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PROPERTIES OF INTUITIONISTIC MULTI-ANTI FUZZY RING

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Abstract

In this chapter, the concept of Cartesian product of intuitionistic multi-anti fuzzy ring of a ring is defined and some of the properties. Cartesian product, homomorphism of an intuitionistic multi-fuzzy ring and intuitionistic multi-anti fuzzy ring of a ring are also discussed.

1. Introduction

Fuzzy set theory introduced by Zadeh [9], 1965 has showed meaningful applications in many fields of study. The idea of fuzzy set is welcome because it handles uncertainty and vagueness which ordinary set could not address. In fuzzy set theory membership function of an element is single value between 0 and 1. Therefore, a generalization of fuzzy set was introduced by Attanassov [1], 1983 called intuitionistic fuzzy set (IFS) which deals with the degree of non-membership function and the degree of hesistation. After several year, Sabu Sebastian introduced the theory of multi-fuzzy sets in terms of multi-dimensional membership function. R. Muthuraj and S. Balamurugan [8] introduced the concept of multi-anti fuzzy subgroup and

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discussed some of its properties.

In this paper, the concept of Cartesian product of an intuitionistic multi-anti fuzzy set is defined and some of the properties of Cartesian product of an intuitionistic multi-anti fuzzy ring of ring and the properties of image of an intuitionistic multi-anti fuzzy ring of a ring under homomorphism and anti homomorphism.

Basic concepts 1.1. The theory of an intuitionistic multi-anti fuzzy set is an extension of theories of multi-fuzzy sets. The membership function of a multi-fuzzy set is an ordered sequence of membership functions of a fuzzy set. The notion of an intuitionistic multi-fuzzy sets provides a new method to represent some problems which are difficult to explain in other extensions of fuzzy set theory.

Definition 1.2. Let R be a ring. Let $G = \{\langle x, A(x), B(x)\rangle / x \in R\}$ be an intuitionistic multi-fuzzy set defined on a ring R. Then G is said to be an intuitionistic multi-anti fuzzy ring on R if the following conditions are satisfied. For all $x, y \in R$,

(i) $A(x-y) \leq \max\{A(x), A(y)\}$,

(ii) $A(xy) \leq \max\{A(x), A(y)\}$,

(iii) $B(x-y) \geq \min\{B(x), B(y)\}$,

(iv) $B(xy) \geq \min\{B(x), B(y)\}$.

Remarks 1.3. For an intuitionistic multi-anti fuzzy subring $G = \{\langle x, A(x), B(x)\rangle / x \in R\}$ of a ring R, the following result is obvious. For all $x, y \in R$,

(i) $A(x) \geq A(0)$ and $A(x) = A(-x)$,

(ii) $A(x-y) = 0$ implies that $A(x) = A(y)$.

(iii) $B(x) \leq B(0)$ and $B(x) = B(-x)$,

(iv) $B(x-y) = 0$ implies that $B(x) = B(y)$.

Example 1.4. Let $(Z, +, \cdot)$ be a ring. Let $G = \{\langle x, A(x), B(x)\rangle / x \in R\}$ be an intuitionistic multi-fuzzy set defined on a ring Z, where

$$A(x) = (A_1(x),\ A_2(x)) = \begin{cases} (0.2,\ 0.3) \text{ if } x = 0 \\ (0.2,\ 0.1) \text{ if } x \neq 0 \end{cases} \text{ and}$$

$$B(x) = (B_1(x),\ B_2(x)) = \begin{cases} (0.8,\ 0.6) \text{ if } x = 0 \\ (0.7,\ 0.9) \text{ if } x \neq 0 \end{cases}$$

Clearly, G is an intuitionistic multi-anti fuzzy ring of dimension 2.

2. Cartesian Product of Intuitionistic Multi-Anti Fuzzy Sets

In this section, the concept of Cartesian product of two intuitionistic multi-anti fuzzy sets defined and discussed some properties of Cartesian product.

