Advances and Applications in Mathematical Sciences
Volume 21, Issue 8, June 2022, Pages 4333-4357

MOLECULAR DESCRIPTORS OF CERTAIN REPURPOSED DRUGS USED FOR TREATING BLACK FUNGUS, WHITE FUNGUS AND PLAGUE (BLACK DEATH)

ANNMARIA BABY, D. ANTONY XAVIER, EDDITH SARAH VARGHESE and DEEPA MATHEW

Department of Mathematics
Loyola College, University of Madras
Chennai, India

Department of Mathematics
St. Joseph's College, Bangalore, India

Abstract

The contemporary world is coping through several infectious diseases and pandemics. These disorders caused by organisms such as bacteria, viruses, fungi or parasites are making its way through humans at a drastic phase. Molecular topology deals with the algebraic description of chemical compounds, which allows a unique and easy characterization of them, and its development had an influential entry in the field of drug design and discovery. One of the most widely used applications of molecular topology is in terms of topological indices. In this paper, we study the molecular descriptors of various antifungal drugs used for treating Black fungus and White fungus. Also, we study the structural properties of antibacterial drugs tested for treating Plagues (black death).

1. Introduction

Topological indices are a kind of molecular descriptors that translate the connectivity of the atoms represented by vertices in a molecule by bonds represented by edges and are essentially graph invariants. Two isomorphic molecular graphs would have the same value for any kind of topological molecular descriptors. However, two different graphs may have the same

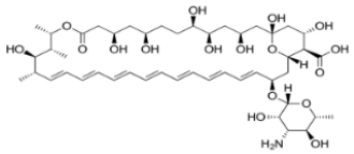
2020 Mathematics Subject Classification: 92C40.
Keywords: Distance-based indices, Degree-based indices, black fungus, white fungus, plaques, antifungal drugs, antibacterial drugs.
Received January 10, 2022; Accepted March 2, 2022

value for a topological descriptor. This generates the necessity of developing topological indices which can give different values for different molecules, which helps in identifying molecular graphs as uniquely as possible [17, 21]. Topological indices are used in the development of quantitative structure activity relationships (QSARs). The IUPAC [19] defines the topological index as "A topological index is a numerical value that is associated with the chemical constitution of a chemical, which helps to correlate the chemical structure with various physical properties, chemical reactivity, or biological activity."

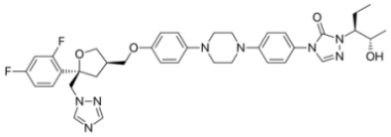
Mucormycosis, commonly referred to as Black fungus is an air borne fungal infection that is naturally present in the air, water and even in food. It was previously called as zygomycosis and is a serious but rare fungal infection caused by a group of molds called mucormycetes. It mainly affects people who are immunocompromised. Amphotericin B. and Posaconazole are the recommended medications. Amphotericin B. is an antifungal medication that fights infections caused by fungus and Posaconazole is a triazole antifungal drug that is used to treat invasive infections by Candida species and Aspergillus species in severely immunocompromised patients. Candida or White fungus is an infection caused by a yeast, a type of fungus called Candida that forms white colour colonies in infected tissues. Fluconazole and Itraconazole are the major antifungal drug compounds used for the treatment. Both belongs to a class of drugs called azole antifungals and is used to prevent and treat a variety of fungal and yeast infections. Plagues (Black death) is a disease caused by the bacterium, Yersinia pestis, which is primarily transmitted by fleas. It is the most fatal pandemic recorded in human history, causing millions of death. There are mainly three types of Plague, namely, bubonic, septicemic and pneumonic, depending on which part of your body is involved. Bubonic plague is the most common variety named after the swollen lymph nodes (buboes). Septicemic plague is a life-threatening infection of the blood, most commonly spread by bites from infected fleas. Pneumonic plague affects the lungs. It's the least common variety of plague but the most dangerous, because it can be spread from person to person via cough droplets. Modern antibiotics are effective in treating plague and the recommended antibacterial drugs are Ciprofloxacin, Doxycycline, Gentamicin, Levofloxacin, Moxifloxacin, Streptomycin,

Tetracycline and Chloramphenicol. Ciprofloxacin, Levofloxacin and Moxifloxacin belongs the class of antibiotics called quinolones and works by killing bacteria that cause infections. Doxycycline is a tetracycline antibiotic that fights bacteria in the body. Gentamicin injection used to treat serious bacterial infections in many different parts of the body belongs to the class of medicines known as aminoglycoside antibiotics. Streptomycin also belongs the class of drugs known as aminoglycoside antibiotics. Tetracycline is in the class of medications called tetracycline antibiotics that prevents the growth and spread of bacteria. Chloramphenicol is a man-made antibiotic. It slows growth of bacteria by preventing them from producing important proteins that they need to survive.

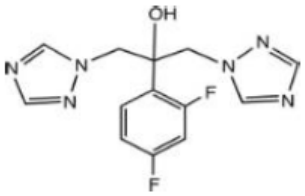
So in this paper we discuss various topological indices of drugs mentioned above, whose molecular structure are represented in Figure 1.



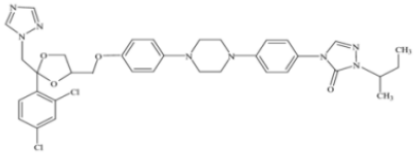
(a) Amphotericin B



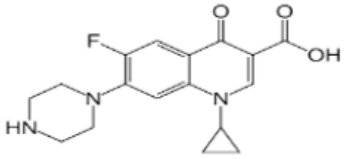
(b) Posaconazole



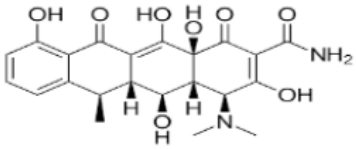
(c) Fluconazole



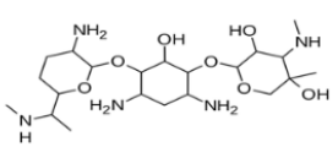
(d) Itraconazole



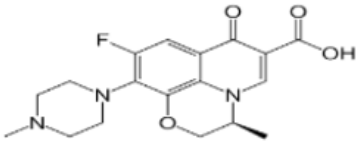
(e) Ciprofloxacin



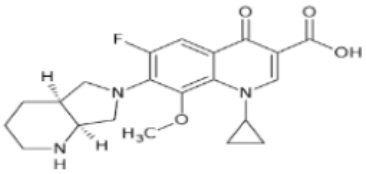
(f) Doxycycline



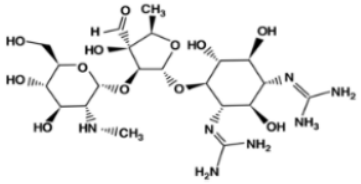
(g) Gentamicin



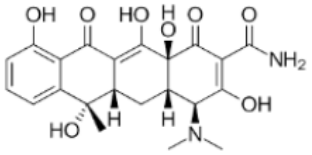
(h) Levofloxacin



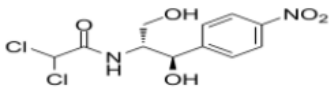
(i) Moxifloxacin



(j) Streptomycin



(k) Tetracycline



(l) Chloramphenicol

Figure 1.

