



SOME NEW RESULTS ON FERMATEAN FUZZY SETS USING IMPLICATION

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Abstract

Fermatean fuzzy sets, involving membership, non membership and hesitancy issues gift mathematically a conventional structure. Owing to these deliberation, it is possible to define several execution of these sets. In the actual information ten distinct operations on such sets are defined. These ten operations on Fermatean fuzzy sets bear interesting properties. In this paper, we have identified and proved various properties, especially those involving the operation $A \rightarrow B$ defined as Fermatean fuzzy implication with other operations.

Introduction

Senapati and Yager [11] outlined basic operation over FFSs and introduced new score functions and accuracy function of FFSs. The generalization of FFSs is the sum of the cubes of the values of the degree of membership and non-membership is not exceeding one.

The idea of an intuitionistic fuzzy matrix (IFM) was presented by Khan et al. [5] also, all the while Im et al. [4] to sum up the idea of Thomason's [12]

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fuzzy matrix. Every component in an intuitionistic fuzzy matrix is communicated by an arranged pair $(\langle \alpha_{a_{ij}}, \beta_{a_{ij}} \rangle)$ with $\alpha_{a_{ij}}, \beta_{a_{ij}} \in [0, 1]$.

Khan and Pal [6] characterized some fundamental tasks and relations of IFMs including maxmin, minmax, supplement, logarithmic whole, arithmetical item and so on and demonstrated equity between IFMs. After the presentation of IFM hypothesis, numerous scientists endeavoured the significant part in IFM hypothesis [3, 8, 9].

For example Let A be the fuzzy set given by $\langle 0.8, 0.65 \rangle$ where the degree of the membership is 0.8 and the degree of the non-membership is 0.65. The sum of the values of the degree of membership and non-membership is $1.45 > 1$. This shows that A is not an Intuitionistic fuzzy set. Consider the sum of the squares of the values of the degree of membership and non-membership is $(0.8)^2 + (0.65)^2 > 1$. exceeding one which shows that A is not a Pythagorean fuzzy set. Consider the sum of the cubes of the values of the degree of membership and non-membership is $(0.8)^3 + (0.65)^3 < 1$. Hence A is known as Fermatean Fuzzy set. FFSs is most effective than PFSs and IFSs.

This is to say that the FFSs have extra unpredictability than IFSs and PFSs, and are skilled to deal with more elevated levels of unpredictability.

The paper is characterized as follows:

In section 2 some basic definitions associated with IFS and FFS theory are conferred and giving the implication operator in Fermatean Fuzzy Sets.

In section 3 we define Fermatean fuzzy implication operator and new results associated with the standard Fermatean fuzzy implication operator are proved.

2. Preliminaries

In this section some definitions of Intuitionistic Fuzzy sets theory and Fermatean fuzzy set theory are given.

Let $Y = \{y_1, y_2, \dots, y_n\}$ be a finite universe of discourse

Definition 2.1 [1]. An intuitionistic fuzzy set in Y is given as

$A = \{\langle y, \alpha_A(y), \beta_A(y) \rangle / y \in Y\}$ where $\alpha_A : Y \rightarrow [0, 1]$ and $\beta_A : Y \rightarrow [0, 1]$ such that $0 \leq \alpha_A(y) + \beta_A(y) \leq 1$. The degree of membership is denoted as $\alpha_a(y)$ and non-membership is denoted as $\beta_A(y)$ of $y \in Y$ to A .

Definition 2.2 [12]. A Fermatean fuzzy set A is defined as $A = \{\langle y, \alpha_A(y), \beta_A(y) \rangle \mid y \in Y\}$ where $\alpha_A : Y \rightarrow [0, 1]$ and $\beta_A : Y \rightarrow [0, 1]$ with the condition $0 \leq (\alpha_A(y))^3 + (\beta_A(y))^3 \leq 1$ for all $y \in Y$.