Definition 2.1. Let $G = \{\langle x,\ A(x),\ B(x)\rangle / x \in R_1\}$ and $H = \{\langle y,\ C(y),\ D(y)\rangle / y \in R_2\}$ be any two intuitionistic multi-fuzzy sets defined on a ring R_1 and R_2 respectively. Then the anti Cartesian product of G and H is defined as

$$G \times H = \{\langle (x,\ y),\ (A \cup C)(x,\ y),\ (B \cap D)(x,\ y)\rangle / (x,\ y) \in R_1 \times R_2\},$$

where, $(A \cup C)(x,\ y) = \max\{A(x),\ C(y)\}$, for all $(x,\ y) \in R_1 \times R_2$,

$$(B \cap D)(x,\ y) = \min\{B(x),\ D(y)\}, \text{ for all } (x,\ y) \in R_1 \times R_2.$$

Theorem 2.2. *Let* $G = \{\langle x,\ A(x),\ B(x)\rangle / x \in R_1\}$ *and* $H = \{\langle y,\ C(y),\ D(y)\rangle / y \in R_2\}$ *be any two intuitionistic multi-anti fuzzy rings defined on a ring* R_1 *and* R_2 *respectively, then,* $G \times H$ *is also an intuitionistic multi-anti fuzzy subring of a ring* $R_1 \times R_2$.

Proof. Let $G = \{\langle x,\ A(x),\ B(x)\rangle / x \in R_1\}$ and $H = \{\langle y,\ C(y),\ D(y)\rangle / y \in R_2\}$ be any two intuitionistic multi-anti fuzzy rings defined on a ring R_1 and R_2 respectively. Then,

$$G \times H = \{\langle (x,\ y),\ (A \cup C)(x,\ y),\ (B \cap D)(x,\ y)\rangle / (x,\ y) \in R_1 \times R_2\},$$

where, $(A \cup C)(x,\ y) = \max\{A(x),\ C(y)\}$, for all $(x,\ y) \in R_1 \times R_2$,

$$(B \cap D)(x, y) = \min\{B(x), D(y)\}, \text{ for all } (x, y) \in R_1 \times R_2.$$

Let $x, y \in R_1 \times R_2$, where $x = (p, q)$, $y = (r, s)$,

(i) $(A \cup C)(x, y) = \max\{A(p - r), C(q - s)\},$

$$\leq \max\{\max\{A(p), A(r)\}, \max\{C(q), C(s)\}\}$$

$$= \max\{\max\{A(p), C(q)\}, \max\{A(r), C(s)\}\}$$

$$= \max\{(A \cup C)(p, q), (A \cup C)(r, s)\}$$

$(A \cup C)(x - y) \leq \max\{(A \cup C)(x), (A \cup C)(y)\}.$

(ii) $(A \cup C)(xy) = (A \cup C)((p, q) \cdot (r, s))$

$$= (A \cup C)(pr, qs)$$

$$= \max\{A(pr), C(qs)\}$$

$$\leq \max\{\max\{A(p), A(r)\}, \max\{C(q), C(s)\}\}$$

$$= \max\{\max\{A(p), C(q)\}, \max\{A(r), C(s)\}\}$$

$$= \max\{(A \cup C)(p, q), (A \cup C)(r, s)\}$$

$(A \cup C)(xy) \leq \max\{(A \cup C)(x), (A \cup C)(y)\}.$

(iii) $(B \cap D)(x, y) = \min\{B(p - r), D(q - s)\},$

$$\geq \min\{\min\{B(p), B(r)\}, \min\{D(q), D(s)\}\}$$

$$= \min\{\min\{B(p), D(q)\}, in\{B(r), D(s)\}\}$$

$$= \min\{(B \cap D)(p, q), (B \cap D)(r, s)\}$$

$(B \cap D)(x - y) \geq \min\{(B \cap D)(x), (B \cap D)(y)\}.$

(iv) $(B \cap D)(xy) = (B \cap D)((p, q) \cdot (r, s))$

$$= (B \cap D)(pr, qs)$$

$$= \min\{B(pr), D(qs)\}$$

$$\geq \min\{\min\{B(p), B(r)\}, \min\{D(q), D(s)\}\}$$

$$= \min\{\min\{B(p), D(q)\}, \min\{B(r), D(s)\}\}$$

$$= \min\{(B \cap D)(p, q), (B \cap D)(r, s)\}$$

$$(B \cap D)(xy) \geq \min\{(B \cap D)(x), (B \cap D)(y)\}.$$

Hence, $G \times H$ is also an intuitionistic multi-anti fuzzy subring of a ring $R_1 \times R_2$.