2. Preliminaries

Let $G = (V(G), E(G))$ be a simple finite connected graph. The degree of a vertex v, denoted as $d_G(v)$ is the number of edges incident to v. The distance between two vertices u and v, $d_G(u, v)$ is the number of edges in the shortest path connecting them. The distance between two edges $e = uv$ and $f = ab$ of G, $D_G(e, f)$ is defined as $\min\{d_G(e, a), d_G(e, b)\}$. The distance between a vertex $u \in V(G)$ and an edge $f = ab \in E(G)$, $d_G(u, f)$, is defined as $\min\{d_G(u, a), d_G(u, b)\}$. The open neighbourhood $N_G(v)$ is the set of vertices adjacent to v. For an edge $e = uv$,

$$N_u(e \mid G) = \{x \in V(G) : d_G(u, x) < d_G(v, x)\}$$

$$M_u(e \mid G) = \{y \in E(G) : d_G(u, y) < d_G(v, y)\}.$$

The cardinality of the sets $N_u(e \mid G)$ and $M_u(e \mid G)$ are denoted as $n_u(e \mid G)$ and $m_u(e \mid G)$ respectively. The values of $n_v(e \mid G)$ and $m_v(e \mid G)$ are analogous.

The strength weighted graph denoted by $G_{sw} = (G, [w_v, s_v], s_e)$, where w_v is the vertex weight, s_v is the vertex strength and s_e is the edge strength was first presented in Arockiaraj et al. [2], where the sets $N_u(e \mid G_{sw}) = N_u(e \mid G)$ and $M_u(e \mid G_{sw}) = M_u(e \mid G)$ are described with cardinality,

$$n_u(e \mid G_{sw}) = \sum_{x \in N_u(e|G_{sw})} w_v(x),$$

$$m_u(e \mid G_{sw}) = \sum_{x \in N_u(e|G_{sw})} s_v(x) + \sum_{y \in M_u(e|G_{sw})} s_e(y)$$

The values of $n_v(e \mid G_{sw})$ and $m_v(e \mid G_{sw})$ are analogous. Moreover $d_{G_{sw}}(u, v) = d_G(u, v)$, $D_{G_{sw}}(e, f) = D_G(e, f)$ and $d_{G_{sw}}(u, f) = d_G(u, f)$. The distance based indices and degree based indices for the strength weighted graphs studied are shown in Table 1 [2, 3, 4, 14] and Table 2 [11, 13, 15] respectively.

Table 1. Distance based indices of strength-weighted graph G_{sw}.

Wiener	$W(G_{sw}) = \sum_{\{u, v\} \subseteq V(G_{sw})} w_v(u) w_v(v) d_{G_{sw}}(u, v)$
Edge-Wiener	$W_e(G_{sw}) = \sum_{\{u, v\} \subseteq V(G_{sw})} s_v(u) s_v(v) d_{G_{sw}}(u, v) + \sum_{\{e, f\} \subseteq E(G_{sw})} s_e(e) s_e(f) d_{G_{sw}}(e, f) + \sum_{u \in V(G_{sw}) f \in E(G_{sw})} s_v(u) s_v(f) d_{G_{sw}}(u, f)$
Szeged	$Sz_v(G_{sw}) = \sum_{e=uv \in E(G_{sw})} s_e(e) n_u(e \mid G_{sw}) n_v(e \mid G_{sw})$

Edge-Szeged	$Sz_e(G_{sw}) = \sum_{e=uv \in E(G_{sw})} s_e(e) m_u(e \mid G_{sw}) m_v(e \mid G_{sw})$
Padmakar-Ivan	$PI(G_{sw}) = \sum_{e=uv \in E(G_{sw})} s_e(e)[m_u(e \mid G_{sw}) + m_v(e \mid G_{sw})]$
Mostar	$Mo(G_{sw}) = \sum_{e=uv \in E(G_{sw})} s_e(e) \mid n_u(e \mid G_{sw}) - n_v(e \mid G_{sw}) \mid$
Edge-Mostar	$Sz_v(G_{sw}) = \sum_{e=uv \in E(G_{sw})} s_e(e) \mid m_u(e \mid G_{sw}) - m_v(e \mid G_{sw}) \mid$

Table 2. Degree based indices of graph.

First and Second Zagreb indices [10]	$M_1(G) = \sum_{e=uv \in E(G)} (d_G(u) + d_G(v))$ $M_2(G) = \sum_{e=uv \in E(G)} (d_G(u) \times d_G(v))$
Harmonic index [7]	$H(G) = \sum_{e=uv \in E(G)} \frac{2}{(d_G(u) + d_G(v))}$
Hyper Zagreb index [18]	$HM(G) = \sum_{e=uv \in E(G)} [d_G(u) + d_G(v)]^2$
Forgotten topological index [9]	$F(G) = \sum_{e=uv \in E(G)} [d_G(u)]^2 + [d_G(v)]^2$
Randic index [16]	$R(G) = \sum_{e=uv \in E(G)} \frac{1}{\sqrt{d_G(u) \times d_G(v)}}$
Reciprocal Randić index [8]	$RR(G) = \sum_{e=uv \in E(G)} \sqrt{d_G(u) \times d_G(v)}$
Sum-connectivity index [22]	$SCI(G) = \sum_{e=uv \in E(G)} \frac{1}{\sqrt{d_G(u) + d_G(v)}}$

Geometric arithmetic index [20]	$GA(G) = \sum_{e=uv \in E(G)} \frac{2\sqrt{d_G(u) \times d_G(v)}}{\sqrt{d_G(u) + d_G(v)}}$
Atom bond connectivity index [6]	$ABC(G) = \sum_{e=uv \in E(G)} \frac{\sqrt{d_G(u) + d_G(v) - 2}}{\sqrt{d_G(u) \times d_G(v)}}$

The cut method is a powerful tool for investigating the distance and degree based molecular structure-descriptors [1]. The concepts of convex subgraph, isometric subgraph, partial cubes and Djoković-Winkler (Θ) relation plays the key roles. A subgraph H of G is said to be convex if for any two vertices $u, v \in H$, any shortest path between them in G lies completely in H. A graph H is an isometric subgraph of G if for every pair of vertices, the distance between them in both G and H are equal. An isometric subgraph of a hypercube is called a partial cube. The Djokovic´-Winkler relation is defined as $d_G(v, b) + d_G(u, a) \neq d_G(u, b) + d_G(v, a)$ for two edges $e = uv$ and $f = ab$ of G. The relation Θ is always reflexive, symmetric and transitive in case of partial cubes but not transitive in general. Hence the Θ partitions the edge set of a partial cube G into classes $F_1, F_2, \ldots, F_r$, called Θ-classes or convex cuts. However, its transitive closure Θ^* forms an equivalence relation in general and partitions the edge set into convex components. In general we describe the cut method as [12], for a given molecular graph G, (i) partition the edge set of G into classes $F_1, F_2, \ldots, F_r$, call them cuts, such that each of the graphs $G - F_i$, $i = 1, 2, \ldots, r$, consists of two (or more) connected components; and (ii) use properties (of the components) of the graphs $G - F_i$ to derive a required property of G.

