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# A FUZZY EOQ MODEL FOR EXPONENTIAL DEMAND, PARABOLIC DETERIORATION WITH PARTIAL BACKLOGGING USING CENTROID METHOD

**K. GEETHA and S. P. RESHMA**

Department of Mathematics
Government Arts and Science College
Gudalur, The Nilgiris, India
E-mail: geethachandru24@gmail.com

Department of Mathematics
Emerald Heights College for Women
Udagamandalam, The Nilgiris, India

## Abstract

This paper deals with fuzzy EOQ model for deteriorating items with exponential function of time. Here the deterioration rate is a parabolic function of time. Shortages are permitted and are partially backlogged. The backlogging rate is depends on the waiting time for the next replenishment. The objective of the model is to develop an optimal solution by centroid method which minimizes the total profit. A numerical example is given to portray the model.

## 1. Introduction

Physical merchandise degradation is a significant factor in any inventory model. Inventory depreciation is indicated by spoilage, destruction, or gradual loss. Chang and Dye [1] developed an EOQ model for decaying merchandise with time-differing request and incomplete excess. Giri, Goswami and Chaudhuri [4] introduced a time-varying demand model for declining goods. Hariga [6] established a time-varying inventory model for deteriorating products. Totan Garai, Dipankar Chakravarthy and Tapan Kumar Roy [16] introduced a fully fuzzy inventory with time varying holding cost, price dependent demand. Sahoo and Tripathy [12] constructed a quadratic model with time dependent holding cost parabolic deterioration

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under partial backlogging. Sasikala [13] developed an inventory model for deteriorating items with time varying demand and shortages using fuzzy environment. Syed and Aziz [15] implemented a fuzzy inventory model without shortages using signed distance method. Skouri and Papachristos [14] set up stock models for declining merchandise with time-differing request and fractional accumulation, and cost correlation is analyzed. Sahoo and Tripathy [12] constructed a quadratic model with time dependent holding cost parabolic deterioration under partial backlogging.

In this paper, a fuzzy model is constructed using exponential demand with parabolic decline rate. Partial backlogging allows for a shortage. The optimum solution is obtained by centroid method in terms of triangular fuzzy numbers. A numerical illustration is adopted to explain the model.

## 2. Assumptions and Notation

(i) The demand rate $D(t) = \begin{cases} ae^{bt}, \ t > 0 \text{ where } a > 0, \ b > 0 \\ D_0, \ t \leq 0 \end{cases}$

(ii) Replenishment rate is infinite.

(iii) Planning horizon is infinite.

(iv) Deterioration follows parabolic function of time $Z(t) = \alpha + \beta t^2$, $\alpha > 0, \ 0 < \beta < 1$.

(v) The products that have deteriorated cannot be replaced during the review period.

(vi) $h$ is the holding cost/unit time.

(vii) $A$-ordering cost/unit.

(viii) $C_1$ - inventory cost/unit.

(ix) $C_2$ - deficiency cost/unit.

(x) $C_3$ - loss of revenue opportunity expense.

(xi) $\widetilde{h}$ - holding cost/unit time in fuzzy sense.

(xii) $\widetilde{A}$- ordering cost/unit in fuzzy sense.

(xiii) Shortages are permitted and are partially backlogged. For negative inventory, the backlog rate is defined as $\frac{1}{1+\theta(T-t)}$ where θ is the backlog parameter.

## 3. Mathematical Model

The model is dealt with exponential demand. When the inventory level reaches its limit at time $t = 0$, refilling begins. Due to demand, the stock decreases and deteriorates at time $(0, t_1)$. When the stock level is zero at time $t_1$, deficiency occurs at the time interval $[t_1, T]$, and demand is partly backlogged.

$$\frac{dQ(t)}{dt} + (\alpha + \beta t^2)Q(t) = -ae^{bt},\ 0 \le t \le t_1. \tag{1}$$

Satisfying the condition $Q(0) = Q_A$ and $Q(t_1) = 0$.

Solving (1), we obtain

$$Q(t) = a[(t_1 - t) + \frac{(b+\alpha)}{2}(t_1^2 - t^2) + \frac{(b\alpha)}{3}(t_1^3 - t^3) + \frac{\beta}{12}(t_1^4 - t^4) + \frac{\beta b}{15}(t_1^5 - t^5)]e^{-\propto t - \frac{\beta t^3}{3}}. \tag{2}$$

Maximum inventory is

$$Q(0) = Q_A = a\left[t_1 + \frac{(b+\alpha)}{2}t_1^2 + \frac{(b\alpha)}{3}t_1^3 + \frac{\beta}{12}t_1^4 + \frac{\beta b}{15}t_1^5\right]. \tag{3}$$

In the deficiency period $[t_1, T]$, at time $t$ the purchasing is partially backlogged at $\frac{1}{1+\theta(T-t)}$.