The numbers $\alpha_A(y)$ and $\beta_A(y)$ denote the degree of membership and non-membership of the element y in A .

Definition 2.3. Let $FFS(Y)$ denote the family of Fermatean Fuzzy Sets on the universe Y .

Let $A, B \in FFS(Y)$ is given as $A = \{\langle y, \alpha_A(y), \beta_A(y) \rangle \mid y \in Y\}$.

$B = \{\langle y, \alpha_B(y), \beta_B(y) \rangle \mid y \in Y\}$. Then the following operators on FFS is defined for $y \in Y$ as

a. $A \cup B = \{\langle y, \max(\alpha_A(y), \alpha_B(y)), \min(\beta_A(y), \beta_B(y)) \rangle / y \in Y\}$.

b. $A \cap B = \{\langle y, \min(\alpha_A(y), \alpha_B(y)), \max(\beta_A(y), \beta_B(y)) \rangle / y \in Y\}$.

c. $A^c = \{\langle y, \beta_A(y), \alpha_A(y) \rangle / y \in Y\}$.

d. $A \oplus_F B =$

$$\{\langle y, \sqrt[3]{(\alpha_A(y))^3 + (\alpha_B(y))^3 - (\alpha_A(y))^3(\alpha_B(y))^3}, \beta_A(y)\beta_B(y) \rangle / y \in Y\}$$

e. $A \odot_F B =$

$$\{\langle y, \alpha_A(y)\alpha_B(y), \sqrt[3]{(\beta_A(y))^3 + (\beta_B(y))^3 - (\beta_A(y))^3(\beta_B(y))^3} \rangle / y \in Y\}$$

f. $A @ B = \left\{ \left\langle y, \sqrt[3]{\frac{(\alpha_A(y))^3 + (\alpha_B(y))^3}{2}}, \sqrt[3]{\frac{(\beta_A(y))^3 + (\beta_B(y))^3}{2}} \right\rangle / y \in Y \right\}$.

g. $A \$ B = \{\langle y, \sqrt[3]{\alpha_A(y) + \alpha_B(y)}, \sqrt[3]{\beta_A(y)\beta_B(y)} \rangle / y \in Y\}$.

$$\text{h. } A \# B = \left\{ \langle y, \frac{\sqrt{2}\alpha_A(y)\alpha_B(y)}{\sqrt[3]{(\alpha_A(y))^3 + (\alpha_B(y))^3}}, \frac{\sqrt{2}\beta_A(y)\beta_B(y)}{\sqrt[3]{(\beta_A(y))^3 + (\beta_B(y))^3}} / y \in Y \rangle \right\}.$$

$$\text{i. } A \rightarrow B = \{ \langle y, \min\{\alpha_B(y), \beta_A(y)\}, \max\{\alpha_A(y)\beta_B(y)\} / y \in Y \rangle \}. \quad [7]$$

Lemma 2.4. [2]. For $\alpha, \beta \in [0, 1]$ then $\alpha \cdot \beta \leq \min(\alpha, \beta) \leq \frac{2(\alpha \cdot \beta)}{\alpha + \beta} \leq \sqrt[3]{\alpha\beta}$
 $\leq \max(\alpha, \beta) \leq \alpha + \beta - \alpha \cdot \beta, \alpha\beta \leq \frac{\alpha + \beta}{2(\alpha + \beta + 1)} \leq \frac{\alpha + \beta}{2}.$

3. Results on Fermatean Fuzzy Implication Operator

In this section we state and demonstrate some new outcomes using ' \rightarrow ' [7]

