Advances and Applications in Mathematical Sciences
Volume 21, Issue 6, April 2022, Pages 3057-3064

RADIO MEAN GRACEFUL LABELING ON DEGREE SPLITTING OF CYCLE RELATED GRAPHS

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Abstract

A Radio Mean Labeling of a connected graph G is a injection φ from the vertex set $V(G)$ to N such that for any two distinct vertices u and v of G satisfying the condition $d(u, v) + \lceil(\varphi(u) + \varphi(v))/2\rceil \geq 1 + diam(G)$. A graph which admits radio mean labeling is called radio mean graph. The radio mean number, $rmn(\varphi)$, is the maximum number assigned to any vertex of G. The radio mean number of G, $rmn(G)$ is the minimum value of $rmn(\varphi)$ taken over all radio mean labeling φ of G. If $rmn(G) = |V(G)|$, then G is known as radio mean graceful. In this paper, we investigate the radio mean graceful labeling on degree splitting of C_n, $C_n + K_1$ and $A(C_{2n})$.

2020 Mathematics Subject Classification: 05C78.
Keywords: Radio mean graceful graph, Cycle graph, Wheel graph, Alternate triangular cycle graph.
Received June 24, 2021; Accepted September 24, 2021

1. Introduction

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Chartrand et al. developed the concept of radio labeling in [1]. S. Somasundaram and Ponraj introduce the notion of mean labeling of graphs in [9]. Radio mean labeling was introduced by Ponraj et al. in [7]. E. Sampathkumar and H. B. Walikar introduced the notion of the splitting graph of a graph in [8]. R. Ponraj and S. Somasundaram developed the concept of degree splitting of graphs in [6]. S. Somasundaram, S. S. Sandhya and S. P. Viji, introduced the concept of Geometric mean labeling on Degree splitting graphs in [10]. K. N. Meera found the notion of Radio Geometric graceful graphs in [5]. Y. Lavanya, Dhanyashree and K. N. Meera raised the ideas about Radio Mean Graceful Graphs in [4]. In this paper we investigated the graceful radio mean labeling on degree splitting of cycle and wheel related graphs. Throughout this paper we consider simple, undirected, finite and connected graphs. $\lceil x \rceil$ is the smallest integer greater than or equal to x, for any real x. For theoretic terminology, we refer to Harary [3] and for a detailed survey of graph labeling we refer to Gallian [2]. The notations $V(G)$ is the vertex set of G, $E(G)$ is the edge set of G, $DS(G)$ is the degree splitting of G, $d(u, v)$ is the distance between the vertices u and v, $\text{diam}(G)$ is the diameter of G, and $|V|$ is the order of a graph G.

Definition 1. The sum of two graphs G and H, denoted by $G + H$, is the graph obtained by taking disjoint copies of G and H and then adding every edge xy, where x is a vertex in G and y is a vertex in H.

Definition 2. An alternate triangular cycle $A(C_{2n})$ is obtained from an even cycle C_{2n} with vertex set $\{v_1, u_1, v_2, u_2, \ldots, v_n, u_n\}$ by joining v_i and u_i to a new vertex w_i, $1 \le i \le n$ i.e. every alternate edge of a cycle is replaced by C_3.

Definition 3. The graph $S(G)$, obtained from G, by taking a new vertex v' for every vertex $v \in V$ and joining v' to all vertices of G adjacent to v, is called a Splitting graph of G.

Definition 4. Let $G = (V, E)$ be a graph with $V = S_1 \cup S_2 \cup \ldots,$

$S_i \cup T$ where each S_i is a set of vertices having at least two vertices and having the same degree and $T = V - \cup S_i$. The degree splitting graph of G denoted by $DS(G)$ is obtained from G by adding $w_1, w_2, \ldots, w_t$ and joining w_i to each vertex of S_i, $1 \leq i \leq t$.