Therefore for the amount of sale backlogged, the differential equation is

$$\frac{dQ(t)}{dt} = \frac{D_0}{1+\theta(T-t)},\ t_1 < t < T \tag{4}$$

Satisfying the condition $Q(t_1) = 0$.

Solving (5) we obtain

$$Q(t) = -\int \frac{D_0}{1 + \theta(T - t)} dt$$

$$= \frac{D_0}{\theta}(\log(1 + \theta(T - t)) - \log(1 + \theta(T - t_1))). \quad (5)$$

The maximum sum of sales backlogged per period is calculated by setting $t = T$ in (5)

$$Q_B = \frac{D_0}{\theta}\log(1 + \theta(T - t_1)). \quad (6)$$

As a result, the EOQ for each cycle is

$$S = Q_A + Q_B$$

$$= a[t_1 + \frac{(b + \alpha)}{2}t_1^2 + \frac{(b\alpha)}{3}t_1^3 + \frac{\beta}{12}t_1^4 + \frac{\beta b}{15}t_1^5] + \frac{D_0}{\theta}\log(1 + \theta(T - t_1)). \quad (7)$$

The cost of inventory storage per period is

$$HC = \int_0^{t_1} hQ(t)dt$$

$$= h\left[\frac{at_1^2}{2} + \frac{a(b + \alpha)}{3}t_1^3 \cdot \frac{ab\alpha}{8}t_1^4 + \frac{a\beta}{60}t_1^5 + \frac{a\beta b}{72}t_1^6\right]. \quad (8)$$

The cost of degradation per cycle is

$$DC = C_1\left(Q_A - \int_0^{t_1} h\,D(t)dt\right)$$

$$= C_1\left(at_1 + \frac{a(b + \alpha)}{2}t_1^2 + \frac{ab\alpha}{3}t_1^3 + \frac{a\beta}{12}t_1^4 + \frac{a\beta b}{15}t_1^5 \cdot \frac{ae^{bt_1}}{b}\right). \quad (9)$$

The cost of shortage per period is

$$SC = C_2\left(-\int_{t_1}^{t} Q(t)dt\right)$$

$$= \frac{C_2 D_0}{\theta}\left(T - t_1 - \frac{1}{\theta}\log(1 + \theta(T - t_1))\right). \quad (10)$$

Due to loss in revenue, the opportunity cost is

$$OC = C_3\left(\int_{t_1}^{T} D_0\left(1 - \frac{1}{1 + \theta(T - t)}\right)dt\right)$$

$$= C_3 D_0\left(T - t_1 - \frac{1}{\theta}\log\left(1 + \theta(T - t_1)\right)\right). \quad (11)$$

As a result, the overall total cost per unit per cycle is as follows

$$TC = \frac{1}{T}[A + h\left(\frac{at_1^2}{2} + \frac{a(b+\alpha)}{3}t_1^3 - \frac{ab\alpha}{8}t_1^4 + \frac{a\beta}{60}t_1^5 + \frac{a\beta b}{72}t_1^6\right) + C_1$$

$$\left(at_1 + \frac{a(b+\alpha)}{2}t_1^2 + \frac{ab\alpha}{3}t_1^3 + \frac{a\beta}{12}t_1^4 + \frac{a\beta b}{15}t_1^5 - \frac{ae^{bt_1}}{b}\right) + \frac{C_2 D_0}{\theta}$$

$$\left(T - t_1 - \frac{1}{\theta}\log\left(1 + \theta(T - t_1)\right)\right) + C_3 D_0\left(T - t_1 - \frac{1}{\theta}\log\left(1 + \theta(T - t_1)\right)\right)] \quad (12)$$

## 4. Fuzzy Model

We shall consider the model in fuzzy sense

Let $\tilde{A} = (\widetilde{A_1}, \widetilde{A_2}, \widetilde{A_3}), \tilde{h} = (\widetilde{h_1}, \widetilde{h_2}, \widetilde{h_3})$

$\tilde{a} = (\widetilde{a_1}, \widetilde{a_2}, \widetilde{a_3}), \tilde{b} = (\widetilde{b_1}, \widetilde{b_2}, \widetilde{b_3})$

$\tilde{\alpha} = (\widetilde{\alpha_1}, \widetilde{\alpha_2}, \widetilde{\alpha_3}), \tilde{\beta} = (\widetilde{\beta_1}, \widetilde{\beta_2}, \widetilde{\beta_3})$ are triangular fuzzy number.