A partition $\mathbb{E} = \{E_1, E_2, \ldots, E_k\}$ of $E(G)$ is said to be coarser than partition $\mathcal{F}$ if each set E_i is the union of one or more Θ^*-classes of G. For any class E_i, the quotient graph G/E_i is formed from the disconnected graph $G - E_i$, where the connected components act as the vertices and the two components C_j^i and C_k^i are adjacent whenever a vertex $x \in C_j^i$ is adjacent to a vertex $x \in C_k^i$ with $xy \in E_i$ [14].

3. Distance Based Indices

In this section, we compute the distance based topological indices of the antifungal and antibacterial drug compounds under consideration. The vertex strength weighted values and the edge strength value of the quotient graph for the convex cut F_i of G is taken as $[a_i, b_i]$, $[c_i, d_i]$ and e_i respectively such that $c_i = |V(G)| - a_i$ and $d_i = |E(G)| - b_i - e_i$ as depicted in figure 2. Some drug compounds in the study contains triangle graph and/or pentagons in their structure. Such compounds does not belong to the family of the partial cubes. In such cases we use the Θ^*-partition to compute the required distance based indices. Here we obtain the quotient graph G/S as depicted in figure 3.

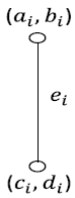


Figure 2. Quotient graph G/F_i.

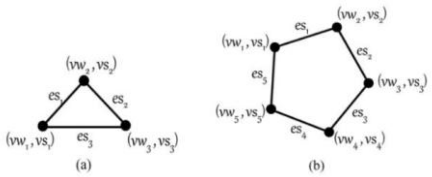


Figure 3. Quotient graph G/S.

Result 1.

$$W(G/F_i) = \sum_i a_i c_i,$$

$$W_e(G/F_i) = \sum_i b_i d_i,$$

$$Sz_v(G/F_i) = \sum_i e_i a_i c_i,$$

$$Sz_e(G/F_i) = \sum_i e_i b_i d_i,$$

$$PI(G/F_i) = \sum_i e_i (b_i + d_i),$$

$$Mo(G/F_i) = \sum_i e_i | a_i - c_i |,$$

$$Mo_e(G/F_i) = \sum_i e_i |\, b_i - d_i \,|$$

Result 2. Let the vertex weight, vertex strength and edge strength values of the quotient graph of the form triangle be (vw_1, vw_2, vw_3), (vs_1, vs_2, vs_3), (es_1, es_2, es_3) respectively (Figure 3(a)), then the following results are obtained:

$$W(G/S) = vw_1vw_2 + vw_2vw_3 + vw_3vw_1$$

$$W_e(G/S) = vs_1vs_2 + vs_2vs_3 + vs_3vs_1 + vs_1es_2 + vs_2es_3 + vs_3es_1$$

$$Sz_v(G/S) = es_1(vw_1vw_2) + es_2(vw_2vw_3) + es_3(vw_3vw_1)$$

$$Sz_e(G/S) = es_1(vs_1 + es_3)(vs_2 + es_2) + es_2(vs_2 + es_1)(vs_3 + es_3) + es_3(vs_3 + es_2)(vs_1 + es_1)$$

$$PI(G/S) = es_1((vs_1 + es_3)(vs_2 + es_2)) + es_2((vs_3 + es_1)(vs_3 + es_3)) + es_3((vs_3 + es_2)(vs_1 + es_1))$$

$$Mo(G/S) = es_1|\, vw_1 - vw_2 \,| + es_2|\, vw_2 - vw_3 \,| + es_3|\, vw_1 - vw_3 \,|$$

$$Mo_e(G/S) = es_1|\, (vs_1 + es_3) - (vs_2 + es_2) \,| + es_2|\, (vs_2 + es_1) - (vs_3 + es_3) \,| + es_3|\, (vs_3 + es_2) - (vs_1 + es_1) \,|$$

Result 3. Consider the vertex weight, vertex strength and edge strength values of the quotient graph of the form pentagon to be $(vw_1, vw_2, vw_3, vw_4, vw_5)$, $(vs_1, vs_2, vs_3, vs_4, vs_5)$, $(es_1, es_2, es_3, es_4, es_5)$ respectively (Figure 3(b)), then the following results hold:

$$W(G/S) = vw_1vw_2 + vw_2vw_3 + vw_3vw_4 + vw_4vw_5 + vw_5vw_1 + 2(vw_1vw_3 + vw_1vw_4 + vw_2vw_4 + vw_2vw_5 + vw_3vw_5)$$

$$W_e(G/S) = vs_1vs_2 + vs_2vs_3 + vs_3vs_4 + vs_4vs_5 + vs_5vs_1 + 2(vs_1vs_3 + vs_1vs_4 + vs_2vs_4 + vs_2vs_5 + vs_3vs_5) + 2(es_1es_3 + es_1es_4 + es_2es_4 + es_2es_5 + es_3es_5) + vs_1es_2 + vs_1es_4$$

$$+vs_2es_3 + vs_2es_5 + vs_3es_4 + vs_3es_1 + +vs_4es_5 + vs_4es_5$$
$$+vs_4es_2 + vs_5es_1 + vs_5es_3 + 2(vs_1es_3 + vs_2es_4 + vs_3es_5 + vs_4es_1$$
$$+vs_5es_2)$$

$$Sz_v(G/S) = es_1(vu_1 + vu_5)(vu_3 + vu_2) + es_2(vu_1 + vu_2)(vu_3 + vu_4)$$
$$+es_3(vu_2 + vu_4)(vu_4 + vu_5) + es_4(vu_3 + vu_4)(vu_1 + vu_5)$$
$$+es_5(vu_4 + vu_5)(vu_1 + vu_2)$$