Theorem 3.1. For $A, B \in \text{FFS}(Y)$

- a. $(A^c \rightarrow B) @ (A \rightarrow B^c)^c = A @ B.$
- b. $(A^c \rightarrow B) \oplus_F (A \rightarrow B^c)^c = A \oplus_F B.$
- c. $(A^c \rightarrow B) \odot_F (A \rightarrow B^c)^c = A \odot_F B.$
- d. $(A^c \rightarrow B) \$ (A \rightarrow B^c)^c = A \$ B.$
- e. $(A^c \rightarrow B) \# (A \rightarrow B^c)^c = A \# B.$
- f. $(A \rightarrow B)^c @ (B \rightarrow A) = A @ B^c.$
- g. $(A \rightarrow B)^c \oplus_F (B \rightarrow A) = A \oplus_F B^c.$
- h. $(A \rightarrow B)^c \odot_F (B \rightarrow A) = A \odot_F B^c.$
- i. $(A \rightarrow B)^c \$ (B \rightarrow A) = A \$ B^c.$
- j. $(A \rightarrow B)^c \# (B \rightarrow A) = A \# B^c.$

Proof. To prove (a):

Using the Definition 2.3 and Lemma 2.4

$$(A^c \rightarrow B) @ (A \rightarrow B^c)^c = \left(y, \sqrt[3]{\frac{(\alpha_A(y))^3 + (\alpha_B(y))^3}{2}}, \sqrt[3]{\frac{(\beta_A(y))^3 + (\beta_B(y))^3}{2}} / y \in Y \right) = A @ B.$$

To prove (f):

$$(A \rightarrow B)^c @ (B \rightarrow A) = \left(y, \sqrt[3]{\frac{(\alpha_A(y))^3 + (\beta_B(y))^3}{2}}, \sqrt[3]{\frac{(\alpha_A(y))^3 + (\beta_B(y))^3}{2}} / y \in Y \right) = A @ B^c.$$

Similarly we can prove (b), (c), (d), (e), (g), (h), (i) and (j).

Theorem 3.2. For any $A, B \in FFS(y)$

- a. $(A \oplus_F B) \rightarrow (A @ B)^c)^c = ((A @ B) \rightarrow (A \oplus_F B)^c)^c = A \oplus_F B.$
- b. $((A \oplus_F B)^c \rightarrow (A @ B)) = ((A @ B)^c \rightarrow (A \oplus_F B)) = A @ B.$
- c. $((A \odot_F B) \rightarrow (A @ B)^c)^c = ((A @ B) \rightarrow (A \odot_F B)^c)^c = A @ B.$
- d. $((A \odot_F B)^c \rightarrow (A @ B)) = ((A @ B)^c \rightarrow (A \odot_F B)) = A \odot_F B.$
- e. $(A \oplus_F B) \rightarrow (A \# B)^c)^c = ((A \# B) \rightarrow (A \oplus_F B)^c)^c = A \oplus_F B.$
- f. $((A \oplus_F B)^c \rightarrow (A \# B)) = ((A \# B)^c \rightarrow (A \oplus_F B)) = A \# B.$
- g. $(A \odot_F B) \rightarrow (A \# B)^c)^c = ((A \# B) \rightarrow (A \odot_F B)^c)^c = A \# B.$
- h. $((A \odot_F B)^c \rightarrow (A \# B)) = ((A \# B)^c \rightarrow (A \odot_F B)) = A \odot_F B.$
- i. $(A \oplus_F B) \rightarrow (A \$ B)^c)^c = ((A \$ B) \rightarrow (A \oplus_F B)^c)^c = A \oplus_F B.$

$$j. ((A \oplus_F B)^c \rightarrow (A \$ B)) = ((A \$ B)^c \rightarrow (A \oplus_F B)) = A \$ B.$$

$$k. (A \odot_F B) \rightarrow (A \$ B)^c = ((A \$ B) \rightarrow (A \odot_F B)^c)^c = A \$ B.$$

$$l. ((A \odot_F B)^c \rightarrow (A \$ B)) = ((A \$ B)^c \rightarrow (A \odot_F B)) = A \odot_F B.$$

$$m. ((A \odot_F B)^c \rightarrow (A \oplus_F B)) = ((A \oplus_F B)^c \rightarrow (A \odot_F B)) = A \odot_F B.$$

