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APPLICATION OF INTUITIONISTIC FUZZY ROUGH MATRICES

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Abstract

Rough set introduced by Pawlak is a mathematical tool to deal with a special type of uncertainty. In this a set with incomplete and insufficient information is represented by set approximations called the lower and upper approximations. Intuitionistic fuzzy rough set is a generalization obtained by combining the notion of intuitionistic fuzzy set and rough set. Intuitionistic fuzzy rough matrices arise when a finite number of intuitionistic fuzzy rough sets are defined over a finite universe. The purpose of this paper is to define intuitionistic fuzzy rough matrices and study some of their theoretical properties including some operations between them. Decision making based on composition of intuitionistic fuzzy rough matrices is developed. An example is presented to illustrate the working of the method.

1. Introduction

The theory of rough sets was initiated by Pawlak [4, 5]. It is an extension of classical set theory for the study of systems characterized by insufficient and incomplete information. A key notion in Pawlak rough set model is the equivalence relation, where equivalence classes serve as the building blocks for the construction of lower and upper approximations. Replacing the equivalence relation by an arbitrary binary relation, different kinds of generalizations in Pawlak rough set models were obtained.

Matrices play an important role in the broad area of science and engineering. The classical matrix theory cannot answer questions involving

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various types of uncertainties. Yang et al. [6] initiated a matrix representation of fuzzy soft set and studied their basic properties. They defined products of fuzzy soft matrices that satisfy commutative law and used them in a decision making method. The notion of fuzzy soft matrix was studied by Borah et al. [1].

In [7] Yang et al. introduced the concept of interval-valued fuzzy soft set. They defined complement, AND, OR operations and proved De Morgan's, associative and distributive laws for the interval-valued fuzzy soft sets. They also developed a decision making method using interval valued fuzzy soft sets. Some numerical examples are employed to substantiate the theoretical arguments.

Lei Zhou et al. [3] proposed a general framework for the study of relation based intuitionistic fuzzy rough approximation operators within which both constructive and axiomatic approaches were used and established some basic properties of intuitionistic fuzzy rough approximation operators. Chetia et al. [2] defined intuitionistic fuzzy soft matrices and some operations on these matrices.

In this paper we define intuitionistic fuzzy rough matrix and and study some of their theoretical properties including some operations between them. Decision making method based on composition of intuitionistic fuzzy rough matrices is developed. Numerical example is given to illustrate the application of the method.

2. Preliminaries

Definition 2. 1 [3]**.** Let U be a nonempty and finite universe of discourse. An intuitionistic fuzzy relation IFR on U is an intuitionistic fuzzy subset of $U \times U$, viz.,

$$I\overline{F} = \{\langle (x, y), \mu(x, y), v(x, y) \mid (x, y) \in U \times U \rangle\}$$

Where $\mu : U \times U \to [0, 1]$ and $v : U \times U \to [0, 1]$ denote the membership and non membership values of (x, y) satisfying the condition $0 \leq \mu(x, y) + v(x, y) \leq 1$ for all $(x, y) \in U \times U$.

Definition 2.2 [3]**.** Let *IFR* be an intuitionistic fuzzy relation defined on

$(U \times U)$. The pair $(U, I\dot{F}R)$ is called an intuitionistic fuzzy rough approximation space. For any $A \in IF(U)$, where $IF(U)$ denotes the intuitionistic fuzzy power set of U, the lower and upper approximations of A with respect to (U, IFR) denoted by $IF\ \underline{R}(A)$ and $IF\ \overline{R}(A)$ are defined as follows:

$$IF\ \underline{R}(A) = \{x, \mu_{IF\ \underline{R}(A)}(x), v_{IF\ \underline{R}(A)}(x)/x \in U\}$$

$$IF\ \overline{R}(A) = \{x, \mu_{IF\overline{R}(A)}(x), v_{IF\overline{R}(A)}(x)/x \in U\},$$

Where

$$\mu_{IF\ \underline{R}(A)}(x) = \wedge_{y \in U}[v_{IF\ \underline{R}}(x, y) \vee \mu_A(y)],$$

$$v_{IF\ \underline{R}(A)}(x) = \vee_{y \in U}[\mu_{IF\ \underline{R}}(x, y) \wedge v_A(y)].$$

$$\mu_{IF\ \overline{R}(A)}(x) = \vee_{y \in U}[\mu_{IF\overline{R}}(x, y) \wedge \mu_A(y)],$$

$$v_{IF\ \overline{R}(A)}(x) = \wedge_{y \in U}[v_{IF\overline{R}}(x, y) \vee v_A(y)].$$

The pair $(IF\ \underline{R}(A), IF\ \overline{R}(A))$ is called the intuitionistic fuzzy rough set associated A denoted by $IFR\ (A)$.