$$Sz_e(G/S) = es_1(vs_1 + vs_5 + es_4 + es_5)(vs_2 + vs_3 + es_2 + es_3)$$
$$+es_2(vs_1 + vs_2 + es_1 + es_5)(vs_3 + vs_4 + es_3 + es_4)$$
$$+es_3(vs_3 + vs_2 + es_1 + es_2)(vs_4 + vs_5 + es_4 + es_5)$$
$$+es_4(vs_3 + vs_4 + es_2 + es_3)(vs_1 + vs_5 + es_5 + es_1)$$
$$+es_5(vs_{14} + vs_5 + es_3 + es_4)(vs_2 + vs_1 + es_1 + es_2)$$

$$PI(G/S) = es_1((vs_1 + vs_5 + es_4 + es_5)(vs_3 + vs_2 + es_2 + es_3))$$
$$+es_2((vs_1 + vs_2 + es_1 + es_5) + (vs_3 + vs_4 + es_3 + es_4))$$
$$+es_3((vs_3 + vs_2 + es_1 + es_2) + (vs_4 + vs_5 + es_4 + es_5))$$
$$+es_4((vs_3 + vs_4 + es_2 + es_3) + (vs_1 + vs_5 + es_5 + es_1))$$
$$+es_5((vs_4 + vs_5 + es_3 + es_4) + (vs_2 + vs_1 + es_1 + es_2))$$

$$Mo(G/S) = es_1(|\,(vu_1 + vu_5) - (vu_3 + vu_2)\,|)$$
$$+es_2|\,(vu_1 + vu_2) - (vu_3 + vu_4)\,|$$
$$+es_3|\,(vu_2 + vu_4) - (vu_4 + vu_5)\,|$$
$$+es_4|\,(vu_3 + vu_4) - (vu_1 + vu_5)\,|$$
$$+es_5|\,(vu_4 + vu_5) - (vu_1 + vu_2)\,|$$

$$Mo_e(G/S) = es_1|\,(vs_1 + vs_5 + es_4 + es_5) - (vs_2 + vs_3 + es_2 + es_3)\,|$$

$$+es_2|\,(vs_1+vs_2+es_1+es_5)-(vs_3+vs_4+es_3+es_4)\,|$$

$$+es_3|\,(vs_3+vs_2+es_1+es_2)-(vs_4+vs_5+es_4+es_5)\,|$$

$$+es_4|\,(vs_3+vs_4+es_2+es_3)-(vs_1+vs_5+es_5+es_1)\,|$$

$$+es_5|\,(vs_4+vs_5+es_3+es_4)-(vs_2+vs_1+es_1+es_2)\,|$$

Result 4. (Refer [2] Theorem 1) Let TI represent the indices such as W, W_e, Sz_v, Sz_e, PI, Mo, Mo_e. Then

$$TI(G)=TI(G/F_i)+\sum_i TI(G/S_i).$$

Theorem 1. *Let G_1 be Amphotericin B compound. Then* $W(G_1)=22036$, $W_e(G_1)=20336$, $Sz_v(G_1)=43690$, $Sz_e(G_1)=41410$, $PI(G_1)=4370$, $Mo(G_1)=2161$, $Mo_e(G_1)=2298$.

Proof. G_1 has 65 vertices and 67 edges. The various convex cuts $\{F_i : 1\le i\le 43\}$ of G_1 is depicted in Figure 4 and the strength-weighted values of quotient graph G_1/F_i are given in Table 3 and from following data obtained by applying Result 1, we compute the indices.

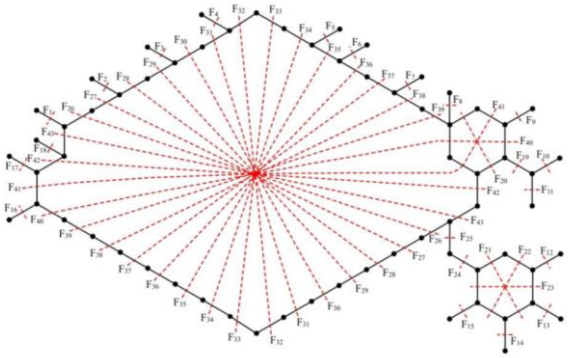


Figure 4. Various cuts of G_1.

Table 3. Strength-weighted values of G_1/F_i from Figure 4

F_i	$1\le i\le 18$	19	20	21	22	23	24	25	26	27	28	29	30

a_i	1	3	7	5	6	6	10	11	22	22	23	23	24
b_i	0	2	6	4	5	5	10	11	21	21	22	22	23
e_i	1	1	2	2	2	2	1	1	2	2	2	2	2

F_i	31	32	33	34	35	36	37	38	39	40	41	42	43
a_i	24	25	25	25	26	27	27	28	28	32	29	31	32
b_i	23	24	24	24	25	26	26	27	27	31	28	31	32
e_i	2	2	2	2	2	2	2	2	2	3	3	2	2

Theorem 2. *Let* G_2 *be Posaconazole compound. Then* $W(G_2) = 13566$, $W_e(G_2) = 14748$, $Sz_v(G_2) = 20599$, $Sz_e(G_2) = 23019$, $PI(G_2) = 3012$, $Mo(G_2) = 1500$, $Mo_e(G_2) = 1722$.

Proof. W have $|V(G_2)| = 51$ and $|E(G_2)| = 57$. G_2 has 3 pentagons and the strength weighted values of the quotient graph G_2/S_1, G_2/S_2 and G_2/S_3 are as follows:

Quotient graph	$(vw_1, vw_2, vw_3, vw_4, vw_5)$	$(vs_1, vs_2, vs_3, vs_4, vs_5)$	$(es_1, es_2, es_3, es_4, es_5)$
G_2/S_1	(15,1,1,33,1)	(16,0,0,36,0)	(1,1,1,1,1)
G_2/S_2	(47,1,1,1,1)	(52,0,0,0,0)	(1,1,1,1,1)
G_2/S_3	(40,2,7,1,1)	(45,1,6,0,0)	(1,1,1,1,1)

Table 4. Strength-weighted values of G_2/F_i from Figure 5.