Theorem 2.3. *Let* $G = \{\langle x, A(x), B(x)\rangle / x \in R_1\}$ *and* $H = \{\langle y, C(y), D(y)\rangle / y \in R_2\}$ *be any two intuitionistic multi-anti fuzzy sets defined on a ring* R_1 *and* R_2 *respectively. Suppose that* 0_1 *and* 0_2 *are identity elements of* R_1 *and* R_2 *respectively. If the anti Cartesian product* $G \times H$ *is an intuitionistic multi-anti fuzzy subring of* $R_1 \times R_2$ *then at least one of the following statements must hold*

(i) $C(0_2) \leq A(x)$ *and* $D(0_2) \geq B(x)$, *for all* $x \in R_1$

(ii) $A(0_1) \leq C(y)$ *and* $B(0_1) \geq D(y)$, *for all* $y \in R_2$.

Proof. Let $G = \{\langle x, A(x), B(x)\rangle / x \in R_1\}$ and $H = \{\langle y, C(y), D(y)\rangle / y \in R_2\}$ be any two intuitionistic multi-fuzzy sets defined on a ring R_1 and R_2 respectively.

Then, $G \times H = \{\langle (x, y), (A \cup C)(x, y), (B \cap D)(x, y)\rangle / (x, y) \in R_1 \times R_2\}$,

where, $(A \cup C)(x, y) = \max\{A(x), C(y)\}$, for all $(x, y) \in R_1 \times R_2$,

$(B \cap D)(x, y) = \min\{B(x), D(y)\}$, for all $(x, y) \in R_1 \times R_2$.

Let $x, y \in R_1 \times R_2$, where $x = (p, q)$, $y = (r, s)$,

Let the anti Cartesian product $G \times H$ be an intuitionistic multi-anti fuzzy subring of $R_1 \times R_2$. By contraposition, suppose that none of the statements (i) and (ii) holds then we can find $x \in R_1$ and $y \in R_2$ such that

(i) $C(0_2) \geq A(x)$ and $D(0_2) \leq B(x)$, for all $x \in R_1$

(ii) $A(0_1) \geq C(y)$ and $B(0_1) \leq D(y)$, for all $y \in R_2$.

We have, $(A \cup C)(x, y) = \max\{A(x), C(y)\}$

$$< \max\{C(0_2), A(0_1)\}$$

$$= \max\{A(0_1), C(0_2)\}$$

$$= (A \cup C)(0_1, 0_2)$$

$$(A \cup C)(x, y) < (A \cup C)(0_1, 0_2).$$

Also, $(B \cap D)(x, y) = \min\{B(x), D(y)\}$

$$> \min\{D(0_2), B(0_1)\}$$

$$= \min\{B(0_1), D(0_2)\}$$

$$= (B \cap D)(0_1, 0_2)$$

$$(B \cap D)(x, y) > (B \cap D)(0_1, 0_2).$$

Thus, $G \times H$ is not an intuitionistic multi-anti fuzzy subring of $R_1 \times R_2$. Hence, either $C(0_2) \leq A(x)$ and $D(0_2) \geq B(x)$, for all $x \in R_1$

or $A(0_1) \leq C(y)$ and $B(0_1) \geq D(y)$, for all $y \in R_2$.

Theorem 2.4. *Let* $G = \{\langle x, A(x), B(x)\rangle / x \in R_1\}$ *and* $H = \{\langle y, C(y), D(y)\rangle / y \in R_2\}$ *be any two intuitionistic multi-fuzzy sets defined on a ring* R_1 *and* R_2 *respectively, such that* $C(0_2) \leq A(x)$ *and* $D(0_2) \geq B(x)$, *for all* $x \in R_1$, 0_2 *being the identity element of* R_2. *If the anti Cartesian product* $G \times H$ *is an intuitionistic multi-anti fuzzy subring of* $R_1 \times R_2$, *then G is an intuitionistic multi-anti fuzzy subring of* R_1.