Example 2.3. Let (U, IFR) be an intuitionistic fuzzy rough approximation space, where $U = \{x_1, x_2, x_3\}$ and

$$A = \{\langle x_1, 0.9, 0.08\rangle, \langle x_2, 1, 0\rangle, \langle x_3, 0.7, 0.3\rangle\},$$

$$IF\overline{R} = \{\langle (x_1, x_1), 0.9, 0.1\rangle, \langle (x_1, x_2), 0.3, 0.6\rangle, \langle (x_1, x_3), 0.4, 0.4\rangle,$$

$$\langle (x_2, x_1), 0.4, 0.6\rangle, \langle (x_2, x_2), 1, 0\rangle, \langle (x_2, x_3), 0.7, 0.3\rangle,$$

$$\langle (x_3, x_1), 0.2, 0.5\rangle, \langle (x_3, x_2), 0.5, 0.5\rangle, \langle (x_3, x_3), 1, 0\rangle\}.$$

Then

$$\mu_{IF\ \underline{R}(A)}(x_1) = 0.7, \mu_{IF\ \underline{R}(A)}(x_2) = 0.7, \mu_{IF\ \underline{R}(A)}(x_3) = 0.7, v_{IF\ \underline{R}(A)}(x_1) = 0.3,$$

$$v_{IF\ \underline{R}(A)}(x_2) = 0.3, v_{IF\ \underline{R}(A)}(x_3) = 0.3,$$

$$\mu_{IF\overline{R}(A)}(x_1) = 0.9, \mu_{IF\overline{R}(A)}(x_2) = 1, \mu_{IF\overline{R}(A)}(x_3) = 0.7, v_{IF\overline{R}(A)}(x_1) = 0.1,$$

$$v_{IF\overline{R}(A)}(x_2) = 0, v_{IF\overline{R}(A)}(x_3) = 0.3.$$

Hence

$$IFR(A) = \{\{\langle x_1, 0.7, 0.3\rangle, \langle x_2, 0.7, 0.3\rangle, \langle x_3, 0.7, 0.3\rangle\}, \{\langle x_1, 0.9, 0.1\rangle, \langle x_2, 1, 0\rangle, \langle x_3, 0.7, 0.3\rangle\}\}.$$

Definition 2.4. Let $U = \{x_1, x_2, \ldots, x_p\}$ be the universal set and $IFR(A_j), j = 1, 2, \ldots, q$ denote the intuitionistic fuzzy rough sets defined on U. An intuitionistic fuzzy rough matrix associated with U and $\{A_j\}$ is a $p \times q$ matrix expressed as

$$IFRM = (IF\underline{R}M, IF\overline{R}M)_{p \times q},$$

where $IFRM = ifrm_{ij}, i = 1, 2, \ldots, p\ j = 1, 2, \ldots, q$ and $ifrm_{ij} = ((\mu_{if\underline{r}m_{ij}}, \mu_{if\overline{r}m_{ij}}), (v_{if\underline{r}m_{ij}}, v_{if\overline{r}m_{ij}}))\ \forall i, j \cdot \mu_{if\underline{r}m_{ij}}, \mu_{if\overline{r}m_{ij}}$ represent the lower and upper approximations of the degree of membership and $v_{if\underline{r}m_{ij}}, v_{if\overline{r}m_{ij}}$ represent the lower and upper approximations of the degree of non-membership satisfying the conditions $\mu_{if\overline{r}m_{ij}} + v_{if\overline{r}m_{ij}} \leq 1$ for all i, j.

Definition 2.5. An intuitionistic fuzzy rough matrix of order $p \times q$ is called intuitionistic fuzzy rough null matrix if all its elements are $((0, 0)(1, 1))$. It is denoted by $IFR\ \phi$.

Definition 2.6. An intuitionistic fuzzy rough matrix of order $p \times q$ is called intuitionistic fuzzy rough absolute matrix if all its elements are $((1, 1)(0, 0))$. It is denoted by *IFRA*.

Definition 2.7. Two intuitionistic fuzzy rough matrices $IFRM = ifrm_{ij}$ and $IFRN = ifrn_{ij}$ of the same order are equal if and only if $\mu_{if\underline{r}m_{ij}} = \mu_{if\underline{r}n_{ij}}, \mu_{if\overline{r}m_{ij}} = \mu_{if\overline{r}n_{ij}}$ and $v_{if\underline{r}m_{ij}} = v_{if\underline{r}n_{ij}}, v_{if\overline{r}m_{ij}} = v_{if\overline{r}n_{ij}}$ for all i, j.