F_i	$1 \le i \le 6$	7	8	9	10	11	12	13	14	15	16	17	18	19	20
a_i	1	3	6	12	15	15	15	18	21	21	21	24	24	24	24
b_i	0	2	5	12	15	15	15	19	22	22	22	26	26	26	26
e_i	1	1	1	1	2	2	2	1	2	2	2	1	2	2	2

F_i	19	20	21	22	23	24	25	26	27	28	29	30

a_i	24	24	21	20	19	5	6	8	5	4	4	2
b_i	26	26	23	22	21	5	6	8	4	3	3	1
e_i	2	2	1	1	1	1	1	1	2	2	2	1

By applying Result 3 thrice and Result 1, followed by using Result 4, we compute the results as follows:

$$W(G_2) = \sum_{i=1}^{30} a_i c_i + W(G_2/S_1) + W(G_2/S_2) + W(G_2/S_3) = 13566,$$

$$W_e(G_2) = \sum_{i=1}^{30} b_i d_i + W_e(G_2/S_1) + W_e(G_2/S_2) + W_e(G_2/S_3) = 14748,$$

$$Sz_v(G_2) = \sum_{i=1}^{30} e_i a_i c_i + Sz_v(G_2/S_1) + Sz_v(G_2/S_2) + Sz_v(G_2/S_3) = 20599,$$

$$Sz_e(G_2) = \sum_{i=1}^{30} e_i b_i d_i + Sz_e(G_2/S_1) + Sz_e(G_2/S_2) + Sz_e(G_2/S_3) = 23019,$$

$$PI(G_2) = \sum_{i=1}^{30} e_i (b_i + d_i) + PI(G_2/S_1) + PI(G_2/S_2) + PI(G_2/S_3) = 3012,$$

$$Mo(G_2) = \sum_{i=1}^{30} e_i | a_i - c_i | + Mo(G_2/S_1) + Mo(G_2/S_2) + Mo(G_2/S_3) = 1500,$$

$$Mo_e(G_2) = \sum_{i=1}^{30} e_i | b_i - d_i | + Mo_e(G_2/S_1) + Mo_e(G_2/S_2) + Mo_e(G_2/S_3) = 1722.$$

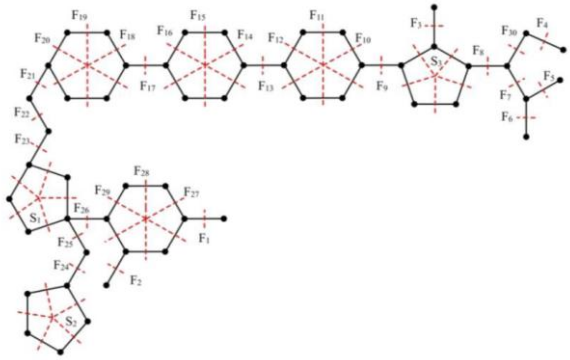


Figure 5. Various cuts of G_2.

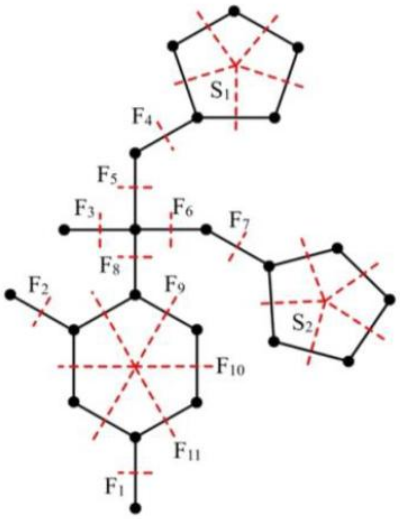


Figure 6. Various cuts of G_3.

Theorem 3. *Let* G_3 *be Fluconazole compound. Then* $W(G_3) = 1000$, $W_e(G_3) = 852$, $Sz_v(G_3) = 1307$, $Sz_e(G_3) = 1220$, $PI(G_3) = 508$, $Mo(G_3) = 326$, $Mo_e(G_3) = 368$.

Proof. We have $|V(G_3)| = 22$ and $|E(G_3)| = 24$. Also G_3 has 2 pentagons and the strength-weighted values of the quotient graph G_3/S_1 and G_3/S_2 are as follows:

Quotient graph	$(vw_1, vw_2, vw_3, vw_4, vw_5)$	$(vs_1, vs_2, vs_3, vs_4, vs_5)$	$(es_1, es_2, es_3, es_4, es_5)$
G_2/S_1	(1,1,1,18,1)	(0,0,0,19,0)	(1,1,1,1,1)
G_2/S_2	(1,1,1,1,18)	0,0,0,0,19)	(1,1,1,1,1)

Table 5. Strength-weighted values of G_3/F_i from Figure 6.

F_i	$1 \le i \le 3$	4	5	6	7	8	9	10	11
a_i	1	5	6	6	5	8	4	4	5
b_i	0	5	6	6	5	8	3	3	4

e_i	1	1	1	1	1	1	2	2	2

We compute the distance based indices by using Result 4 and applying Result 3 twice and Result 1.

Theorem 4. *Let* G_4 *be Itraconazole compound. Then* $W(G_4) = 12096$, $W_e(G_4) = 13260$, $Sz_v(G_4) = 18445$, $Sz_e(G_4) = 20771$, $PI(G_4) = 2796$, $Mo(G_4) = 1378$, $Mo_e(G_4) = 1586$.

Proof. $|V(G_4)| = 49$ and $|E(G_4)| = 55$. Also G_4 has 3 pentagons and the strength weighted values of the quotient graph G_4/S_1, G_4/S_2 and G_4/S_3 are as follows:

Quotient graph	$(vw_1, vw_2, vw_3, vw_4, vw_5)$	$(vs_1, vs_2, vs_3, vs_4, vs_5)$	$(es_1, es_2, es_3, es_4, es_5)$
G_2/S_1	(45,1,1,1,1)	(50,0,0,0,0)	(1,1,1,1,1)
G_2/S_2	(1,31,1,15,1)	(0,34,0,16,0)	(1,1,1,1,1)
G_2/S_3	(40,2,5,1,1)	(45,1,4,0,0)	(1,1,1,1,1)

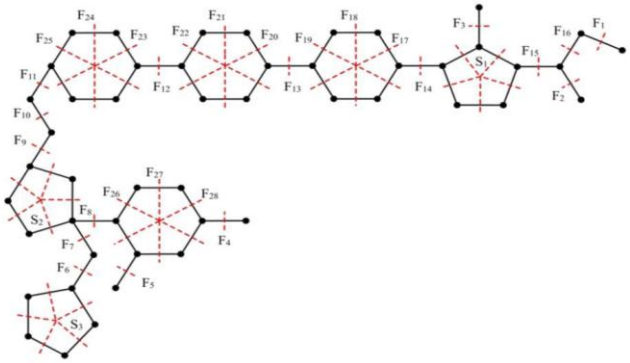


Figure 7. Various cuts of G_4.

Table 6. Strength-weighted values of G_4/F_i from Figure 7

F_i	$1 \le i \le 5$	6	7	8	9	10	11	12	13	14	15	16
a_i	1	5	6	8	19	20	21	22	16	10	4	2

b_i	0		5	6	8	21	22	23	24	17	10	3	1
e_i	1		1	1	1	1	1	1	1	1	1	1	1
F_i	17	18	19	20	21	22	23	24	25	26	27	27	28
a_i	13	13	13	19	19	19	24	24	24	4	4	4	5
b_i	13	13	13	20	20	20	26	26	26	3	3	3	4
e_i	2	2	2	2	2	2	2	2	2	2	2	2	2

By applying Result 3 thrice and Result 1, followed by using Result 4 we compute the results from the obtained data.