Proof. Let $G = \{\langle x, A(x), B(x)\rangle / x \in R_1\}$ and $H = \{\langle y, C(y), D(y)\rangle / y \in R_2\}$ be any two intuitionistic multi-fuzzy sets defined on a ring R_1 and R_2 respectively.

Then, $G \times H = \{\langle (x, y), (A \cup C)(x, y), (B \cap D)(x, y)\rangle / (x, y) \in R_1 \times R_2\}$,

where, $(A \cup C)(x, y) = \max\{A(x), C(y)\}$, for all $(x, y) \in R_1 \times R_2$,

$$(B \cap D)(x,\ y) = \min\{B(x),\ D(y)\},\ \text{for all}\ (x, y) \in R_1 \times R_2.$$

Let the anti Cartesian product $G \times H$ be an intuitionistic multi-anti fuzzy subring of $R_1 \times R_2$ and $x,\ y \in R_1$ then $(x,\ 0_2), (y,\ 0_2) \in R_1 \times R_2$.

Given, $C(0_2) \leq A(x)$ and $D(0_2) \geq B(x)$, for all $x \in R_1$, 0_2 being the identity element of R_2.

(i) $A(x - y) = \max\{A(x - y),\ C(0_2 - 0_2)\}$

$$= (A \cup C)(x - y,\ 0_2 - 0_2)$$

$$= (A \cup C)((x,\ 0_2) - (y,\ 0_2))$$

$$\leq \max\{(A \cup C)(x,\ 0_2),\ (A \cup C)(y,\ 0_2)\}$$

$$= \max\{\max\{A(x),\ C(0_2)\},\ \max\{A(y),\ C(0_2)\}\}$$

$$= \max\{A(x),\ A(y)\}$$

$$A(x - y) \leq \max\{A(x),\ A(y)\}$$

(ii) $A(xy) = \max\{A(xy),\ C(0_2 0_2)\}$

$$= (A \cup C)(xy,\ 0_2 0_2)$$

$$= (A \cup C)((x,\ 0_2) \cdot (y,\ 0_2))$$

$$\leq \max\{(A \cup C)(x,\ 0_2),\ (A \cup C)(y,\ 0_2)\}$$

$$= \max\{\max\{A(x),\ C(0_2)\},\ \max\{A(y),\ C(0_2)\}\}$$

$$= \max\{A(x),\ A(y)\}$$

$$A(xy) \leq \max\{A(x),\ A(y)\}$$

(iii) $B(x - y) = \min\{B(x - y),\ D(0_2 - 0_2)\}$

$$= (B \cap D)(x - y,\ 0_2 - 0_2)$$

$$= (B \cap D)((x,\ 0_2) - (y,\ 0_2))$$

$$\geq \min\{(B \cap D)(x,\ 0_2),\ (B \cap D)(y,\ 0_2)\}$$

$$= \min\{\min\{B(x), D(0_2)\}, \min\{B(y), D(0_2)\}\}$$

$$= \min\{B(x), B(y)\}$$

$$B(x - y) \geq \min\{B(x), B(y)\}$$

(iv) $B(xy) = \min\{B(xy), D(0_2 0_2)\}$

$$= (B \cap D)(xy, 0_2 0_2)$$

$$= (B \cap D)((x, 0_2)(y, 0_2))$$

$$\geq \min\{(B \cap D)(x, 0_2), (B \cap D)(y, 0_2)\}$$

$$= \min\{\min\{B(x), D(0_2)\}, \min\{B(y), D(0_2)\}\}$$

$$= \min\{B(x), B(y)\}$$

$$B(xy) \geq \min\{B(x), B(y)\}$$

Hence, G is an intuitionistic multi-anti fuzzy subring of R_1.

