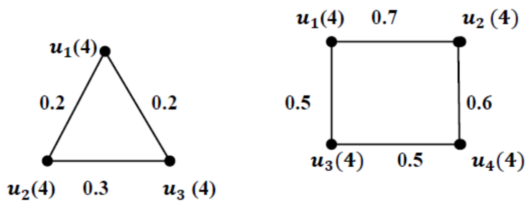
$$\alpha_{0-a-\text{supp}}(G) = \alpha_{0-a-\text{supp}} = \alpha_0 = 1.$$

Example 2

The strong support vertex cover is $S = \{u_2, u_3\}$ and anti-strong support vertex cover is $\{u_1, u_2, u_3\}$

$$\alpha_{0-a-\text{supp}}(G) = 2 \text{ and } \alpha_{0-a-\text{supp}} = 3.$$

Example 3.



Figure(i) Figure(ii)

In figure (i) and figure (ii), all the arcs are strong.

Since strong support of each vertices are same (constant), this fuzzy graph is called strong support regular fuzzy graph.

From figure (i),

The strong (anti-strong) support vertex covers are $S = \{u_1, u_3\}$ and $S = \{u_1, u_3\}$

$$\alpha_{0-\text{supp}}(G) = \alpha_{0-a-\text{supp}} = 2.$$

From figure (ii), the strong (anti-strong) strong support vertex covers are $S = \{u_1, u_4\}$ and $S = \{u_1, u_4\}$

$$\alpha_{0-\text{supp}}(G) = \alpha_{0-a-\text{supp}} = 2.$$

Example 4.

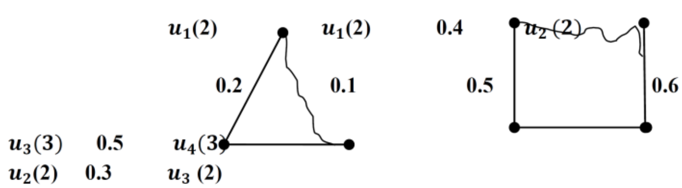


Figure (i) Figure (ii)

Now, arc (u_1, u_2) is a non strong arc in figure (i) and figure (ii) while other arcs are strong arc.

Since (u_1, u_2) is a non strong arc, the strong support of vertex can be varied and strong support are pointed on each vertex.

From figure (ii), the strong support vertex cover is $S = \{u_1, u_2, u_4\}$ and Anti-strong support vertex cover is $\{u_1, u_2, u_4\}$.

$$\alpha_{0-\text{supp}}(G) = \alpha_{0-a-\text{supp}} = 3.$$

Remark 2. From the above two examples, it is observed that strong support of vertices are varied in a right place with respect to the non strong existence in a fuzzy graph and cardinality of $\alpha_{0-\text{supp}}(G)$ and $\alpha_{0-a-\text{supp}}$ are also increased.

Theorem 3.10. *Let $G(\sigma, \mu)$ be a fuzzy graph. Let u in $V(G)$. If fuzzy graph $\langle N_s[u] \rangle$ is complete with n vertices, then for every $v \in N_s[u]$ has full strong degree of $n - 1$.*

Proof. Let fuzzy graph $\langle N_s[u] \rangle$ be a complete with *n* vertices, Since all the arcs are strong in $\langle N_s[u] \rangle$ and complete with *n* vertices, each vertex has same strong degree $n - 1$ (By def. of Strong degree). Hence, $n - 1$ is the full strong degree of $v \in N_s[u]$ (By Def. of full strong degree).

Theorem 3.11. *Let $G(\sigma, \mu)$ be a simple fuzzy graph with n vertex and let $u \in G(\sigma, \mu)$ be a full strong degree vertex. Then, strong $\sup p\ (u) \geq$ strong $\sup p\ (v)$, for every $v \in V(G)$.*

Proof. Let $u \in G(\sigma, \mu)$ be full strong degree vertices, then it must be adjacent to all vertices of $V(G)$ by strong arc (By def. of full strong degree). Hence strong degree of u is $n - 1$ (By theorem 1). We claim that $\text{Strong supp}\,(u) \geq \text{Strong supp}\,(v)$, for every $v \in V(G)$.

Suppose, *u* has full strong degree such that

$$\text{Strong supp}\,(u) \geq \text{Strong supp}\,(v), \text{ for some } v \in V(G).$$

Then *v* has more strong arc neighborhood than *u* and strong degree of *v* is more than $n - 1$.

Since $G(\sigma, \mu)$ be a simple fuzzy graph with n vertex and u is the full strong degree vertices, it is not possible to have strong degree of v is more than $n - 1$.