Definition 6. A radio mean graceful labeling of a connected graph G is bijection φ from $V(G)$ to $\{1, 2, 3, \ldots, |V(G)|\}$ such that for any two distinct vertices u and v of G satisfying the condition, $d(u, v) + \lceil(\varphi(u) + \varphi(v))/2\rceil \geq 1 + \text{diam}(G)$. A graph which admits radio mean graceful labeling is called radio mean graceful graph.

2. Main Result

Theorem 2.1. *$DS(C_n)$ is a radio mean graceful graph.*

Proof. Let $s_1, s_2, \ldots, s_n$ be the vertices of cycle C_n. Introduce a new vertex t and join it with the vertices of C_n graph. Then the new graph obtained is $DS(C_n)$, whose vertex set is $V = \{s_i, 1 \leq i \leq n\} \cup \{t\}$. Clearly the

$$\text{diam}(DS(C_n)) = \begin{cases} 1, & n = 3 \\ 2, & n > 3 \end{cases}$$

Define a bijection $\varphi : V(DS(C_n)) \to \{1, 2, \ldots | V(DS(C_n))|\}$ by

$\varphi(s_i) = i + 1, 1 \leq i \leq n,$

$\varphi(t) = 1.$

To check the radio mean graceful condition for φ,

Case 1: $n = 3$

Subcase (i): Examine the pair (s_i, s_j), $1 \leq i \leq n - 1$, $i + 1 \leq j \leq n$;

$$d(s_i, s_j) + \lceil(\varphi(s_i)) + (\varphi(s_j))/2\rceil \geq 1 + \lceil(i + j + 2)/2\rceil \geq 2 = 1 + \text{diam}(\text{DS}(\text{C}_\text{n})).$$

Subcase (ii): Examine the pair (s_i, t), $1 \leq i \leq n$;

$$d(s_i, t) + \lceil(\varphi(s_i)) + (\varphi(t))/2\rceil \geq 1 + \lceil(i + 2)/2\rceil \geq 2$$

Case 2: $n > 3$

Subcase (i): Examine the pair $(s_i, s_j), 1 \leq i \leq n-1, i+1 \leq j \leq n;$

$$d(s_i, s_j) + \lceil(\varphi(s_i)) + (\varphi(s_i))/2\rceil \geq 1 + \lceil(i + j + 2)/2\rceil \geq 3 = 1 + diam(DS(\mathrm{C_n})).$$

Subcase (ii): Examine the pair $(s_i, t), 1 \leq i \leq n;$

$$d(s_i, t) + \lceil(\varphi(s_i)) + (\varphi(t))/2\rceil \geq 1 + \lceil(i + 2)/2\rceil \geq 3.$$

Thus, all the pair of vertices satisfies the radio mean graceful condition. Hence $DS(C_n)$ is a radio mean graceful graph.

Example 2.2.

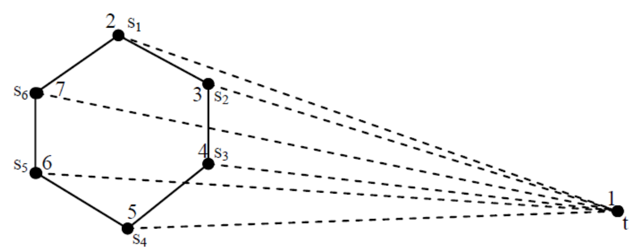

Figure 1. Radio Mean Graceful Labeling of $DS(C_6)$.

Theorem 2.3. $DS(C_n + K_1)$ *is a radio mean graceful graph.*

Proof. Let $s_1, s_2, \ldots, s_n$ be the vertices of cycle C_n. Let t be the vertex of K_1. Joining each vertices of the cycle C_n with t, results in $C_n + K_1$ graph. Introduce a new vertex u and join it with the vertices of $C_n + K_1$ graph of degree three. The new graph so obtained is $DS(C_n + K_1)$, whose vertex set is $V = \{s_i, 1 \leq i \leq n-1\} \cup \{u\}$. Clearly the diameter of $DS(C_n + L_1)$ $= \begin{cases} 1, & n = 3 \\ 2, & n > 3 \end{cases}$