Theorem 2.5. *Let* $G = \{\langle x, A(x), B(x)\rangle / x \in R_1\}$ *and* $H = \{\langle y, C(y), D(y)\rangle / y \in R_2\}$ *be any two intuitionistic multi-fuzzy sets defined on a ring* R_1 *and* R_2 *respectively, such that* $A(0_1) \leq C(y)$ *and* $B(0_1) \geq D(y)$, *for all* $y \in R_2$, 0_1 *being the identity element of* R_1. *If anti Cartesian product* $G \times H$ *is an intuitionistic multi-anti fuzzy subring of* $R_1 \times R_2$, *then* H *is an intuitionistic multi-anti fuzzy subring of* R_2.

Proof. Let $G = \{\langle x, A(x), B(x)\rangle / x \in R_1\}$ and $H = \{\langle y, C(y), D(y)\rangle / y \in R_2\}$ be any two intuitionistic multi-fuzzy sets defined on a ring R_1 and R_2 respectively.

Then, $G \times H = \{\langle (x, y), (A \cup C)(x, y), (B \cap D)(x, y)\rangle / (x, y) \in R_1 \times R_2\}$,

where, $(A \cup C)(x, y) = \max\{A(x), C(y)\}$, for all $(x, y) \in R_1 \times R_2$,

$(B \cap D)(x, y) = \min\{B(x), D(y)\}$, for all $(x, y) \in R_1 \times R_2$.

Let $G \times H$ be an intuitionistic multi-anti fuzzy subring of $R_1 \times R_2$ and $x, y \in R_2$ then $(0_2, x), (0_2, y) \in R_1 \times R_2$.

Given, $A(0_1) \leq C(y)$ and $B(0_1) \geq D(y)$, for all $y \in R_2$, 0_1 being the identity element of R_1.

(i) $C(x-y) = \max\{A(0_1 - 0_1), C(x-y)\}$

$$= (A \cup C)(0_1 - 0_1, x - y)$$

$$= (A \cup C)((0_1, x) - (0_1, y))$$

$$\leq \max\{(A \cup C)(0_1, x), (A \cup C)(0_1, y)\}$$

$$= \max\{\max\{A(0_1), C(x)\}, \max\{A(0_1), C(y)\}\}$$

$$= \max\{C(x), C(y)\}$$

$C(x-y) \leq \max\{C(x), C(y)\}$

(ii) $C(xy) = \max\{A(0_1 0_1), C(xy)\}$

$$= (A \cup C)(0_1 0_1, xy)$$

$$= (A \cup C)((0_1, x) \cdot (0_1, y))$$

$$\leq \max\{(A \cup C)(0_1, x), (A \cup C)(0_1, y)\}$$

$$= \max\{\max\{A(0_1), C(x)\}, \max\{A(0_1), C(y)\}\}$$

$$= \max\{C(x), C(y)\}$$

$C(xy) \leq \max\{C(x), C(y)\}$

(iii) $D(x-y) = \min\{B(0_1 - 0_1), D(x-y)\}$

$$= (B \cap D)(0_1 - 0_1, x - y)$$

$$= (B \cap D)((x, 0_2) - (y, 0_2))$$

$$\geq \min\{(B \cap D)(0_1, x), (B \cap D)(0_1, y)\}$$

$$= \min\{\min\{B(0_1), D(x)\}, \min\{B(0_1), D(x)\}\}$$

$$= \min\{D(x), D(y)\}$$

$D(x-y) \geq \min\{D(x), D(y)\}$

(iv) $D(xy) = \min\{B(0_1 0_1), D(xy)\}$

$$= (B \cap D)(0_1 0_1, xy)$$

$$= (B \cap D)((x, 0_2) \cdot (y, 0_2))$$

$$\geq \min\{(B \cap D)(0_1, x), (B \cap D)(0_1, y)\}$$

$$= \min\{\min\{B(0_1), D(x)\}, \min\{B(0_1), D(x)\}\}$$

$$= \min\{D(x), D(y)\}$$

$D(xy) \geq \min\{D(x), D(y)\}$

Hence, H is an intuitionistic multi-anti fuzzy subring of R_2.

Remark 2.6. Let $G = \{\langle x, A(x), B(x)\rangle / x \in R_1\}$ and $H = \{\langle y, C(y), D(y)\rangle / y \in R_2\}$ be any two intuitionistic multi-fuzzy sets defined on a ring R_1 and R_2 respectively. If the anti Cartesian product $G \times H$ is an intuitionistic multi-anti fuzzy subring of $R_1 \times R_2$, then either G is an intuitionistic multi-anti fuzzy subring of R_1 or H is an intuitionistic multi-anti fuzzy subring of R_2.