Contradiction (since, u has full strong degree of $n - 1$).

Hence strong supp $(u) \geq$ strong supp (v), for every $v \in V(G)$.

Theorem 3.12. *Let* $G(\sigma, \mu)$ *be a simple fuzzy graph, Then there exist a vertex in G have a full strong degree iff* $n[\gamma_{s \text{ supp}}(G)] = 1$.

Proof. Let $u \in G(\sigma, \mu)$ be full strong degree vertex. Then it must be adjacent to all vertices of $V(G)$. by strong arc.

By theorem 2, clearly it satisfy that condition that strong strong supp $(u) \geq$ strong supp (v), where $v \in V(G)$.

Hence this u is enough to form a minimum strong support dominating set.

Therefore, $n[\gamma_{s \text{ supp}}(G)] = 1$.

Converse part can be proved in similar way by reversing the statement.

Counterexample 3

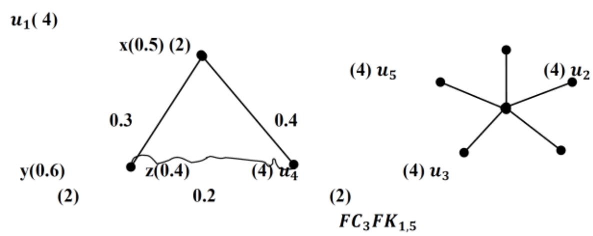


Figure (i) Figure (ii)

Here, Arcs $(x, y), (x, z)$ are strong arc and (y, z) is non-strong arc.

Hence strong degree of vertices y and z are 1(one), strong degree of vertex x is 2. Therefore strong support of vertices x, y and z are 2(two).

Minimal strong support dominating set $D = \{x\}$.

$n[\gamma_{s \text{ supp}}(G)] = 1$ and $\gamma_{s \text{ supp}}(G) = 2 + 0.5 = 2.5$.

Remark 3. In $FK_{1,5}$ and FC_3 all the vertices have same strong support and $n[\gamma_{s\,\text{supp}}(G)] = 1$.

Note that only one vertex in $FK_{1,5}$ is a full strong degree while others are not.

Therefore, though vertices have same strong support which cannot be full strong degree,

Theorem 3.13. *Fuzzy graph G is strong support regular iff* $\alpha_{0-s\,\text{supp}}(G) = \alpha_{0-a-s\,\text{supp}} = \alpha_0$.

Proof.

Let $G(\sigma, \mu)$ be a strong support regular with n vertices.

Claim: $\alpha_{0-s\,\text{supp}}(G) - \alpha_{0-a-s\,\text{supp}} = \alpha_0$.

Suppose $\alpha_{0-s\,\text{supp}}(G) = \alpha_{0-a-s\,\text{supp}} \neq \alpha_0$.

Then two cases can arise (i) $\alpha_{0-s\,\text{supp}}(G) = \alpha_{0-a-s\,\text{supp}} < \alpha_0$. (or)

(ii) $\alpha_{0-s\,\text{supp}}(G) = \alpha_{0-a-s\,\text{supp}} > \alpha_0$.

If Case (i) true then $\alpha_{0-s\,\text{supp}}(G) = \alpha_{0-a-s\,\text{supp}} < \alpha_0$.

That is cardinality of vertex cover is greater than strong support vertex cover.

There no restriction are given for vertex cover while compare to strong support vertex cover. Surely case (i) is not possible to exit (Refer example 3).

If case (ii) true then $\alpha_{0-s\,\text{supp}}(G) = \alpha_{0-a-s\,\text{supp}}(G) > a_0$. Since given fuzzy graph strong support regular, strong supp (u) is constant for all u in $V(G)$. Hence cardinality of minimal vertex cover can exactly equal to the strong support vertex cover since the entire vertex have same strong support.

Clearly case (ii) is also not possible.

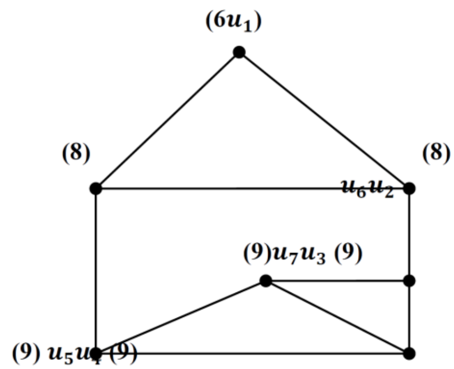
Therefore, fuzzy graph is strong support regular then $\alpha_{0-s\,\text{supp}}(G) = \alpha_{0-a-s\,\text{supp}}(G) = \alpha_0$.