3. Operations on Intuitionistic Fuzzy Rough Matrices

Definition 3.1. The sum of two intuitionistic fuzzy rough matrices *IFRM* and *IFRN* of the same order is defined as

$$IFRM + IFRN = [\max(\mu_{if\,\underline{r}m_{ij}}, \mu_{if\,\underline{r}n_{ij}}), \max(\mu_{if\,\bar{r}m_{ij}}, \mu_{if\,\bar{r}n_{ij}})]$$

$$= [\min(v_{if\,\underline{r}m_{ij}}, v_{if\,\underline{r}n_{ij}}), \min(v_{if\,\bar{r}m_{ij}}, v_{if\,\bar{r}n_{ij}})].$$

Example 3.2. The intuitionistic fuzzy rough matrix associated with $U = \{X_1, X_2, X_3\}$ and $\{A_1, A_2, A_3\}$ is given by

$$IFRM = \begin{array}{c} \\ X_1 \\ X_2 \\ X_3 \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\ \begin{pmatrix} (0.6, 0.9)(0.4, 0.1) & (0.6, 0.75)(0.4, 0.25) & (0.7, 0.75)(0.3, 0.25) \\ (0.7, 0.8)(0.3, 0.2) & (0.7, 0.7)(0.3, 0.3) & (0.7, 0.7)(0.3, 0.29) \\ (0.76, 0.9)(0.14, 0.1) & (0.76, 0.81)(0.16, 0.16) & (0.8, 0.88)(0.16, 0.09) \end{pmatrix} \end{array}$$

For the same universal set $U = \{X_1, X_2, X_3\}$ the intuitionistic fuzzy rough matrix associated with $\{B_1, B_2, B_3\}$ is given by

$$IFRM = \begin{array}{c} \\ X_1 \\ X_2 \\ X_3 \end{array} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{pmatrix} (0.5, 0.8)(0.5, 0.2) & (0.8, 0.8)(0.19, 0.19) & (0.5, 0.8)(0.5, 0.19) \\ (0.6, 0.6)(0.4, 0.4) & (0.75, 0.79)(0.25, 0.15) & (0.6, 0.84)(0.4, 0.16) \\ (0.75, 0.8)(0.25, 0.2) & (0.75, 0.9)(0.25, 0.1) & (0.8, 0.89)(0.2, 0.11) \end{pmatrix} \end{array}$$

Now

$$IFRM + IFRN = \begin{pmatrix} (0.6, 0.9)(0.4, 0.1) & (0.8, 0.8)(0.19, 0.19) & (0.7, 0.8)(0.3, 0.19) \\ (0.7, 0.8)(0.3, 0.2) & (0.75, 0.79)(0.25, 0.15) & (0.7, 0.84)(0.3, 0.16) \\ (0.76, 0.9)(0.14, 0.1) & (0.76, 0.9)(0.16, 0.1) & (0.81, 0.89)(0.16, 0.09) \end{pmatrix}.$$

Definition 3.3. The difference between two intuitionistic fuzzy rough matrices *IFRM* and *IFRN* of the same order is defined as

$$IFRM - IFRN = [\min(\mu_{if\,\underline{r}m_{ij}}, \mu_{if\,\underline{r}n_{ij}}), \min(\mu_{if\,\bar{r}m_{ij}}, \mu_{if\,\bar{r}n_{ij}})]$$

$$= [\max(v_{if\,\underline{r}m_{ij}}, v_{if\,\underline{r}n_{ij}}), \max(v_{if\,\bar{r}m_{ij}}, v_{if\,\bar{r}n_{ij}})].$$

for all i, j.

Example 3.4. Consider the intuitionistic fuzzy rough matrices *IFRM* and *IFRN* as in Example 3.2.

$$IFRM - IFRN = \begin{pmatrix} (0.5, 0.8)(0.5, 0.2) & (0.6, 0.75)(0.4, 0.25) & (0.5, 0.75)(0.5, 0.25) \\ (0.6, 0.6)(0.4, 0.4) & (0.7, 0.7)(0.3, 0.3) & (0.6, 0.7)(0.4, 0.29) \\ (0.75, 0.8)(0.25, 0.2) & (0.75, 0.81)(0.25, 0.16) & (0.8, 0.88)(0.2, 0.11) \end{pmatrix}$$