Proof. Using the Definition (2.3) and Lemma (2.4), to prove (a):

$$((A \oplus_F B) \rightarrow (A @ B)^c)^c =$$

$$\left(y, \sqrt[3]{(\alpha_A(y))^3 + (\alpha_B(y))^3 - (\alpha_A(y))^3(\alpha_B(y))^3}, \beta_A(y)\beta_B(y)/y \in Y \right) \\ = A \oplus_F B \quad (1)$$

$$((A @ B) \rightarrow (A \oplus_F B)^c)^c =$$

$$\left(y, \sqrt[3]{(\alpha_A(y))^3 + (\alpha_B(y))^3 - (\alpha_A(y))^3(\alpha_B(y))^3}, \beta_A(y)\beta_B(y)/y \in Y \right) \\ = A \oplus_F B \quad (2)$$

From (1) and (2), we obtain the result (a).

To prove (b):

$$((A \oplus_F B)^c \rightarrow (A @ B)) =$$

$$\left(y, \sqrt[3]{\frac{(\alpha_A(y))^3 + (\alpha_B(y))^3}{2}}, \sqrt[3]{\frac{(\beta_A(y))^3 + (\beta_B(y))^3}{2}}/y \in Y \right) \\ = A @ B \quad (3)$$

$$((A @ B) \rightarrow (A \oplus_F B)) =$$

$$\left(y, \sqrt[3]{\frac{(\alpha_A(y))^3 + (\alpha_B(y))^3}{2}}, \sqrt[3]{\frac{(\beta_A(y))^3 + (\beta_B(y))^3}{2}}/y \in Y \right) \\ = A @ B \quad (4)$$

From (3) and (4), we obtain the result (b).

To prove (c):

$$\begin{aligned}
 & ((A \odot_F B) \rightarrow (A @ B)^c)^c = \\
 & \left(y, \sqrt[3]{\frac{(\alpha_A(y))^3 + (\alpha_B(y))^3}{2}}, \sqrt[3]{\frac{(\beta_A(y))^3 + (\beta_B(y))^3}{2}} / y \in Y \right) \\
 & = A @ B \tag{5}
 \end{aligned}$$

$$\begin{aligned}
 & ((A @ B) \rightarrow (A \odot_F B)^c)^c = \\
 & \left(y, \sqrt[3]{\frac{(\alpha_A(y))^3 + (\alpha_B(y))^3}{2}}, \sqrt[3]{\frac{(\beta_A(y))^3 + (\beta_B(y))^3}{2}} / y \in Y \right) \\
 & = A @ B \tag{6}
 \end{aligned}$$

From (5) and (6), we obtain the result (c).

To prove (d):

$$\begin{aligned}
 & ((A \odot_F B)^c \rightarrow (A @ B)) = \\
 & (y, \alpha_A(y)\alpha_B(y), \sqrt{(\beta_A(y))^3 + (\beta_A(y))^3 - (\beta_A(y))^3(\beta_A(y))^3} / y \in Y) \\
 & = A \odot_F B \tag{7}
 \end{aligned}$$

$$\begin{aligned}
 & ((A @ B) \rightarrow (A \odot_F B)^c) = \\
 & (y, \alpha_A(y)\alpha_B(y), \sqrt[3]{(\beta_A(y))^3 + (\beta_A(y))^3 - (\beta_A(y))^3(\beta_A(y))^3} / y \in Y) \\
 & = A \odot_F B \tag{8}
 \end{aligned}$$

From (7) and (8), we obtain the result (d).