Converse part is explained by counterexample.

Counterexample 4

Assume that all the arcs of G are strong arc.

Now, the fuzzy graph G drawn and calculated strong support for all vertex of G.



Here $\{u_2, u_4, u_5, u_6\}$ is a minimum vertex cover and $\alpha_0(G) = 4$

$\{u_2, u_3, u_4, u_5, u_6\}$ minimum strong support vertex cover of G and Hence, $\alpha_{0-s\,\text{supp}}(G) = 5$.

$\{u_1, u_2, u_4, u_6, u_7\}$ is a minimum Anti-strong support vertex cover of G and $\alpha_{0-a-s\,\text{supp}}(G) = 5$.

Therefore $\alpha_{0-s\sup p}(G) = \alpha_{0-a-s\,\text{supp}}(G) \neq \alpha_0(G)$.

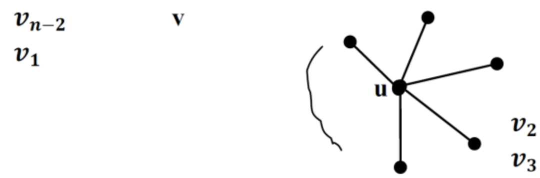
This fuzzy graph is not strong support regular.

Hence $\alpha_{0-s\sup p}(G) = \alpha_{0-a-s\,\text{supp}}(G) = \alpha_0$ implies fuzzy graph is strong support regular.

Theorem 3.15. *The sum of strong degree of a fuzzy graph is twice the number of strong arc.*

Proof. Let G be a simple fuzzy graph and let $u, v_1, v_2, v_3, \ldots, v_{n-2}$ be the n vertices of G.

Let $u \in G$. Suppose u only adjacent with $v, v_1, v_2, v_3, \ldots, v_{n-2}$ by strong arc and $\{v, v_1, v_2, v_3, \ldots, v_{n-2}\}$ set of vertices have no adjacency as in the below figure.



Since each strong arc (u, v) incident with two distinct ends u and v.

That is, one strong arc contribute one strong degree to each.

Assume that

Suppose u adjacent with $v, v_1, v_2, v_3, \ldots, v_{n-2}$ by strong arc.

Then, strong degree of u is $n-1$ and others have one each.

Hence, Sum of strong degree these n vertices $\{u, v, v_1, v_2, v_3, \ldots, v_{n-2}\}$

$$= n - 1 + \{1 + \ldots n - 2 \text{ times}\}$$

$$= n + n - 2$$

$$= 2n - 2$$

$$2(n-1)$$

$$= 2 \text{ times of number of strong arcs in } G.$$

Therefore sum of strong degree of a fuzzy G is equal to twice the number of strong arcs of G.

Theorem 3.16. *Let G be a simple fuzzy graph with n vertices. Then, sum of strong support of entire vertex of G is even sum. That is $\sum_{u \in V(G)} \text{strong supp}(u) = \text{eveun sum}$. In particular, if the fuzzy graph G is strong support regular then $\sum_{u \in V(G)} \text{strong supp}(u) = n, p$, where p is the strong support of a vertex.*

Proof. Take same consideration as in the above theorem.

By definition of Strong support of vertices,

$$\text{Strong Supp}(u) = \sum_{v \in N_s(u)} \text{strong degree of } v$$

$$= 1 + 1 + \ldots (n - 1)\,\text{times}$$

$$= n + 1.$$

Similarly, Strong Supp (v) = Strong Supp $(v_1) \ldots$ Strong Supp $(v_{n-2}) = n - 1$.

Adding all strong support value of n vertices $= n(n-1) =$ even sum.

Moreover, the entire vertices of fuzzy graph have same strong support value (say, $n - 1$).

Therefore this fuzzy graph is as called strong support regular.

Take $p = n - 1$, where p is the strong support of a vertex

$\sum_{u \in V(G)}$ strong supp $(u) = np$, where p is the strong support of a vertex.

Evidently, this is true for any arbitrary fuzzy graph.