3. Intuitionistic Multi-Anti Fuzzy Ring of a Ring under Homomorphism and Anti Homomorphism

In this section, the properties of homomorphic and anti homomorphic anti image and anti pre-image of an intuitionistic multi-anti fuzzy subring of a ring R is discussed.

Definition 3.1. Let R_1 and R_2 be any two rings. Let $G = \{\langle x, A(x), B(x)\rangle / x \in R_1\}$ and $H = \{\langle y, C(y), D(y)\rangle / y \in R_2\}$ be any two intuitionistic multi-fuzzy subsets in R_1 and R_2 respectively. Let $f : R_1 \to R_2$ be a mapping then the anti image of G denoted as $f(G) = \{\langle f(y), f(A)(y), f(A)(y)\rangle / y \in R_2\}$ is an intuitionistic multi-fuzzy subset of R_2 defined as for each $y \in R_2$,

$$(f(A))(y) = \begin{cases} \inf\{A(x) : x \in f^{-1}(y)\}, & \text{if } f^{-1}(y) \neq \phi \\ 1, & \text{otherwise} \end{cases}$$

$$(f(B))(y) = \begin{cases} \sup\{B(x) : x \in f^{-1}(y)\}, & \text{if } x \in f^{-1}(y) \neq \phi \\ 0, & \text{otherwise} \end{cases}$$

Also the anti pre-image of Hunder f denoted as $f^{-1}(H) = \{\langle f^{-1}(x), f^{-1}(C)(x), f^{-1}(D)(x)\rangle / x \in R_1\}$ is an intuitionistic multi-fuzzy subset of R_1 defined as for each $x \in R_1$, $(f^{-1}(C))(x) = C(f(x))$ and $(f^{-1}(D))(x) = D(f(x))$.

Theorem 3.2. *Let R_1 and R_2 be any two rings. Let $f : R_1 \to R_2$ be a homomorphism onto rings. Let $G = \{\langle x, A(x), B(x)\rangle / x \in R_1\}$ be an intuitionistic multi-anti fuzzy subring of R_1 then $f(G)$ is an intuitionistic multi-anti fuzzy subring of R_2, if G has a* inf *property and G is f-invariant.*

Proof. Let $G = \{\langle x, A(x), B(x)\rangle / x \in R_1\}$ be an intuitionistic multi-anti fuzzy subring of R_1

Then, $f(G) = \{\langle f(y), f(A)(f(y)), f(B)(f(y))\rangle / y \in R_2\}$.

There exist $x, y \in R_1$ such that $f(x), f(y) \in R_2$,

(i) $(f(A))(f(x) - f(y)) = (f(A))(f(x - y))$,

$$= A(x - y)$$

$$\leq \max\{A(x), A(y)\}$$

$$= \max\{(f(A))(f(x)), (f(A))(f(y))\}$$

$(f(A))(f(x) - f(y)) \leq \max\{(f(A))(f(x)), (f(A))(f(y))\}$.

(ii) $(f(A))(f(x) f(y)) = (f(A))(f(xy))$,

$$= A(xy)$$

$$\leq \max\{A(x), A(y)\}$$

$$= \max\{(f(A))(f(x)), (f(A))(f(y))\}$$

$$(f(A))(f(x)f(y)) \le \max\{(f(A))(f(x)), (f(A))(f(y))\}.$$

(iii) $(f(B))(f(x)-f(y)) = (f(B))(f(x-y)),$

$$= B(x-y)$$

$$\ge \min\{B(x), B(y)\}$$

$$= \min\{(f(B))(f(x)), (f(B))(f(y))\}$$

$$(f(B))(f(x)-f(y)) \ge \min\{(f(B))(f(x)), (f(B))(f(y))\}.$$

(iv) $(f(B))(f(x)f(y)) = (f(B))(f(xy)),$

$$= B(xy)$$

$$\ge \min\{B(x), B(y)\}$$

$$= \min\{(f(B))(f(x)), (f(B))(f(y))\}$$

$$(f(B))(f(x)f(y)) \ge \min\{(f(B))(f(x)), (f(B))(f(y))\}.$$

Hence, $f(G)$ is an intuitionistic multi-anti fuzzy subring of R_2.