Definition 3.5. The *AND* operator between two intuitionistic fuzzy rough matrices *IFRM* and *IFRN* denoted by $IFRN \wedge IFRN$ is the intuitionistic fuzzy rough matrix

$$IFRM \wedge IFRN = (ifrm_{ij}, ifrn_{ij})_{p^2 \times q} \forall i, j = 1, 2, \ldots, p$$

and

$$ifrm_{ij} \times ifrn_{ij} = [\min(\mu_{if\underline{r}m_{ij}}, \mu_{if\underline{r}n_{ij}}), \min(\mu_{if\bar{r}m_{ij}}, \mu_{if\bar{r}n_{ij}})]$$

$$[\max(\upsilon_{if\underline{r}m_{ij}}, \upsilon_{if\underline{r}n_{ij}}), \max(\upsilon_{if\bar{r}m_{ij}}, \upsilon_{if\bar{r}n_{ij}})], i, j = 1, 2, \ldots, p.$$

Example 3.6. Consider the intuitionistic fuzzy rough matrices *IFRM* and *IFRN* as in Example 3.2.

$$IFRM \wedge IFRN = (ifrm_{ij}, ifrn_{ij})_{p^2 \times q} \forall i, j = 1, 2, \ldots, p$$

$$\begin{pmatrix} (0.5, 0.8)(0.5, 0.2) & (0.6, 0.75)(0.4, 0.25) & (0.5, 0.75)(0.5, 0.25) \\ (0.6, 0.6)(0.4, 0.4) & (0.6, 0.75)(0.4, 0.25) & (0.6, 0.75)(0.4, 0.25) \\ (0.6, 0.8)(0.4, 0.2) & (0.6, 0.75)(0.4, 0.25) & (0.7, 0.75)(0.3,. 0.25) \\ (0.5, 0.8)(0.5, 0.2) & (0.7, 0.7)(0.3, 0.3) & (0.5, 0.7)(0.5, 0.29) \\ (0.6, 0.6)(0.4, 0.4) & (0.7, 0.7)(0.3, 0.3) & (0.6, 0.7)(0.4, 0.29) \\ (0.7, 0.8)(0.3, 0.2) & (0.7, 0.7)(0.3, 0.3) & (0.7, 0.7)(0.3, 0.29) \\ (0.5, 0.8)(0.5, 0.2) & (0.76, 0.8)(0.19, 0.19) & (0.5, 0.8)(0.5, 0.19) \\ (0.6, 0.6)(0.4, 0.4) & (0.75, 0.79)(0.25, 0.16) & (0.6, 0.84)(0.4, 0.16) \\ (0.75, 0.8)(0.25, 0.2) & (0.75, 0.81)(0.25, 0.16) & (0.8, 0.88)(0.2, 0.11) \end{pmatrix}$$

Definition 3.7. The OR operator between two intuitionistic fuzzy rough matrices *IFRM* and *IFRN* denoted by $IFRM \vee IFRN$ is the intuitionistic fuzzy rough matrix

$$IFRM \vee IFRN = (ifrm_{ij}, ifrn_{ij})_{p^2 \times q} \forall i, j = 1, 2, \ldots, p$$

and

$$ifrm_{ij} \times ifrn_{ij} = [\max(\mu_{if\underline{r}m_{ij}}, \mu_{if\underline{r}n_{ij}}), \max(\mu_{if\bar{r}m_{ij}}, \mu_{if\bar{r}n_{ij}})]$$

$$[\min(\upsilon_{if\underline{r}m_{ij}}, \upsilon_{if\underline{r}n_{ij}}), \min(\upsilon_{if\bar{r}m_{ij}}, \upsilon_{if\bar{r}n_{ij}})], i, j = 1, 2, \ldots, p.$$

Example 3.8. Consider the intuitionistic fuzzy rough matrices *IFRM* and *IFRN* as in Example 3.2.

$$IFRM \vee IFRN = (ifrm_{ij}, ifrn_{ij})_{p^2 \times q} \; \forall i, j = 1, 2, \ldots, p$$

$$\begin{pmatrix} (0.6, 0.9)(0.4, 0.1) & (0.8, 0.8)(0.19, 0.19) & (0.7, 0.8)(0.3, 0.19) \\ (0.6, 0.9)(0.4, 0.1) & (0.75, 0.79)(0.25, 0.15) & (0.7, 0.84)(0.3, 0.16) \\ (0.75, 0.9)(0.25, 0.1) & (0.75, 0.9)(0.25, 0.1) & (0.8, 0.79)(0.2, 0.11) \\ (0.7, 0.8)(0.3, 0.2) & (0.8, 0.8)(0.19, 0.19) & (0.7, 0.8)(0.3, 0.19) \\ (0.75, 0.8)(0.3, 0.2) & (0.75, 0.79)(0.25, 0.15) & (0.7, 0.84)(0.3, 0.16) \\ (0.75, 0.8)(0.25, 0.2) & (0.75, 0.9)(0.25, 0.1) & (0.8, 0.89)(0.2, 0.11) \\ (0.76, 0.9)(0.14, 0.1) & (0.8, 0.81)(0.16, 0.16) & (0.81, 0.88)(0.16, 0.09) \\ (0.76, 0.9)(0.14, 0.1) & (0.76, 0.81)(0.16, 0.15) & (0.81, 0.88)(0.16, 0.09) \\ (0.76, 0.9)(0.14, 0.1) & (0.76, 0.9)(0.16, 0.1) & (0.81, 0.89)(0.16, 0.09) \end{pmatrix}.$$