Theorem 5. *Let* G_5 *be Ciprofloxacin compound. Then* $W(G_5) = 1234$, $W_e(G_5) = 1092$, $Sz_v(G_5) = 2237$, $Sz_e(G_5) = 2085$, $PI(G_5) = 658$, $Mo(G_5) = 346$, $Mo_e(G_5) = 412$.

Proof. $|V(G_5)| = 24$ and $|V(G_5)| = 27$.

Table 7. Strength-weighted values of G_5/F_i from Figure 8.

F_i	$1 \le i \le 4$	5	6	7	8	9	10	11	12	13	14	15
a_i	1	3	3	6	7	9	10	10	10	3	3	3
b_i	0	2	3	6	6	9	10	10	9	2	2	2
e_i	1	1	1	1	2	2	2	2	3	2	2	2

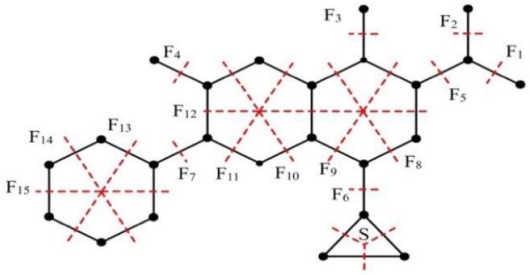


Figure 8. Various cuts of G_5.

Also G_5 contains one triangle and the strength-weighted values of the

quotient graph G/S are $(vw_1, vw_2, vw_3) = (22, 1, 1)$, $(vs_1, vs_2, vs_3) = (24, 0, 0)$, $(es_1, es_2, es_3) = (1, 1, 1)$. So by applying Result 1 and Result 2 followed by using Result 4, we get the results.

Theorem 6. *Let G_6 be Doxycycline compound. Then $W(G_6) = 2336$, $W_e(G_6) = 1878$, $Sz_v(G_6) = 4891$, $Sz_e(G_6) = 4300$, $PI(G_6) = 1154$, $Mo(G_5) = 642$, $Mo_e(G_6) = 722$.*

Proof. Doxycycline compound has 32 vertices and edges. We compute the results from the obtained data by applying Result 1.

Table 8. Strength-weighted values of G_6/F_i from Figure 9.

F_i	$1 \le i \le 10$	11	12	13	14	15	16	17	18	19	20	21	22	23
a_i	1	3	3	10	8	15	15	9	9	4	3	15	1	1
b_i	0	2	2	9	7	16	16	9	9	3	2	14	0	0
e_i	1	1	1	2	2	2	2	2	2	2	2	5	1	1

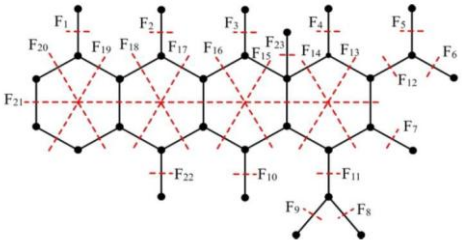


Figure 9. Various cuts of G_6.

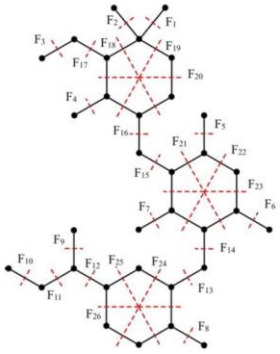


Figure 10. Various cuts of G_7.

Theorem 7. *Let G_7 be Gentamicin compound. Then $W(G_7) = 3234$, $W_e(G_7) = 2727$, $Sz_v(G_7) = 4920$, $Sz_e(G_7) = 4261$, $PI(G_7) = 1172$, $Mo(G_7) = 727$, $Mo_e(G_7) = 798$.*

Proof. $|V(G_7)| = 33$ and $|E(G_7)| = 35$. By calculations based on Result 1, the results are obtained from the following data.

Table 9. Strength-weighted values of G_7/F_i from Figure 10.

F_i	$1 \le i \le 10$	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
a_i	1	2	4	11	12	12	11	2	5	8	7	5	16	17	7	7	4
b_i	0	1	3	11	12	12	11	1	4	7	6	4	16	17	6	6	3
e_i	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2

Theorem 8. *Let* G_8 *be Levofloxacin compound. Then* $W(G_8) = 1484$, $W_e(G_8) = 1255$, $Sz_v(G_8) = 3098$, $Sz_e(G_8) = 2777$, $PI(G_8) = 782$, $Mo(G_8) = 394$, $Mo_e(G_8) = 468$.

Proof. $|V(G_8)| = 33$ and $|E(G_8)| = 35$. By applying Result 1 on the following data we can compute the required results.

Table 10. Strength-weighted values of G_8/F_i from Figure 11.

F_i	$1 \le i \le 6$	7	8	9	10	11	12	13	14	15	16	17
a_i	1	3	7	4	4	4	10	13	11	9	7	4
b_i	0	2	7	3	3	3	9	13	11	8	6	3
e_i	1	1	1	2	2	2	3	3	2	3	2	2

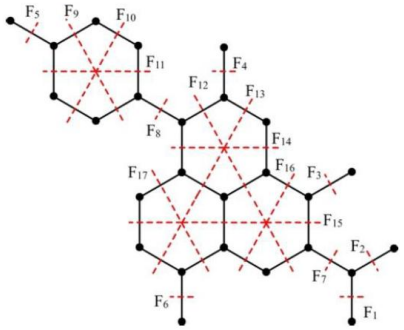


Figure 11. Various cuts of G_8

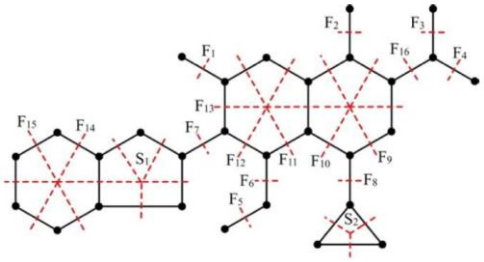


Figure 12. Various cuts of G_9

Theorem 9. *Let* G_9 *be Moxifloxacin compound. Then* $W(G_9) = 2006$, $W_e(G_9) = 1908$, $Sz_v(G_9) = 3443$, $Sz_e(G_9) = 3544$, $PI(G_9) = 956$, $Mo(G_9) = 516$, $Mo_e(G_9) = 532$.

Proof. G_9 has 29 vertices and 33 edges.

Table 11. Strength-weighted values of G_9/F_i from Figure 12.