Theorem 3.3. *Let R_1 and R_2 be any two rings. Let $f : R_1 \to R_2$ be a homomorphism onto rings. Let $H = \{\langle y, C(y), D(y)\rangle / y \in R_2\}$ be an intuitionistic multi-anti fuzzy subring of R_2 then $f^{-1}(H)$ is an intuitionistic multi-anti fuzzy subring of R_1.*

Proof. $H = \{\langle y, C(y), D(y)\rangle / y \in R_2\}$ be an intuitionistic multi-anti fuzzy subring of R_2.

Then, $f^{-1}(H) = \{\langle f^{-1}(x), f^{-1}(C)(x), f^{-1}(D)(x)\rangle / x \in R_1\}.$

For any $x, y \in R_1$, $f(x), f(y) \in R_2$,

(i) $(f^{-1}(C))(x-y) = C(f(x-y))$

$$= C(f(x)-f(y))$$

$$\leq \min\{C(f(x)), C(f(y))\}$$

$$= \min\{(f^{-1}(C))(x), (f^{-1}(C))(y)\}$$

$$(f^{-1}(C))(x-y) \leq \min\{(f^{-1}(C))(x), (f^{-1}(C))(y)\}.$$

(ii) $(f^{-1}(C))(xy) = C(f(xy))$

$$= C(f(x)f(y))$$

$$\leq \min\{C(f(x)), C(f(y))\}$$

$$= \min\{(f^{-1}(C))(x), (f^{-1}(C))(y)\}$$

$$(f^{-1}(C))(xy) \leq \min\{(f^{-1}(C))(x), (f^{-1}(C))(y)\}.$$

(iii) $(f^{-1}(D))(x-y) = D(f(x-y))$

$$= D(f(x) - f(y))$$

$$\leq \max\{D(f(x)), D(f(y))\}$$

$$= \max\{(f^{-1}(D))(x), (f^{-1}(D))(y)\}$$

$$(f^{-1}(D))(x-y) \leq \max\{(f^{-1}(D))(x), (f^{-1}(D))(y)\}.$$

(iv) $(f^{-1}(D))(xy) = D(f(xy))$

$$= D(f(x)f(y))$$

$$\leq \max\{D(f(x)), D(f(y))\}$$

$$= \max\{(f^{-1}(D))(x), (f^{-1}(D))(y)\}$$

$$(f^{-1}(D))(xy) \leq \max\{(f^{-1}(D))(x), (f^{-1}(D))(y)\}.$$

Hence, $f^{-1}(H)$ is an intuitionistic multi-anti fuzzy subring of R_1.

Theorem 3.4. *Let R_1 and R_2 be any two rings. Let $f : R_1 \to R_2$ be an anti homomorphism onto rings. Let $G = \{\langle x, A(x), B(x)\rangle / x \in R_1\}$ be an* *intuitionistic multi-anti fuzzy subring of* R_1 *then* $f(G)$ *is an intuitionistic multi-anti fuzzy subring of* R_2, *if G has a* $\inf$ *property and G is f-invariant.*

Proof. Let $G = \{\langle x, A(x), B(x)\rangle / x \in R_1\}$ be an intuitionistic multi-anti fuzzy subring of R_1

Then, $f(G) = \{\langle f(y), f(A)(f(y)), f(B)(f(y))\rangle / y \in R_2\}$.

There exist $x, y \in R_1$ such that $f(x), f(y) \in R_2$,

(i) $(f(A))(f(x) - f(y)) = (f(A))(f(y - x))$,

$$= A(y - x)$$

$$= A(x - y)$$

$$\leq \max\{A(x), A(y)\}$$

$$= \max\{(f(A))(f(x)), (f(A))(f(y))\}$$

$(f(A))(f(x) - f(y)) \leq \max\{(f(A))(f(x)), (f(A))(f(y))\}$.