To prove (e):

$$\begin{aligned}
 & ((A \oplus_F B)^c \rightarrow (A \# B)^c) = \\
 & (y, \sqrt[3]{(\alpha_A(y))^3 + (\alpha_A(y))^3 - (\alpha_A(y))^3(\alpha_A(y))^3}, \beta_A(y)\beta_B(y) / y \in Y)
 \end{aligned}$$

$$= A \oplus_F B \quad (9)$$

$$\begin{aligned} & ((A \# B) \rightarrow (A \oplus_F B)^c)^c = \\ & (y, \sqrt[3]{(\alpha_A(y))^3 + (\alpha_B(y))^3 - (\alpha_A(y))^3(\alpha_B(y))^3}, \beta_A(y)\beta_B(y)/y \in Y) \\ & = A \oplus_F B \end{aligned} \quad (10)$$

From (9) and (10), we obtain the result (e).

In a similar manner, we can prove the results (f), (g), (h), (i), (j), (k), (l) and (m).

Corollary 3.3. For any $AB \in FFS(y)$

$$\begin{aligned} & ((A \odot_F B)^c \rightarrow (A @ B)) = ((A @ B)^c \rightarrow (A \odot_F B)) \\ & = ((A \odot_F B)^c \rightarrow (A \# B)) = ((A \# B)^c \rightarrow (A \odot_F B)) \\ & = ((A \odot_F B)^c \rightarrow (A \$ B)) = ((A \$ B)^c \rightarrow (A \odot_F B)) \\ & = ((A \odot_F B)^c \rightarrow (A \oplus_F B)) = ((A \oplus_F B)^c \rightarrow (A \odot_F B)) = A \odot_F B. \end{aligned}$$

Corollary 3.4. For any $A, B \in FFS(y)$

$$\begin{aligned} & (A \oplus_F B)^c \rightarrow (A @ B)^c = ((A @ B) \rightarrow (A \oplus_F B)^c)^c \\ & = (A \oplus_F B) \rightarrow (A \# B)^c = ((A \# B) \rightarrow (A \oplus_F B)^c)^c \\ & = (A \oplus_F B) \rightarrow (A \$ B)^c = ((A \$ B) \rightarrow (A \oplus_F B)^c)^c = A \oplus_F B. \end{aligned}$$

Theorem 3.5. For any $A, B \in FFS(y)$

$$((A^c \rightarrow B) \oplus_F (A \rightarrow B^c)^c) @ ((A^c \rightarrow B) \odot_F (A \rightarrow B^c)^c) = A @ B.$$

Proof. Using (b) and (c) of theorem 3.1 we have

$$(A^c \rightarrow B) \oplus_F (A \rightarrow B^c)^c A \oplus_F B \quad (11)$$

$$(A^c \rightarrow B) \oplus_F (A \rightarrow B^c)^c A \oplus_F B \quad (12)$$

Taking @ with (11) and (12), i.e.

$$(11)@(12) =$$

$$\left(y, \sqrt[3]{\frac{(\alpha_A(y))^3 + (\alpha_B(y))^3 - (\alpha_A(y))^3(\alpha_B(y))^3 + (\alpha_A(y))^3(\alpha_B(y))^3}{2}}, \sqrt[3]{\frac{(\beta_A(y))^3 + (\beta_B(y))^3 - (\beta_A(y))^3(\beta_B(y))^3 + (\beta_A(y))^3(\beta_B(y))^3}{2}} \right)$$

$$= \left(y, \sqrt[3]{\frac{(\alpha_A(y))^3 + (\alpha_A(y))^3}{2}}, \sqrt[3]{\frac{(\beta_A(y))^3 + (\beta_B(y))^3}{2}} / y \in Y \right)$$

$$= A @ B.$$

Theorem 3.6. For any $A, B \in FFS(y)$

$$((A^c \rightarrow B) \oplus_F (A \rightarrow B^c)^c) \cap ((A^c \rightarrow B) \odot_F (A \rightarrow B^c)^c) @$$

$$((A^c \rightarrow B) \oplus_F (A \rightarrow B^c)^c) \cup ((A^c \rightarrow B) \odot_F (A \rightarrow B^c)^c) = A @ B.$$