Verification From the examples

1. From example 1, since FP_3 is strong support regular and $p = 2$

$$\sum_{u \in V(G)} \text{strong supp}\,(u) = np = 3(2) = 6\,(\text{even}).$$

2. From example 2, it not strong support regular

$$\text{But} \sum_{u \in V(G)} \text{strong supp}\,(u) = 10\,(\text{even}).$$

3. From example 3, since figure (i) and (ii) are strong support regular and $p = 4$

$$\sum_{u \in V(G)} \text{strong supp}\,(u) = np = 3(4) = 12\,(\text{even}) \text{ and}$$

$$\sum_{u \in V(G)} \text{strong supp}\,(u) = np = 4(4) = 16\,(\text{even}).$$

4. From example 4, it not strong support regular

for figure (i) $\sum_{u \in V(G)}$ strong supp $(u) = 6\,(\mathit{even})$

for figure (ii) $\sum_{u \in V(G)}$ strong supp $(u) = 10\,(\text{even})$

Corollary 3.17. *If fuzzy graph is strong support regular with n vertices, where $n \geq 2$, then strong support of entire vertex is of even.*

Proof. Obviously proved by the above theorem.

Result on Fuzzy Paths FP_n and Fuzzy cycle

Result 1. Let FP_n be a fuzzy path with n vertices

(i) $\alpha_0(FP_n) = \left\lfloor \frac{n}{2} \right\rfloor$

$\alpha_0(G)$ is vertex covering of fuzzy graph,

$$\text{(ii) } \alpha_{0-s\sup p}(FP_n) = \begin{cases} 1 & \text{if } n = 1, 2 \\ n-2 & \text{if } n = 3, 4, 5 \\ \frac{n+2}{2} & \text{if } n = 6, 8, 10, 12\ldots \\ \frac{n+3}{2} & \text{if } n = 7, 9, 11, 13\ldots \end{cases}$$

$\alpha_{0-s\,\text{supp}}(G)(G)$ is a strong support vertex covering of fuzzy graph.

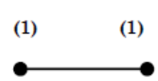
$$\text{(iii) } \alpha_{0-s\sup p}(FP_n) = \begin{cases} 1 & \text{if } n = 1, 2, 3 \\ n-1 & \text{if } n = 4 \\ \frac{n+3}{2} & \text{if } n = 5, 7, 9, 11\ldots \\ \frac{n+3}{2} & \text{if } n = 6, 8, 10, 12\ldots \end{cases}$$

$\alpha_{0-a-s\,\text{supp}}(G)$ is a anti-strong support vertex covering of fuzzy graph.

Proof. Let be a fuzzy path, where $n = 2, 3, 4, \ldots$ are drawn below.

(Since entire arcs of fuzzy path are strong arc, vertex cover, strong support vertex cover and anti-strong support vertex are easily found).

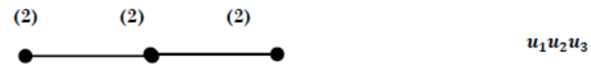
Fuzzy path FP_2 is drawn below.



u_1u_2

$\alpha_{0-s\,\text{supp}} = \alpha_{0-a-s\,\text{supp}} = \alpha_0 = 1.$

Fuzzy path FP_3 is drawn below



$\alpha_{0-s\,\text{supp}} = \alpha_{0-a-s\,\text{supp}}(G) = \alpha_0 = 1.$

Fuzzy path FP_4 is drawn below

$\alpha_0 = 2$, $\alpha_{0-s\,\text{supp}} = 2$ and $\alpha_{0-a-s\,\text{supp}}(G) = 3$.

Similarly for various $n = 5, 6, 7 \ldots$ the values are listed in the table.

N	$\alpha_0(G)$	$\alpha_{0-s\,\text{supp}}$	$\alpha_{0-a-s\,\text{supp}}(G)$
2	1	1	1
3	1	1	1
4	2	2	3
5	2	3	4
6	3	4	5
7	3	5	5
8	4	5	6
9	4	6	6
10	5	6	7
11	5	7	7
12	6	7	8
13	6	8	8
And so on...			

Hence general form for

(i) $\alpha_0(FP_n) = \left\lfloor \frac{n}{2} \right\rfloor$ for all n

(ii) $$\alpha_{0-s\sup p}(FP_n) = \begin{cases} 1 & \text{if } n = 1, 2 \\ n-2 & \text{if } n = 3, 4, 5 \\ \frac{n+2}{2} & \text{if } n = 6, 8, 10, 12\ldots \\ \frac{n+3}{2} & \text{if } n = 7, 9, 11, 13\ldots \end{cases}$$

(iii) $$\alpha_{0-a-s\sup p}(FP_n) = \begin{cases} 1 & \text{if } n = 1, 2, 3 \\ n-1 & \text{if } n = 4 \\ \frac{n+3}{2} & \text{if } n = 5, 7, 9, 11\ldots \\ \frac{n+4}{2} & \text{if } n = 6, 8, 10, 12\ldots \end{cases}.$$