Definition 3.9. The max-min composition of the two intuitionistic fuzzy rough matrices $IFRM_{p \times q}$ and $IFRN_{p \times q}$ denoted by $IFRM * IFRN$ is defined as

$$IFRM * IFRN = IFRD_{p \times r} = [\max_j \min(\mu_{if\underline{r}m_{ij}}, \mu_{if\underline{r}n_{jk}}), \max_j \min(\mu_{if\bar{r}m_{ij}}, \mu_{if\bar{r}n_{jk}})]$$

$$[\min_j \max(\upsilon_{if\underline{r}m_{ij}}, \upsilon_{if\underline{r}n_{jk}}), \min_j \max(\upsilon_{if\bar{r}m_{ij}}, \upsilon_{if\bar{r}n_{jk}})] \forall i, j, k.$$

Definition 3.10. The intuitionistic fuzzy rough complement of *IFRM* denoted by *IFRM* is defined as $IFRM' = (ifrm_{ij})$, where

$$ifrm'_{ij} = ((\upsilon_{if\underline{r}m_{ij}}, \upsilon_{if\bar{r}m_{ij}}), (\mu_{if\underline{r}m_{ij}}, \mu_{if\bar{r}m_{jk}})) \forall i, j$$

where

$$\mu_{if\bar{r}m_{ij}} + \upsilon_{if\bar{r}m_{ij}} \leq 1.$$

Theorem 3.11. *Commutative law holds for intuitionistic fuzzy rough matrices over the universe U.*

(i) $IFRM \wedge IFRN = IFRN \wedge IFRM$

(ii) $IFRM \vee IFRN = IFRN \vee IFRM$.

Proof.

(i) $IFRM \wedge IFRN = \min(\mu_{ifrm_{ij}}, \mu_{ifrn_{ij}}), \max(\upsilon_{ifrm_{ij}}, \upsilon_{ifrn_{ij}})$

$= \min(\mu_{ifrn_{ij}}, \mu_{ifrm_{ij}}), \max(v_{ifrn_{ij}}, v_{ifrm_{ij}})$

$= IFRN \wedge IFRM$.

(ii) $IFRM \vee IFRN = \max(\mu_{ifrm_{ij}}, \mu_{ifrn_{ij}}), \min(v_{ifrm_{ij}}, v_{ifrn_{ij}})$

$= \max(\mu_{ifrn_{ij}}, \mu_{ifrm_{ij}}), \min(v_{ifrn_{ij}}, v_{ifrm_{ij}})$

$= IFRN \vee IFRM$.

Theorem 3.12. *Associative law holds for intuitionistic fuzzy rough matrices over the universe U.*

(i) $(IFRM \wedge IFRN) \wedge IFRO = IFRM \wedge (IFRN \wedge IFRO)$

(ii) $(IFRM \vee IFRN) \vee IFRO = IFRM \vee (IFRN \vee IFRO)$

Proof.

(i)

$(IFRM \wedge IFRN) \wedge IFRO = (\min(\mu_{ifrm_{ij}}, \mu_{ifrn_{ij}}), \max(v_{ifrm_{ij}}, v_{ifrn_{ij}})) \wedge ((\mu_{ifro_{ij}}),$

$(v_{ifro_{ij}}))$

$= (\min(\mu_{ifrm_{ij}}, \mu_{ifrn_{ij}}, \mu_{ifro_{ij}}), \max(v_{ifrm_{ij}}, v_{ifrn_{ij}}, v_{ifro_{ij}}))$

$= ((\mu_{ifrm_{ij}}), (v_{ifrm_{ij}})) \wedge (\min(\mu_{ifrn_{ij}}, \mu_{ifro_{ij}}),$

$\max(v_{ifrn_{ij}}, v_{ifro_{ij}}))$

$= IFRM \wedge (IFRN \wedge IFRO)$.