Define a bijection $\varphi : V(DS(C_n + K_1)) \to \{1, 2, \ldots |V(DS(C_n + K_1))|\}$ by

$\varphi(s_i) = i, 1 \leq i \leq n,$

$\varphi(t) = n + 1,$

$\varphi(u) = n + 2.$

To check the radio mean graceful condition for φ,

Case 1. $n = 3$

Subcase (i): Examine the pair (s_i, s_j), $1 \leq i \leq n-1$, $i+1 \leq j \leq n$;

$d(s_i, s_j) + \lceil (\varphi(s_i)) + (\varphi(s_j)/2 \rceil \geq 1 + \lceil (i + j + 2)/2 \rceil \geq 2 = 1$

$+ diam(DS(C_n + K_1)).$

Subcase (ii): Examine the pair (s_i, t), $1 \leq i \leq n$,

$d(s_i, t) + \lceil (\varphi(s_i)) + (\varphi(t))/2 \rceil \geq 1 + \lceil (i+2)/2 \rceil \geq 2.$

Subcase (iii): Examine the pair (s_i, u), $1 \leq i \leq n$,

$d(s_i, u) + \lceil (\varphi(s_i)) + (\varphi(u))/2 \rceil \geq 1 + \lceil (n + i + 2)/2 \rceil \geq 2.$

Subcase (iv): Examine the pair (u, t),

$d(u, t) + \lceil (\varphi(u)) + (\varphi(t))/2 \rceil \geq 1 + \lceil (2n + 3)/2 \rceil \geq 2$

Case 2. $n > 3$

Subcase (i): Examine the pair (s_i, s_j), $1 \leq i \leq n-1$, $i+1 \leq j \leq n$,

$d(s_i, s_j) + \lceil (\varphi(s_i)) + (\varphi(s_i))/2 \rceil \geq 1 + \lceil (i + j)/2 \rceil \geq 2 = 1$

$+ diam(DS(C_n + K_1)).$

Subcase (ii): Examine the pair (s_i, t), $1 \leq i \leq n$;

$d(s_i, t) + \lceil (\varphi(s_i)) + (\varphi(t))/2 \rceil \geq 1 + \lceil (n + i + 1)/2 \rceil \geq 3.$

Subcase (iii): Examine the pair (s_i, u), $1 \leq i \leq n$,

$d(s_i, u) + \lceil (\varphi(s_i)) + (\varphi(u))/2 \rceil \geq 1 + \lceil (n + i + 2)/2 \rceil \geq 3.$

Subcase (iv): Examine the pair (u, t);

$d(u, t) + \lceil (\varphi(u)) + (\varphi(t))/2 \rceil \geq 1 + \lceil (2n + 3)/2 \rceil \geq 3.$

Thus, all the pair of vertices satisfies the radio mean graceful condition.

Hence $DS(C_n + K_1)$ is a radio mean graceful graph.

Example 2.4.

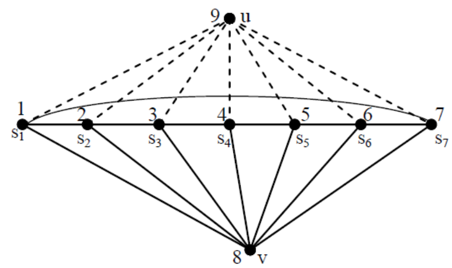

Figure 2. Radio Mean Graceful Labeling of $DS(C_n + K_1)$.

Theorem 2.5. *$DS(A(C_{2n}))$ is a radio mean graceful graph.*

Proof. Let $s_1, s_2, \ldots, s_{2n}$ be the vertices of even cycle C_{2n}. Join s_{2i-2}, s_{2i-1} to $t_{i-1}, 2 \leq i \leq n$ and s_1, s_{2n} to t_n. The resultant graph is $A(C_{2n})$. Introduce two new vertices u and v and join them with the vertices of $A(C_{2n})$ graph of degree three and two respectively. The new graph so obtained is $DS(A(C_{2n}))$ whose vertex set is $V = \{s_i, 1 \leq i \leq 2n, t_i, 1 \leq i \leq n\} \cup \{u, v\}$. Clearly the diameter of $DS(A(C_{2n})) = 3$.