Proof. By taking and of (11) and (12), we get

$$((A^c \rightarrow B) \oplus_F (A \rightarrow B^c)^c) \cap ((A^c \rightarrow B) \odot_F (A \rightarrow B^c)^c)$$

$$= (A \oplus_F B) \cap (A \odot_F B) \tag{13}$$

$$((A^c \rightarrow B) \oplus_F (A \rightarrow B^c)^c) \cup ((A^c \rightarrow B) \odot_F (A \rightarrow B^c)^c)$$

$$= (A \oplus_F B) \cup (A \odot_F B) \tag{14}$$

Using @ for (13) and (14) i.e. (13) @ (14)

$$= \left(y, \sqrt[3]{\frac{(\alpha_A(y))^3 + (\alpha_B(y))^3}{2}}, \sqrt[3]{\frac{(\beta_A(y))^3 + (\beta_B(y))^3}{2}} / y \in Y \right) = A @ B.$$

Theorem 3.7. For any $A, B \in FFS(y)$

$$[((A \oplus_F B) \rightarrow (A @ B)^c)^c \cup ((A \odot_F B) \rightarrow (A @ B)^c)^c]$$

$$\cup [((A \oplus_F B) \rightarrow (A @ B)^c)^c \cap ((A \odot_F B) \rightarrow (A @ B)^c)^c] = A \oplus_F B.$$

Proof. Using (a) and (c) of theorem 3.2, we have

$$((A \oplus_F B) \rightarrow (A @ B)^c)^c = A \oplus_F B \quad (15)$$

$$((A \odot_F B) \rightarrow (A @ B)^c)^c = A @ B \quad (16)$$

Take \cup with (15), (16)

$$\begin{aligned} & ((A \oplus_F B) \cup (A @ B)) \cup ((A \oplus_F B) \cap (A @ B)) \\ & (y, \sqrt[3]{(\alpha_A(y))^3 + (\alpha_B(y))^3 - (\alpha_A(y))^3(\alpha_B(y))^3}, \beta_A(y)\beta_B(y)/y \in Y) \\ & = A \oplus_F B \end{aligned}$$

Theorem 3.8. For any $A, B \in FFS(y)$

$$\begin{aligned} & [((A \oplus_F B) \rightarrow (A @ B)^c)^c \cup ((A \odot_F B) \rightarrow (A @ B)^c)^c] \\ & \cap [((A \oplus_F B) \rightarrow (A @ B)^c)^c \cap ((A \odot_F B) \rightarrow (A @ B)^c)^c] = A \oplus_F B. \end{aligned}$$

Proof. Take \cap with (15) and (16),

$$\begin{aligned} & ((A \oplus_F B) \cup (A @ B)) \cup ((A \oplus_F B) \cap (A @ B)) \\ & = \left(y, \sqrt[3]{\frac{(\alpha_A(y))^3 + (\alpha_B(y))^3}{2}}, \sqrt[3]{\frac{(\beta_A(y))^3 + (\beta_B(y))^3}{2}}/y \in Y \right) = A @ B. \end{aligned}$$

Theorem 3.9. For any $A, B \in FFS(y)$

$$((A \oplus_F B)^c \rightarrow (A @ B)) @ ((A \odot_F B) \rightarrow (A @ B)^c)^c = A @ B.$$

Proof. Using (b) and (c) of Theorem 3.1,

$$((A \oplus_F B)^c \rightarrow (A @ B)) = (A @ B) \quad (17)$$

$$((A \odot_F B) \rightarrow (A @ B)^c)^c = (A @ B) \quad (18)$$

Take @ of (17) and (18), we get,

$$(A @ B) @ (A @ B) = (A @ B)$$

Theorem 3.10. For any $A, B \in FFS(y)$

$$= ((A \oplus_F B)^c \rightarrow (A \# B)) @ ((A \odot_F B) \rightarrow (A \# B)^c)^c = A @ B.$$