(ii) $(IFRM \vee IFRN) \vee IFRO = (\max(\mu_{ifrm_{ij}}, \mu_{ifrn_{ij}}), \min(v_{ifrm_{ij}}, v_{ifrn_{ij}}))$

$\vee ((\mu_{ifro_{ij}}), (v_{ifro_{ij}}))$

$= (\max(\mu_{ifrm_{ij}}, \mu_{ifrn_{ij}}, \mu_{ifro_{ij}}), \min(v_{ifrm_{ij}}, v_{ifrn_{ij}}, v_{ifro_{ij}}))$

$= ((\mu_{ifrm_{ij}}), (v_{ifrm_{ij}})) \vee (\max(\mu_{ifrn_{ij}}, \mu_{ifro_{ij}}), \min(v_{ifrn_{ij}}, v_{ifro_{ij}}))$

$= IFRM \vee (IFRN \vee IFRO)$.

Theorem 3.13. *Distributive law holds for intuitionistic fuzzy rough matrices over the universe U.*

(i) $IFRM \wedge (IFRN \vee IFRO) = (IFRM \wedge IFRN) \vee (IFRM \wedge IFRO)$

(ii) $IFRM \vee (IFRN \wedge IFRO) = (IFRM \vee IFRN) \wedge (IFRM \vee IFRO)$.

Proof. Proof is straight forward.

4. Application of Intuitionistic Fuzzy Rough Matrices

In this section some applications of intuitionistic fuzzy rough matrices in real life situations are presented.

4.1. Statement of the problem:

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a given universal set and $Y = \{y_1, y_2, \ldots, y_m\}$ be a given set of properties based on which the elements of the universal set are described in the form of intuitionistic fuzzy rough matrices. Let $Z = \{z_1, z_2, \ldots, z_p\}$ be the set of possible conclusions that can be drawn on the individual elements of *U* based on the set of properties *Y*. The effects of the properties on the conclusion are given in the form of another intuitionistic fuzzy rough matrix of appropriate order. The problem is to arrive at the best conclusion for each element of the universal set based on the properties.

4.2. The Method

Let $X = \{x_1, x_2, \ldots, x_p\}$ be the given set of objects. Let $A = \{A_1, A_2, \ldots, A_q\}$ be the set of properties associated with the given set of objects and let $B = \{B_1, B_2, \ldots, B_r\}$ be the set of conclusions arrived on *U* based on *A*. Let $IFRM = ((\mu_{if\underline{r}m_{ij}}, \mu_{if\bar{r}m_{ij}})(v_{if\underline{r}m_{ij}}, v_{if\bar{r}m_{ij}}))$ be the intuitionistic fuzzy rough matrix associated with $U \times A$ and $IFRN = ((\mu_{if\underline{r}n_{jk}}, \mu_{if\bar{r}n_{ij}})(v_{if\underline{r}n_{ij}}, v_{if\bar{r}n_{jk}}))$ be the intuitionistic fuzzy rough matrix associated with $A \times B$. Let

$$IFRT = ((\mu_{if\underline{r}t_{ik}}, \mu_{if\bar{r}t_{ik}})(v_{if\underline{r}t_{ik}}, v_{if\bar{r}t_{ik}})) = IFRM * IFRN$$

be the composition of *IFRM* and *IFRN*. Let

$$IFRS = ((\mu_{if\,\underline{r}s_{ik}}, \mu_{if\,\bar{r}s_{ik}})(\upsilon_{if\,\underline{r}s_{ik}}, \upsilon_{if\,\bar{r}s_{ik}})) = IFRM * IFRN$$

be the composition of *IFRM* and *IFRN*, where *IFRN* denotes the complement of $IFRN$. *IFRT* and IFRS are $p \times r$ intuitionistic fuzzy rough matrices giving the conclusions on the elements of the universal set *U* based on the influence of *A* and *B*.

Let $X_{ik} = (\max(\mu_{if\,\bar{r}t_{ik}}, \mu_{if\,\underline{r}s_{ik}}), \min(\upsilon_{if\,\bar{r}t_{ik}}, \upsilon_{if\,\underline{r}s_{ik}}))$

And $Y_{ik} = (\max(\mu_{if\,\underline{r}t_{ik}}, \mu_{if\,\underline{r}s_{ik}}), \min(\upsilon_{if\,\underline{r}t_{ik}}, \upsilon_{if\,\bar{r}s_{ik}}))$.

$$Z_{ik} = \left(\left\{\frac{\mu_{ifrx_{jk}} + \mu_{ifry_{jk}}}{2}\right\}, \left\{\frac{\upsilon_{ifrx_{jk}} - \upsilon_{ifry_{jk}}}{2}\right\}\right).$$

The elements of Z_{ik} give the maximum intuitionistic fuzzy membership and minimum intuitionistic fuzzy non membership values of each element of the universal set based on the effect of *A* in relation with *B*. It is a $p \times r$ matrix whose rows are labeled by elements of the universal set and the columns are labeled by the elements of *B*. To arrive at a conclusion regarding each element of the universal set, we compare between the elements of first column and conclude that the element of the column which has maximum intuitionistic fuzzy membership and minimum intuitionistic fuzzy non membership values is the best among the elements of the universal set in respect of the first element of *B*. Similar conclusions can be drawn with respect to other elements of *B*.