(ii) $(f(A))(f(x)f(y)) = (f(A))(f(yx))$,

$$= A(yx)$$

$$\leq \max\{A(x), A(y)\}$$

$$= \max\{(f(A))(f(x)), (f(A))(f(y))\}$$

$(f(A))(f(x)f(y)) \leq \max\{(f(A))(f(x)), (f(A))(f(y))\}$.

(iii) $(f(B))(f(x) - f(y)) = (f(B))(f(y - x))$,

$$= B(y - x)$$

$$= B(x - y)$$

$$\geq \min\{B(x), B(y)\}$$

$$= \min\{(f(B))(f(x)), (f(B))(f(y))\}$$

$(f(B))(f(x) - f(y)) \geq \min\{(f(B))(f(x)), (f(B))(f(y))\}$.

(iv) $(f(B))(f(x)f(y)) = (f(B))(f(yx)),$

$$= B(yx)$$

$$\geq \min\{B(x), B(y)\}$$

$$= \min\{(f(B))(f(x)), (f(B))(f(y))\}$$

$(f(B))(f(x)f(y)) \geq \min\{(f(B))(f(x)), (f(B))(f(y))\}.$

Hence, $f(G)$ is an intuitionistic multi-anti fuzzy subring of R_2.

Theorem 3.5. *Let R_1 and R_2 be any two rings. Let $f : R_1 \to R_2$ be an anti homomorphism onto rings. Let $H = \{\langle y, C(y), D(y)\rangle / y \in R_2\}$ be an intuitionistic multi-anti fuzzy subring of R_2 then $f^{-1}(H)$ is an intuitionistic multi-anti fuzzy subring of R_1.*

Proof. $H = \{\langle y, C(y), D(y)\rangle / y \in R_2\}$ be an intuitionistic multi-anti fuzzy subring of R_2.

Then, $f^{-1}(H) = \{\langle f^{-1}(x), f^{-1}(C)(x), f^{-1}(D)(x)\rangle / x \in R_1\}.$

For any $x, y \in R_1$, $f(x), f(y) \in R_2$,

(i) $(f^{-1}(C))(x - y) = C(f(x - y))$

$$= C(f(y) - f(x))$$

$$= C(f(x) - f(y))$$

$$\leq \max\{C(f(x)), C(f(y))\}$$

$$= \max\{(f^{-1}(C))(x), (f^{-1}(C))(y)\}$$

$(f^{-1}(C))(x - y) \leq \max\{(f^{-1}(C))(x), (f^{-1}(C))(y)\}.$

(ii) $(f^{-1}(C))(xy) = C(f(xy))$

$$= C(f(y)f(x))$$

$$\leq \min\{C(f(x)), C(f(y))\}$$

$$= \min\{(f^{-1}(C))(x), (f^{-1}(C))(y)\}$$

$$(f^{-1}(C))(xy) \leq \min\{(f^{-1}(C))(x), (f^{-1}(C))(y)\}.$$

(iii) $(f^{-1}(D))(x-y) = D(f(x-y))$

$$= D(f(y) - f(x))$$

$$= D(f(x) - f(y))$$

$$\geq \min\{D(f(x)), D(f(y))\}$$

$$= \min\{(f^{-1}(D))(x), (f^{-1}(D))(y)\}$$

$$(f^{-1}(D))(x-y) \geq \min\{(f^{-1}(D))(x), (f^{-1}(D))(y)\}.$$

(iv) $(f^{-1}(D))(xy) = D(f(xy))$

$$= D(f(x)f(y))$$

$$\geq \max\{D(f(x)), D(f(y))\}$$

$$= \max\{(f^{-1}(D))(x), (f^{-1}(D))(y)\}$$

$$(f^{-1}(D))(xy) \geq \max\{(f^{-1}(D))(x), (f^{-1}(D))(y)\}.$$

Hence, $f^{-1}(H)$ is an intuitionistic multi-anti fuzzy subring of R_1.

4. Conclusion

In this paper we have defined the Cartesian product of intuitionistic multi-anti fuzzy ring and analyze some of the properties of Cartesian product of intuitionistic multi-anti fuzzy ring and homomorphism of a ring.

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