F_i	$1 \le i \le 5$	6	7	8	9	10	11	12	13	14	15	16
a_i	1	2	9	3	7	9	14	13	10	3	3	3
b_i	0	1	10	3	6	9	15	14	9	2	2	2
e_i	1	1	1	2	2	2	2	3	2	2	1	1

Also it contains a triangle and a pentagon. The strength-weighted values of quotient graph G_9/S_1 are $(vw_1, vw_2, vw_3) = (27, 1, 1)$, $(vs_1, vs_2, vs_3) = (30, 0, 0)$, $(es_1, es_2, es_3) = (1, 1, 1)$ and that of the quotient graph G_9/S_2 are $(vw_1, vw_2, vw_3, vw_4, vw_5) = (1, 21, 1, 3, 3)$, $(vs_1, vs_2, vs_3, vs_4, vs_5) = (0, 23, 0, 2, 2)$, $(es_1, es_2, es_3, es_4, es_5) = (1, 1, 1, 2, 1)$. So by using Result 4, we compute the results by applying Result 1, Result 2 and Result 3.

Theorem 10. *Let* G_{10} *be Streptomycin compound. Then* $W(G_{10}) = 5024$, $W_e(G_{10}) = 4134$, $Sz_v(G_{10}) = 6744$, $Sz_e(G_{10}) = 5801$, $PI(G_{10}) = 1673$, $Mo(G_{10}) = 1178$, $Mo_e(G_{10}) = 1275$.

Proof. $|V(G_{10})| = 40$ and $|E(G_{10})| = 42$. Also G_{10} has one pentagon and the strength-weighted values of the quotient graph G_{10}/S are $(vw_1, vw_2, vw_3, vw_4, vw_5) = (4, 2, 1, 19, 14)$, $(vs_1, vs_2, vs_3, vs_4, vs_5) = (3, 1, 0, 19, 14)$, $(es_1, es_2, es_3, es_4, es_5) = (1, 1, 1, 1, 1)$.

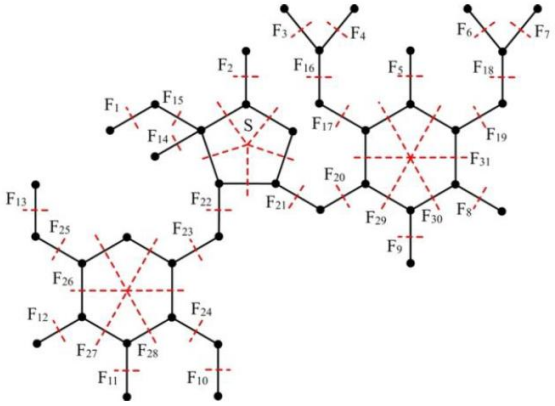


Figure 13. Various cuts of G_{10}.

Table 12. Strength-weighted values of G_{10}/F_i from Figure 13.

F_i	$1 \le i \le 14$	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
a_i	1	3	3	4	3	4	17	18	13	12	2	2	7	6	7	9	9	12
b_i	0	2	2	3	2	3	17	18	13	12	1	1	6	5	6	8	8	11
e_i	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2

By applying Result 1 and Result 3, followed by using Result 4, we compute the results.

Theorem 11. *Let* G_{11} *be Tetracycline compound. Then* $W(G_{11}) = 2263$, $W_e(G_{11}) = 1828$, $Sz_v(G_{11}) = 4832$, $Sz_e(G_{11}) = 4264$, $PI(G_{11}) = 1120$, $Mo(G_{11}) = 608$, $Mo_e(G_{11}) = 688$.

Proof. We have $|V(G_{11})| = 32$ and $|E(G_{11})| = 35$.

Table 13. Strength-weighted values of G_{11}/F_i from Figure 14.

F_i	$1 \le i \le 12$	13	14	15	16	17	18	19	20	21	22
a_i	1	3	8	10	15	16	10	9	3	4	15
b_i	0	2	7	9	16	17	10	9	2	3	14

e_i	1	1	2	2	2	2	2	2	2	2	5

By applying Result 1, we get the results

Theorem 12. *Let* G_{12} *be Chloramphenicol compound. Then* $W(G_{12}) = 880$, $W_e(G_{12}) = 589$, $Sz_v(G_{12}) = 1132$, $Sz_e(G_{12}) = 784$, $PI(G_{12}) = 374$, $Mo(G_{12}) = 240$, $Mo_e(G_{12}) = 252$.

Proof. Chloramphenicol compound has 20 vertices and 20 edges. By applying Result 1on following data, we obtain the results.

Table 14. Strength-weighted values of G_{12}/F_i from Figure 15.

F_i	$1 \le i \le 7$	8	9	10	11	12	13	14	15	16	17
a_i	1	3	9	9	2	6	5	3	6	6	6
b_i	0	2	9	8	1	5	4	2	5	5	5
e_i	1	1	1	1	1	1	1	1	2	2	2

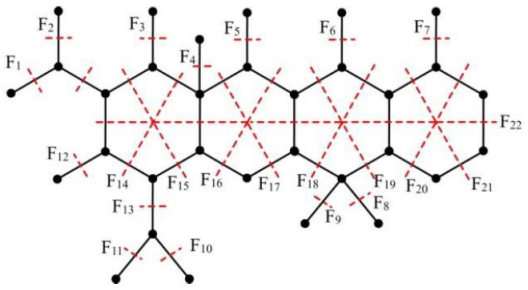


Figure 14. Various cuts of G_{11}.

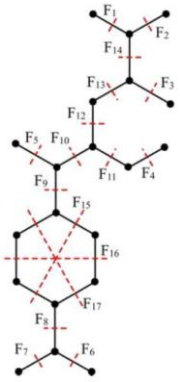


Figure 15. Various cuts of G_{12}.

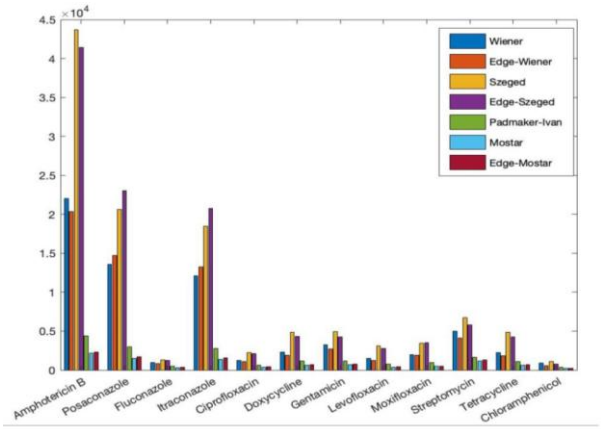


Figure 16. Graphical representation of the distance based indices of the drugs.

4. Degree Based Indices

Several degree based indices (given in Table 2) of the antifungal and antibacterial drug compounds under consideration are discussed in this section. For we use degree counting methods based on vertices and edges of the chemical structures. Edge partition of the drug compounds are detailed in Table 15 and the results are tabulated in Table 16.

Table 15. Edge Partition of the drug compounds.