Result 2. Let FC_n be a fuzzy cycle with n vertices

Two type of results are exist in the fuzzy cycle

Type I

If no strong arc in the fuzzy cycle then

$$\alpha_0(FC_n) = \alpha_{0-s\,\mathrm{supp}}(FC_n) = \alpha_{0-a-s\,\mathrm{supp}}(FC_n) = \left\lceil \frac{n}{2} \right\rceil \text{ for all } n \geq 3.$$

Type II

If non-strong arc in the fuzzy cycle then

$$\alpha_0(FC_n) = \left\lceil \frac{n}{2} \right\rceil \text{ or all } n \geq 3$$

$$\alpha_{0-s\sup p}(FC_n) = \alpha_{0-a-s\sup p}(FC_n) = \begin{cases} n-1 & \text{if } n = 3, 4 \\ n - \left\lceil \frac{n}{2} \right\rceil - 2 & \text{if } n \geq 5 \end{cases}$$

Where $\alpha_0(FC_n)$ is vertex covering of a fuzzy cycle with n vertices $\alpha_{0-s\sup p}(FC_n)$ is strong support vertex covering of a fuzzy cycle $\alpha_{0-a-s\sup p}(FC_n)$ is anti-strong support vertex covering of fuzzy cycle.

Proof for Type I.

Let us assume that no non-strong arc in a fuzzy cycle FC_n.

By example 3, $\alpha_0(FC_3) = \alpha_{0-s\,\text{supp}}(FC_3) = \alpha_{0-a-s\,\text{supp}}(FC_3) = \left\lceil \frac{3}{2} \right\rceil = 1.$

And $\alpha_0(FC_4) = \alpha_{0-s\,\text{supp}}(FC_4) = \alpha_{0-a-s\,\text{supp}}(FC_4) = \left\lceil \frac{4}{2} \right\rceil = 2.$

Similarly for various $n = 5, 6, 7, \ldots$ are found that are listed as in the table below.

N	$\alpha_0(G)$	$\alpha_{0-s\,\text{supp}}$	$\alpha_{0-a-s\,\text{supp}}(G)$
3	2	2	2
4	2	2	2
5	3	3	3
6	3	3	3
7	4	4	4
8	4	4	4
9	5	5	5
10	5	5	5
11	6	6	6
12	6	6	6
13	7	7	7
And so on...			

Hence general form for $\alpha_0(FC_n) = \alpha_{0-s\,\text{supp}}(FC_n) = \alpha_{0-a-s\,\text{supp}}(FC_n) = \left\lceil \frac{n}{2} \right\rceil$ for all $n \geq 3$.

Proof for Type II.

Maximum number non-strong in a fuzzy cycle is one. (Refer [14]). Since non-strong arc appear in the fuzzy cycle, there is small variations in vertex covering and strong support vertex covering of a fuzzy cycle FC_n compared with Type I.

Verified by example 4 of figure (i) and figure (ii) for $n = 3$ and $n = 4$.

Similarly for $n = 5, 6, 7 \ldots$ are found that are listed as in the below table.

N	$\alpha_0(G)$	$\alpha_{0-s\,\text{supp}}$	$\alpha_{0-a-s\,\text{supp}}(G)$
3	2	2	2
4	2	3	3
5	3	4	4
6	3	5	5
7	4	5	5
8	4	6	6
9	5	6	6
10	5	7	7
11	6	7	7
12	6	8	8
13	7	8	8
14	7	9	9
And so on…			

Hence general form for $\alpha_0(FC_n) = \left\lceil \frac{n}{2} \right\rceil$ for all $n \geq 3$

$$\alpha_{0-s\,\text{supp}}(FC_n) = \alpha_{0-a-s\,\text{supp}}(FC_n) = \begin{cases} n-1 & \text{if } n = 3, 4 \\ n - \left\lceil \frac{n}{2} \right\rceil - 2 & \text{if } n \geq 5 \end{cases}$$

Conclusion

In this paper, definition of strong degree and strong full degree, strong support and strong support regular of a vertex of fuzzy graph are newly introduced by using strong arc. Further, definition of strong support covering and anti-strong support covering of the fuzzy graphs are newly introduced by using strong arc.

Moreover, some standard theorems on strong full degree and strong support regular for the fuzzy graph are proved with example. Finally, results on strong support covering number and anti-strong support covering number for the standard fuzzy graph like fuzzy Paths FP_n, fuzzy cycle FC_n are found. In future, anti-strong support domination by using strong arc, strong arc (edge) covering of the fuzzy graph and strong independent will be discussed.

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