Advances and Applications in Mathematical Sciences
Volume 20, Issue 4, February 2021, Pages 641-648

INVERSE ECCENTRIC DOMINATION IN GRAPHS

R. JAHIR HUSSAIN and A. FATHIMA BEGAM

PG and Research Department of Mathematics
Jamal Mohamed College (Autonomous)
Tiruchirappalli-620 020, Tamilnadu, India
E-mail: mathfab06@gmail.com

Abstract

In this paper, we discussed some results of inverse eccentric domination number of graphs in graph theory.

1. Introduction

Definition 1.1. A set $D \subseteq V(G)$ is an eccentric dominating set if *D* is a dominating set of *G* and also for every *v* in $V - D$ there exist at least one eccentric point of *v* in *D*. The eccentric dominating set is a minimal eccentric dominating set if no proper subset D' of *D* is an eccentric dominating set. The minimal eccentric dominating set with minimum cardinality is called as a minimum eccentric dominating set. The cardinality of minimum eccentric dominating set is known as eccentric domination number and is denoted by $\gamma_{ed}(G)$.

Definition 1.2. A spider is a tree on $2n + 1$ vertices obtained by subdividing each edge of a star $K_{1,n}$ where $n \geq 3$.

2. Inverse Eccentric Domination in Graphs

Definition 2.1. Let *D* be the minimum eccentric dominating set in a graph *G*. If $V - D$ contains an eccentric dominating set D' of *G* then D' is called as an inverse eccentric dominating set with respect to *D*.

2010 Mathematics Subject Classification: 03E72; 54H25; 47H10; 54A40.
Keywords: inverse eccentric dominating set; minimum inverse eccentric dominating set; inverse eccentric domination number.
Received January 21, 2020; Accepted May 13, 2020

Definition 2.2. An inverse eccentric dominating set D' is called a minimum inverse eccentric dominating set if D' consists of minimum number of vertices among all inverse eccentric dominating set.

Definition 2.3. The number of vertices in a minimum inverse eccentric dominating set is known as the inverse eccentric domination number of a graph G and is denoted by $\gamma_{ed}^{-1}(G)$.

Theorem 2.1. *Let D be a minimum eccentric dominating set of a graph G. Then there exist inverse eccentric domination number of G with respect to D iff every vertex in D has at least one eccentric vertex in* $V - D$.

Proof. Let D be a minimum eccentric dominating set of a graph G and also G has inverse eccentric domination number with respect to D. Let $D' = V - D$.

Then D' contains minimum eccentric dominating set. That is D' is also eccentric dominating set.

Therefore, every vertex in D has at least one eccentric vertex in D'.

Conversely, let D be a minimum eccentric dominating set of G and every vertex in D has at least one eccentric vertex in $V - D$.

We know that $V - D$ is also dominating set of G. Therefore, $V - D$ is an eccentric dominating set of G with respect to D.

$\Rightarrow V - D$ contains minimum eccentric dominating set D'. Then D' is the minimum inverse eccentric domination set with respect to D whose cardinality is the inverse eccentric domination number of G.

Theorem 2.2. $\gamma_{ed}^{-1}(K_n) = 1$ *for all* $n \geq 2$.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of K_n where $n \geq 2$. We know that $\gamma_{ed}(K_n) = 1$.

Let $D = \{v_1\}$ be the minimum eccentric dominating set of K_n.

Any vertex v_i in $V - D$ is a minimum inverse eccentric dominating set. That is $D' = \{v_i \in V - \{v_1\}\}$ and anyone v_i in D' is a minimum inverse eccentric dominating set of G with respect to D. Therefore, $\gamma_{ed}^{-1}(K_n) = 1$ for all $n \geq 2$.

Theorem 2.3. $\gamma_{ed}^{-1}(K_{m,n}) = 2$ *for all* $m \geq 2$ *and* $n \geq 2$.

Proof. Let $G = K_{m,n}$. Then $V(G) = V_1 \cup V_2$. $|V_1| = m$ and $|V_2| = n$ where $m \geq 2$ and $n \geq 2$. We know that $\gamma_{ed}(K_{m,n}) = 2$.

Let $D = \{u_1, v_1\}$ where $u_1 \in V_1$ and $v_1 \in V_2$ is a minimum eccentric dominating set. Then any vertex $u_i \in V_1 - \{u_1\}$ dominates all the vertices of V_2 and it is an eccentric vertex of all the vertices in $V_1 - \{u_i\}$. Similarly for $V_2 - \{v_1\}$.

Let $D' = \{u_i, v_i\}$ where $u_i \in V_1 - \{u_1\}$ and $v_i \in V_2 - \{v_1\}$. Then any two vertices $\{u_i, v_i\} \subseteq D'$ where $u_i \in V_1 - \{u_1\}$ and $v_i \in V_2 - \{v_1\}$ is a minimum inverse eccentric dominating set of G with respect to D. Therefore, $\gamma_{ed}^{-1}(K_{m,n}) = 2$ for all $m \geq 2$ and $n \geq 2$.

Theorem 2.4.