	No. of edges							
Edge partition (d_u, d_v)	(1,2)	(1,3)	(1,4)	(2,2)	(2,3)	(2,4)	(3,3)	(3,4)
Amphotericin B	0	17	1	14	21	3	11	0
Posaconazole	1	5	0	13	26	3	8	1
Fluconazole	0	2	1	7	10	2	1	1
Itraconazole	1	4	0	13	26	3	7	1
Ciprofloxacin	0	4	0	5	10	0	8	0
Doxycycline	0	11	1	2	2	0	16	3

Gentamicin	2	6	2	2	13	1	8	1
Levofloxacin	0	6	0	3	10	0	10	0
Moxifloxacin	1	4	0	4	13	0	11	0
Streptomycin	3	10	1	0	14	1	11	2
Tetracycline	0	9	3	2	4	0	12	5
Chloramphenicol	1	6	0	2	7	0	4	0

Table 16. Degree based indices of the drug compounds.

Drugs	$M_1(G)$	$M_2(G)$	H(G)	HM(G)	F(G)	R(G)	RR(G)	SCI(G)	GA(G)	ABC (G)
Amphotericin B	318	360	28.97	1550	830	30.62	152.37	31.05	63.93	48.95
Posaconazole	278	333	24	1392	726	24.7	136	26.1	55.6	40.5
Fluconazole	116	135	10.19	576	306	10.57	56.08	11.02	23.21	17.25
Itraconazole	268	321	23.19	1340	698	23.81	130.98	25.17	53.7	38.98
Ciprofloxacin	134	164	11.17	682	354	11.56	65.42	12.24	26.26	19.21
Doxycycline	184	237	13.89	1006	532	14.87	88.34	15.51	33.26	25.28
Gentamicin	174	208	14.62	894	478	15.49	83.36	15.92	33.35	25.34
Levofloxacin	1892	2238	162.91	9516	5040	170.88	913.04	176.96	371.88	279.36
Moxifloxacin	166	207	13.53	858	444	13.99	81.19	14.88	32.14	23.33
Streptomycin	210	255	17.57	1092	582	18.71	100.61	19.1	39.93	30.38
Tetracycline	186	239	13.73	1028	550	14.77	88.71	15.42	33.06	25.42
Chloramphenicol	94	106	8.80	456	244	9.36	44.95	9.34	19	14.64

5. Conclusion

Human lives are being drastically affected by various infectious diseases. New drugs are to be discovered for various diseases. Topological indices open

ups a path to drug discovery. The study we made in this paper on the concepts of topological indices of various drugs recommended for treating Black fungus, White fungus and Black death are explained in section 3 and section 4. Hopefully this may help chemists in the further study and future developments in the area of drug discovery.

References

[1] M. Arockiaraj and A. J. Shalini, Extended cut method for edge Wiener, Schultz and Gutman indices with applications, MATCH Commun. Math. Comput. Chem. 76(1) (2016), 233-250.

[2] M. Arockiaraj J. Clement and K. Balasubramanian, Topological indices and their applications to circumcised donut benzenoid systems, kekulenes and drugs, Polycyclic Aromatic Compounds 40(2) (2020), 280-303.

[3] M. Arockiaraj, J. Clement, N. Tratnik, S. Mushtaq and K. Balasubramanian, Weighted Mostar indices as measures of molecular peripheral shapes with applications to graphene, graphyne and graphdiyne nanoribbons, SAR and QSAR in Environmental Research 31(3) (2020), 187-208.

[4] M. Arockiaraj, S. Klavžar, S. R. J. Kavitha, S. Mushtaq and K. Balasubramanian, Relativistic structural characterization of molybdenum and tungsten disulfide materials, International Journal of Quantum Chemistry 121(5) (2021), e26492.

[5] D. Bonchev, Chemical graph theory: introduction and fundamentals 1 CRC Press 4, (1991).

[6] E. Estrada, L. Torres, L. Rodriguez and I. Gutman, An atom-bond connectivity index: Modelling the enthalpy of formation of alkanes, (1998).

[7] S. Fajtlowicz, On Conjectures of Grafitti II, Congr. Number. 60 (1987), 189-197.

[8] O. Favaron, M. Mahéo and J. F. Saclé, Some eigenvalue properties in graphs (conjectures of Graffiti-II), Discrete mathematics 111(1-3) (1993), 197-220.

[9] B. Furtula and I. Gutman, A forgotten topological index, Journal of mathematical chemistry 53(4) (2015), 1184-1190.

[10] I. Gutman, Degree-based topological indices, Croatica Chemica Acta 86(4) (2013), 351-361.

[11] M. R. Kanna and R. Jagadeesh, Topological Indices of Paracetamol, International Journal of Pure and Applied Mathematics 117(13) (2017), 343-352.

[12] S. Klavzar, A bird's eye view of the cut method and a survey of its applications in chemical graph theory, MATCH Commun. Math. Comput. Chem 60(2) (2008), 255-274.

[13] A. K. KN, B. NS, S. MC and S. KC, Degree-Based Topological Indices on Asthma Drugs with QSPR Analysis during Covid-19, European Journal of Molecular and Clinical Medicine 7(10) (2020), 53-66.

[14] J. B. Liu, M. Arockiaraj, M. Arulperumjothi and S. Prabhu, Distance based and bond additive topological indices of certain repurposed antiviral drug compounds tested for treating COVID-19, International Journal of Quantum Chemistry 121(10) (2021), e26617.

[15] S. Mondal, N. De and A. Pal, Topological indices of some chemical structures applied for the treatment of COVID-19 Patients, Polycyclic Aromatic Compounds (2020), 1-15.

[16] M. Randic, Characterization of molecular branching, Journal of the American Chemical Society 97(23) (1975), 6609-6615.

[17] C. Raychaudhury and D. Pal, Discrimination of small molecules using topological molecular descriptors, In Advances in Mathematical Chemistry and Applications Bentham Science Publishers (2015), 55-73.

[18] G. H. Shirdel, H. Rezapour and A. M. Sayadi, The hyper-Zagreb index of graph operations, (2013).

[19] H. Van de Waterbeemd, R. E. Carter, G. Grassy, H. Kubinyi, Y. C. Martin, M. S. Tute and P. Willett, Glossary of terms used in computational drug design (IUPAC Recommendations 1997), Pure and applied chemistry 69(5) (1997), 1137-1152.

[20] D. Vukičević and B. Furtula, Topological index based on the ratios of geometrical and arithmetical means of end-vertex degrees of edges, Journal of Mathematical Chemistry 46(4) (2009), 1369-1376.

[21] H. Wiener, Structural determination of paraffin boiling points, Journal of the American Chemical Society 69(1) (1947), 17-20.

[22] B. Zhou and N. Trinajstić, On general sum-connectivity index, Journal of Mathematical Chemistry 47(1) (2010), 210-218.