Proof. Using Theorem 3.1.

$$(A \oplus_F B)^c \rightarrow (A \# B) = (A \# B) \tag{19}$$

$$(A \odot_F B)^c \rightarrow (A \# B)^c = (A \# B) \tag{20}$$

Take @ of (19) and (20),

$$\begin{aligned} (A \# B) @ (A \# B) &= \\ & \left(y, \frac{\sqrt{2}\alpha_A(y)\alpha_B(y)}{\sqrt[3]{(\alpha_A(y))^3 + (\alpha_B(y))^3}}, \frac{\sqrt{2}\beta_A(y)\beta_B(y)}{\sqrt[3]{(\beta_A(y))^3 + (\beta_B(y))^3}} / y \in Y \right) @ \\ &= \left(\left\langle y, \frac{\sqrt{2}\alpha_A(y)\alpha_B(y)}{\sqrt[3]{(\alpha_A(y))^3 + (\alpha_B(y))^3}}, \frac{\sqrt{2}\beta_A(y)\beta_B(y)}{\sqrt[3]{(\beta_A(y))^3 + (\beta_B(y))^3}} \right\rangle / y \in Y \right) \\ &= \left(\left\langle y, \frac{\sqrt{2}\alpha_A(y)\alpha_B(y)}{\sqrt[3]{(\alpha_A(y))^3 + (\alpha_B(y))^3}}, \frac{\sqrt{2}\beta_A(y)\beta_B(y)}{\sqrt[3]{(\beta_A(y))^3 + (\beta_B(y))^3}} \right\rangle / y \in Y \right) \\ &= A \# B. \end{aligned} \tag{21}$$

Theorem 3.11. For any $A, B \in FFS(y)$

$$((A \oplus_F B)^c \rightarrow (A \$ B)) @ ((A \odot_F B) \rightarrow (A \$ B)^c)^c = A \$ B.$$

Proof. Using the results of (j) and (k) of Theorem 3.2,

$$(A \oplus_F B)^c \rightarrow (A \$ B) = (A \$ B) \tag{22}$$

$$((A \odot_F B) \rightarrow (A \$ B)^c)^c = (A \$ B) \tag{23}$$

Using @ of (22) and (23), we get $A \$ B$.

Theorem 3.12. For any $A, B \in FFS(y)$

$$((A \odot_F B)^c \rightarrow (A \oplus_F B)) @ ((A \oplus_F B) \rightarrow (A \odot_F B)^c)^c = A @ B.$$

Proof. Using the result (m) of theorem 3.2

$$\begin{aligned} & ((A \odot_F B)^c \rightarrow_1 (A \oplus_F B)) = \\ & (y, \alpha_A(y)\alpha_B(y), \sqrt[3]{(\beta_A(y))^3 + (\beta_B(y))^3 - (\beta_A(y))^3(\beta_B(y))^3} / y \in Y) \end{aligned} \quad (24)$$

$$\begin{aligned} & ((A \oplus_F B) \rightarrow (A \odot_F B)^c)^c = \\ & (y, \sqrt[3]{(\alpha_A(y))^3 + (\alpha_B(y))^3 - (\alpha_A(y))^3(\alpha_B(y))^3}, \beta_A(y)\beta_B(y)) \end{aligned} \quad (25)$$

Take @ of (24) and (25)

$$= \left(y, \sqrt[3]{\frac{(\alpha_A(y))^3 + (\alpha_B(y))^3}{2}}, \sqrt[3]{\frac{(\beta_A(y))^3 + (\beta_B(y))^3}{2}} / y \in Y \right) = A @ B.$$

Conclusion

The properties demonstrated here give a knowledge into the FFSs, under the set activities characterized before in the writing. Our investigation prompts for additional properties as additionally for characterizing potentially new activities. Accordingly there remains scope for contemplating more properties of these sets emerging from those other characterizing set activities that might be considered utilizing alternate methods of consolidating the capacities

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