(i) $\gamma_{ed}^{-1}(W_4) = 1$

(ii) $\gamma_{ed}^{-1}(W_5) = 2$

(iii) $\gamma_{ed}^{-1}(W_6) = 3$

(iv) $\gamma_{ed}^{-1}(W_7) = 2$

(v) $\gamma_{ed}^{-1}(W_n) = \left\lceil \frac{n}{3} \right\rceil n \geq 8$ *and* $n \neq 3m + 1$ *where* $m \geq 3$.

$\gamma_{ed}^{-1}(W_n) = \frac{(n-1)}{3} n \geq 8$ *and* $n = 3m + 1$ *where* $m \geq 3$.

Proof.

(i) Let $G = W_4 = K_4$. Hence by Theorem 2.2 $\gamma_{ed}^{-1}(W_4) = 1$.

(ii) Let $G = W_5$. We know that $\gamma_{ed}(W_5) = 2$.

Let $D = \{u_1, u_2\}$ is a minimum eccentric dominating set. Then $D' = V - D = \{u_3, u_4, v\}$ where v is the central vertex of W_4. Consider $D'' = \{u_3, u_4\} \subseteq D'$ which is a minimum inverse eccentric dominating set of G with respect to D. Therefore, $\gamma_{ed}^{-1}(W_5) = 2$.

(iii) Let $G = W_6$. We know that $\gamma_{ed}(W_6) = 3$

Let $D = \{u_1, u_2, v\}$ where u_1 and u_2 are adjacent non-central vertices and v is a central vertex. Consider $D' = V - D = \{u_3, u_4, u_5\}$ which is a minimum inverse eccentric dominating set. Therefore, $\gamma_{ed}^{-1}(W_6) = 3$

(iv) Let $G = W_7$. We know that $\gamma_{ed}^{-1}(W_7) = 2$.

Let $D = \{u_1, u_4\}$ is a minimum eccentric dominating set. Then $D' = V - D = \{u_2, u_3, u_5, u_6, v\}$ where v is the central vertex. Consider $D'' = \{u_2, u_5\}$ where u_2 dominates u_1, u_3, v and u_5 dominates u_6, u_4, v and also where u_2 is an eccentric point of u_6, u_4, v and u_5 is an eccentric point of u_1, u_3, v. Therefore, D'' is the minimum inverse eccentric dominating set of G with respect to D. Hence $\gamma_{ed}^{-1}(W_7) = 2$.

(v) Let $G = W_n$ for $n \geq 8$. We know that $\gamma_{ed}(W_n) = 3$ for $n \geq 7$.

Let $D = \{u_1, u_2, v\}$ is a minimum eccentric dominating set of G where v is the central vertex and u_1, u_2 are adjacent non-central vertices. Then $D' = V - D = \{u_3, u_4, \ldots, u_{n-1}\}$. In W_n each vertex dominates two adjacent non-central vertices and the central vertex and also each non-central vertices is the eccentric vertex of all other non-adjacent non-central vertices and adjacent central vertex.

Case (i): n is even

(a) If $n = 2k = 3m + 2$ (m is even)

$\Rightarrow k = 3\frac{m}{2} + 1$

$\Rightarrow k = 3l + 1$ [since let $\frac{m}{2} = -1$]

Consider $D'' = \{u_3, u_6, \ldots, u_{k-1}, u_{k+2}, u_{k+5}, \ldots, u_{2k-2}, u_{2k-1}\}$ is a minimum inverse eccentric dominating set.

Therefore, $\gamma_{ed}^{-1}(W_n) = \left\lceil \frac{n}{3} \right\rceil$ when $n = 2k = 3m + 2$ (1)

(b) If $n = 2k = 3m + 1$ (m is odd)

$\Rightarrow 2k = 3(m - 1) + 4$

$\Rightarrow k = 3\frac{(m - 1)}{2} + 2$

$\Rightarrow k = 3l + 2$ [since let $\frac{(m - 1)}{2} = 1$].

Consider $D'' = \{u_3, u_6, \ldots, u_{k-2}, u_{k+1}, u_{k+4}, \ldots, u_{2k-4}, u_{2k-1}\}$ is a minimum inverse eccentric dominating set of G with respect to D.

Therefore, $\gamma_{ed}^{-1}(W_n) = \frac{(n - 1)}{3}$ when $n = 2k = 3m + 1$ (2)

(c) If $n = 2k = 3m$ (m is even)

$\Rightarrow k = 3l$ [since let $1 = \frac{m}{2}$]

Consider $D'' = \{u_3, u_6, \ldots, u_{k-3}, u_k, u_{k+3}, \ldots, u_{2k-3}, u_{2k-1}\}$ is a minimum inverse eccentric dominating set.

Therefore, $\gamma_{ed}^{-1}(W_n) = \left\lceil \frac{n}{3} \right\rceil$ when $n = 2k = 3m$ (3)

From (1), (3) and (2)

$\gamma_{ed}^{-1}(W_n) = \left\lceil \frac{n}{3} \right\rceil$ when n is even and $n \geq 8$ and $n \neq 3m + 1$ where $m \geq 3$.