4.3. Algorithm

The steps of the algorithm based on the method explained above are as follows:

Step 1: Input the intuitionistic fuzzy rough matrix *IFRM* on $U \times A$.

Step 2: Input the intuitionistic fuzzy rough matrix *IFRN* on $A \times B$.

Step 3: Compute the matrices $IFRS = IFRM * IFRN$ and $IFRS = IFRM * IFRN$.

Step 4: Compute the values X_{ik} and Y_{ik} where,

$$X_{ik} = (\max(\mu_{if\bar{r}t_{ik}}, \mu_{if\underline{r}s_{ik}}), \min(\upsilon_{if\bar{r}t_{ik}}, \upsilon_{if\underline{r}s_{ik}}))$$

$$Y_{ik} = (\max(\mu_{if\underline{r}t_{ik}}, \mu_{if\underline{r}s_{ik}}), \min(\upsilon_{if\underline{r}t_{ik}}, \upsilon_{if\bar{r}s_{ik}}))$$ and

$$Z_{ik} = \left(\left\{\frac{\mu_{ifrx_{jk}} + \mu_{ifry_{jk}}}{2}\right\}, \left\{\frac{\upsilon_{ifrx_{jk}} - \upsilon_{ifry_{jk}}}{2}\right\}\right).$$

Step 5: Compare and conclude. The entry Z_{ik} that corresponds to the maximum membership and minimum non-membership values under each column identifies the best element of the universal set with respect to that column.

4.4. Example

Suppose there are four patients P_1, P_2, P_3 and P_4 in a hospital. Let $U = \{S_1, S_2, S_3, S_4\}$ be the universal set representing the symptoms weight loss, headache, fatigue and rashes, respectively.

Step 1: The intuitionistic fuzzy rough matrix *IFRM* representing Patients symptoms is given below:

$$IFRM = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \\ \end{array}$$

$$\begin{pmatrix} (0.6,0.8)(0.29,0.1) & (0.75,0.9)(0.14,0.07) & (0.68,0.69)(0.29,0.2) & (0.88,0.9)(0.1,0.07) \\ (0.6, 0.7)(0.4,0.29) & (0.7,0.8)(0.3,0.2) & (0.68,0.69)(0.29,0.2) & (0.73,0.88)(0.22,0.12) \\ (0.75,0.9)(0.2,0.03) & (0.78,0.87)(0.19,0.1) & (0.8,0.85)(0.12,0.08) & (0.79,0.92)(0.16,0.06) \\ (0.7, 0.9)(0.19,0.1) & (0.6,0.85)(0.2,0.08) & (0.65,0.76)(0.17,0.15) & (0.79,0.83)(0.15,0.09) \end{pmatrix}$$

$U = \{D_1, D_2, D_3, D_4\}$ be the set of diseases under consideration.

Let D_1 = chicken pox, D_2 = small pox, D_3 = mumps and D_4 = measles.

Step 2: The intuitionistic fuzzy rough matrix IFRN representing symptoms diseases is given below:

$$IFRN = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \\ \end{array}$$

$$\begin{pmatrix} (0.65,0.9)(0.25,0.1) & (0.8,0.8)(0.15,0.15) & (0.69,0.73)(0.18,0.13) & (0.7,0.74)(0.22,0.17) \\ (0.69, 0.9)(0.3,0.1) & (0.66,0.75)(0.27,0.21) & (0.75, 0.8)(0.15,0.12) & (0.82,0.88)(0.18,0.11) \\ (0.69,0.9)(0.19,0.07) & (0.75,0.75)(0.25,0.24) & (0.77,0.8)(0.24, 0.2) & (0.81,0.82)(0.18,0.14) \\ (0.73, 0.85)(0.22,0.13) & (0.68,0.8)(0.3,0.16) & (0.79,0.9)(0.19, 0.08) & (0.81,0.85)(0.18,0.14) \end{pmatrix}$$

$$IFRN' = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{c} \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} \\ \begin{pmatrix} (0.25,0.1)(0.65,0.9) & (0.15,0.15)(0.8,0.8) & (0.18,0.13)(0.69,0.75) & (0.2,0.17)(0.7,0.74) \\ (0.3, 0.1)(0.67,0.9) & (0.27,0.21)(0.66,0.75) & (0.15, 0.12)(0.75,0.8) & (0.18,0.11)(0.82,0.88) \\ (0.19,0.07)(0.69,0.9) & (0.25,0.24)(0.75,0.75) & (0.24,0.2)(0.77, 0.8) & (0.18,0.14)(0.81,0.82) \\ (0.22, 0.13)(0.73,0.85) & (0.3,0.16)(0.68,0.8) & (0.19,0.08)(0.79, 0.9) & (0.18,0.14)(0.81,0.85) \end{pmatrix} \end{array}$$