Define a bijection $\varphi : V(DS(A(C_{2n}))) \to \{1, 2, \ldots | V(DS(A(C_{2n})))|\}$ by

$\varphi(t_i) = i, 1 \leq i \leq n,$

$\varphi(s_i) = n + 1 + 1, 1 \leq i \leq 2n,$

$\varphi(u) = n + 1,$

$\varphi(v) = 3n + 2.$

To check the radio mean graceful condition for φ,

Case (i). Examine the pair $(s_i, s_j), 1 \leq i \leq 2n - 1, i + 1 \leq j \leq 2n,$

$d(s_i, s_j) + \lceil(\varphi(s_i) + \varphi(s_i))/2\rceil \geq 1 + \lceil(2n + i + j + 2)/2\rceil \geq 4 = 1$

$+ diam(DS(C_n + K_1)).$

Case (ii). Examine the pair (s_i, u), $1 \le i \le 2n$,

$$d(s_i, u) + \lceil(\varphi(s_i)) + (\varphi(u))/2\rceil \ge 1 + \lceil(2n + i + 2)/2\rceil \ge 4.$$

Case (iii). Examine the pair (s_i, v), $1 \le i \le 2n$,

$$d(s_i, v) + \lceil(\varphi(s_i)) + (\varphi(v))/2\rceil \ge 2 + \lceil(4n + i + 3)/2\rceil \ge 4.$$

Case (iv). Examine the pair (t_i, t_j), $1 \le i \le n - 1$, $i + 1 \le j \le n$,

$$d(t_i, t_j) + \lceil(\varphi(t_i)) + (\varphi(t_j))/2\rceil \ge 2 + \lceil(i + j)/2\rceil \ge 4.$$

Case (v). Examine the pair (t_i, u), $1 \le i \le n$,

$$d(t_i, u) + \lceil(\varphi(t_i)) + (\varphi(u))/2\rceil \ge 2 + \lceil(n + i + 1)/2\rceil \ge 4.$$

Case (vi). Examine the pair (t_i, v), $1 \le i \le n$,

$$d(t_i, v) + \lceil(\varphi(t_i)) + (\varphi(v))/2\rceil \ge 1 + \lceil(3n + i + 2)/2\rceil \ge 4.$$

Case (vii). Examine the pair (u, v),

$$d(u, v) + \lceil(\varphi(u)) + (\varphi(v))/2\rceil \ge 3 + \lceil(4n + 3)/2\rceil \ge 4.$$

Case (viii). Examine the pair (s_i, t_j), $1 \le i \le 2n$, $1 \le j \le n$,

$$d(s_i, t_j) + \lceil(\varphi(s_i)) + (\varphi(t_j)/2\rceil \ge 1 + \lceil n + (i + j + 1)/2\rceil \ge 4.$$

Thus, all the pair of vertices satisfies the radio mean graceful condition. Hence $DS(A(C_{2n}))$ is a radio mean graceful graph.

Example 2.6.

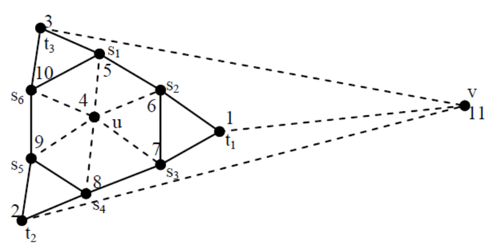

Figure 3. Radio Mean Graceful Labeling of $DS(A(C_{2n}(3)))$.

3. Conclusion

The labeling of graphs is an interesting and vast research area which is very useful and it is extended in various topics by several people. Radio mean graceful labeling is discussed in this paper and some of the results are obtained. More results will be done in the further research article. This paper will help the beginners in research.

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