$\gamma_{ed}^{-1}(W_n) = \frac{(n-1)}{3}$ when n is even and $n \geq 8$ and $n = 3m+1$ where $m \geq 3$. (4)

Case(ii): n is odd

(a) If $n = 2k+1 = 3m+1$ (m is even)

$\Rightarrow 2k = 3m$

$\Rightarrow k = 3l$ [since let $l = \frac{m}{2}$]

Consider $D'' = \{u_3, u_6, \ldots, u_{k-3}, u_k, u_{k+3}, \ldots, u_{2k-3}, u_{2k}\}$ is a minimum inverse eccentric dominating set of G with respect to D.

Therefore, $\gamma_{ed}^{-1}(W_n) = \frac{(n-1)}{3}$ when $n = 2k+1 = 3m+1$ (5)

(b) If $n = 2k+1 = 3m+2$ (m is odd)

$\Rightarrow 2k = 3(m-1)+4$

$\Rightarrow k = 3\frac{(m-1)}{2} + 2$

$\Rightarrow k = 3l + 2$ [since let $\frac{(m-1)}{2} = l$]

Consider $D'' = \{u_3, u_6, \ldots, u_{k-2}, u_{k+1}, u_{k+4}, \ldots, u_{2k-4}, u_{2k-1}, u_{2k}\}$ is a minimum inverse eccentric dominating set of G with respect to D.

Therefore, $\gamma_{ed}^{-1}(W_n) = \left\lceil \frac{n}{3} \right\rceil$ when $n = 2k+1 = 3m+2$ (6)

(c) If $n = 2k+1 = 3m$ (m is odd)

$\Rightarrow 2k = 3(m-1)+2$

$\Rightarrow k = 3\frac{(m-1)}{2} + 1$

$\Rightarrow k = 3l + 1$ [since let $\frac{(m-1)}{2} = l$]

Consider $D'' = \{u_3, u_6, \ldots, u_{k-4}, u_{k-1}, u_{k+2}, \ldots, u_{2k-5}, u_{2k-2}, u_{2k}\}$ is a minimum inverse eccentric dominating set of G with respect to D.

Therefore, $\gamma_{ed}^{-1}(W_n) = \left\lceil \frac{n}{3} \right\rceil$ when $n = 2k + 1 = 3m$ (7)

From (6), (7) and (5)

$\gamma_{ed}^{-1}(W_n) = \left\lceil \frac{n}{3} \right\rceil$ when n is odd and $n \geq 8$ and $n \neq 3m + 1$ where $m \geq 3$.

$\gamma_{ed}^{-1}(W_n) = \frac{(n-1)}{3}$ when n is odd and $n \geq 8$ and $n = 3m + 1$ where $m \geq 3$]. (8)

From (4) and (8)

$\gamma_{ed}^{-1}(W_n) = \left\lceil \frac{n}{3} \right\rceil n \geq 8$ and $n \neq 3m + 1$ where $m \geq 3$.

$\gamma_{ed}^{-1}(W_n) = \frac{(n-1)}{3} n \geq 8$ and $n = 3n + 1$ where $m \geq 3$.

Theorem 2.5. *For a spider graph* T, $\gamma_{ed}^{-1}(T) = n - \Delta(T) - 1$ *when* $n = 2k + 1 \geq 9$ *is the number of vertices of T.*

Proof.

Le T be a spider graph with $n = 2k + 1 \geq 9$. Then $r(T) = 2$ and $diam\ (T) = 4$.

We know that $\gamma_{ed}(T) = n - \Delta(T) - 1 = k = |N(u)|$ where u is the central vertex of T.

Let D be a minimum eccentric dominating set containing $k - 2$ vertices of $N(u)$ and 2 pendent vertices of T which are not adjacent to the vertex which we have selected from $N(u)$ to form D. Then $D' = V - D$ contains remaining $k - 2$ pendent vertices, and 2 vertices from $N(u)$ and the central vertex u.

Let D'' be the subset of D' containing $k - 2$ pendent vertices and 2 vertices of $N(u)$. Then D'' is the minimum inverse eccentric dominating set of T with respect to D.

Therefore, $\gamma_{ed}^{-1}(T) = |D''| = k - 2 + 2 = k = n - \Delta(T) - 1$.

References

[1] F. Buckley and F. Harary, Distance in graphs, Addison-Wesley Publishing Company, 1990.

[2] T. N. Janakiraman, M. Bhanumathi and S. Muthammai, Eccentric domination in graphs, International Journal of Engineering Science, Advanced Computing and Bio-technology 1(2) (2010), 55-70.

[3] T. N. Janakiraman, On some Eccentricity properties of the graphs, Thesis, Madras University, Tamil Nadu, India, 1991.

[4] V. R. Kulli and S. C. Sigarkanti, Inverse domination in graphs, Nat. Acad. Sci. Lett. 14 (1991), 473-475.

[5] V. R. Kulli, Theory of Domination in Graphs, Vishwa International Publications, 2010.