Step 3: The composition of the intuitionistic fuzzy rough matrices $IFRM * IFRN$ and $IFRM * IFRN$ give two intuitionistic fuzzy rough matrices *IFRT* and *IFRS* respectively, as given below:

$$IFRT = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{array}{c} \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} \\ \begin{pmatrix} (0.73,0.9)(0.22,0.1) & (0.68,0.8)(0.27,0.15) & (0.79,0.9)(0.15,0.08) & (0.81,0.88)(0.18,0.11) \\ (0.73, 0.9)(0.19,0.1) & (0.75,0.8)(0.25,0.16) & (0.77, 0.8)(0.22,0.12) & (0.8,0.85)(0.18,0.14) \\ (0.73,0.9)(0.19,0.08) & (0.75,0.8)(0.2,0.15) & (0.79,0.9)(0.19, 0.08) & (0.79,0.87)(0.18,0.11) \\ (0.73, 0.9)(0.19,0.1) & (0.7,0.8)(0.19,0.15) & (0.79,0.83)(0.19, 0.09) & (0.79,0.85)(0.18,0.11) \end{pmatrix} \end{array}$$

$$IFRS = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{array}{c} \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} \\ \begin{pmatrix} (0.3,0.13)(0.65,0.85) & (0.3,0.24)(0.66,0.75) & (0.24,0.2)(0.69,0.75) & (0.22,0.17)(0.7,0.74) \\ (0.3, 0.13)(0.65,0.85) & (0.3,0.24)(0.66,0.75) & (0.24, 0.2)(0.69,0.75) & (0.22,0.17)(0.7,0.74) \\ (0.3,0.13)(0.65,0.85) & (0.3,0.24)(0.66,0.75) & (0.24,0.2)(0.69, 0.75) & (0.22,0.17)(0.7,0.74) \\ (0.3, 0.13)(0.65,0.85) & (0.3,0.24)(0.66,0.75) & (0.24,0.2)(0.69, 0.75) & (0.22,0.17)(0.7,0.74) \end{pmatrix} \end{array}$$

Step 4: Computation of X_{ik}, Y_{ik} and Z_{ik}.

$$X_{ik} = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{array}{c} \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} \\ \begin{pmatrix} (0.9,0.1) & (0.8,0.15) & (0.9,0.08) & (0.88,0.11) \\ (0.9,0.1) & (0.8,0.16) & (0.8,0.12) & (0.85,0.14) \\ (0.9,0.08) & (0.8,0.15) & (0.9, 0.08) & (0.87,0.11) \\ (0.73,0.1) & (0.8,0.15) & (0.83,0.09) & (0.85,0.11) \end{pmatrix} \end{array}$$

$$Y_{ik} = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{array}{c} \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} \\ \begin{pmatrix} (0.73,0.22) & (0.68,0.27) & (0.79,0.15) & (0.81,0.18) \\ (0.73, 0.19) & (0.75,0.25) & (0.77,0.22) & (0.8,0.18) \\ (0.73,0.19) & (0.75,0.2) & (0.79,0.19) & (0.79,0.18) \\ (0.73,0.19) & (0.7,0.19) & (0.79,0.19) & (0.79,0.18) \end{pmatrix} \end{array}$$

$$Z_{ik} = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{array}{c} \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} \\ \begin{pmatrix} (0.815,-0.06) & (0.74,-0.035) & (0.845,-0.035) & (0.845,-0.025) \\ (0.815, -0.045) & (0.775,-0.045) & (0.825,-0.05) & (0.825,-0.015) \\ (0.815,-0.055) & (0.775,-0.025) & (0.845,-0.055) & (0.835,-0.025) \\ (0.815,-0.045) & (0.75,-0.02) & (0.81,-0.05) & (0.82,-0.03) \end{pmatrix} \end{array}$$

Step 5: By comparing the elements of first column we conclude that patient P_1 is affected by chicken pox. Similarly we conclude that P_2 is affected by small pox, P_3 is affected by mumps and P_2 is affected by measles while P_4 is also affected by measles.

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