Total cost per unit time in fuzzy environment is given by

$$\widetilde{TC} = \frac{1}{T}[\tilde{A} + \tilde{h}\left(\frac{\tilde{a}t_1^2}{2} + \frac{\tilde{a}(\tilde{b}+\tilde{\alpha})}{3}t_1^3 - \frac{\tilde{a}\tilde{b}\tilde{\alpha}}{8}t_1^4 + \frac{\tilde{a}\tilde{\beta}}{60}t_1^5 + \frac{\tilde{a}\tilde{\beta}\tilde{b}}{72}t_1^6\right) + C_1$$

$$\left(\tilde{a}t_1 + \frac{\tilde{a}(\tilde{b}+\tilde{\alpha})}{2}t_1^2 + \frac{\tilde{a}\tilde{b}\tilde{\alpha}}{3}t_1^3 + \frac{\tilde{a}\tilde{\beta}}{12}t_1^4 + \frac{\tilde{a}\tilde{\beta}\tilde{b}}{15}t_1^5 - \frac{\tilde{a}e^{\tilde{b}t_1}}{\tilde{b}}\right) + \frac{C_2 D_0}{\theta}$$

$$\left(T - t_1 - \frac{1}{\theta}\log\left(1 + \theta(T - t_1)\right)\right) + C_3 D_0\left(T - t_1 - \frac{1}{\theta}\log\left(1 + \theta(T - t_1)\right)\right)].$$

Fuzzy total cost is defuzzified by centroid method.

$$TC_{dc} = \frac{1}{3}[TC_{dC_1} + TC_{dC_2} + TC_{dC_3}]$$

$$\widetilde{TC} = \frac{1}{3T}[[\tilde{A}_1 + \tilde{h}_1\left(\frac{\tilde{a}_1 t_1^2}{2} + \frac{\tilde{a}_1(\tilde{b}_1 + \tilde{\alpha}_1)}{3}t_1^3 - \frac{\tilde{a}_1\tilde{b}_2\tilde{\alpha}_1}{8}t_1^4 + \frac{\tilde{\alpha}_1\tilde{\beta}_2}{60}t_1^5 + \frac{\tilde{\alpha}_1\tilde{b}_1\tilde{\beta}_1}{72} + t_1^6\right)$$

$$+C_1\left(\widetilde{a_1}t_1 + \frac{\widetilde{a_1}(\widetilde{b_1} + \widetilde{a_1})}{2}t_1^2 + \frac{\widetilde{a_1}\widetilde{b_1}\widetilde{\alpha_1}}{3}t_1^3 + \frac{\widetilde{a_1}\widetilde{\beta_1}}{12}t_1^4 + \frac{\widetilde{a_1}\widetilde{b_1}\widetilde{\beta_1}}{15}t_1^5 - \frac{\widetilde{a_1}e^{\widetilde{b_1}t_1}}{\widetilde{b_1}}\right)$$

$$\frac{C_2 D_0}{\theta}\left(T - t_1 - \frac{1}{\theta}\log(1 + \theta(T - t_1))\right) + C_3 D_0\left(T - t_1 - \frac{1}{\theta}\log(1 + \theta(T - t_1))\right)]$$

$$[\widetilde{A_2} + \widetilde{h_2}\left(\frac{\widetilde{a_2}t_1^2}{2} + \frac{\widetilde{a_2}(\widetilde{b_2} + \widetilde{a_2})}{3}t_1^3 - \frac{\widetilde{a_2}\widetilde{b_2}\widetilde{a_2}}{8}t_1^4 + \frac{\widetilde{a_2}\widetilde{\beta_2}}{60}t_1^5 + \frac{\widetilde{a_2}\widetilde{b_2}\tilde{\alpha}}{72}t_1^6\right)$$

$$+C_1\left(\widetilde{a_2}t_1 + \frac{\widetilde{a_2}(\widetilde{b_2} + \widetilde{a_2})}{2}t_1^2 + \frac{\widetilde{a_2}\widetilde{b_2}\widetilde{\alpha_2}}{3}t_1^3 + \frac{\widetilde{a_2}\widetilde{\beta_2}}{12}t_1^4 + \frac{\widetilde{a_2}\widetilde{b_2}\widetilde{\beta_2}}{15}t_1^5 - \frac{\widetilde{a_2}e^{\widetilde{b_2}t_2}}{\widetilde{b_2}}\right)$$

$$+\frac{C_2 D_0}{\theta}\left(T - t_1 - \frac{1}{\theta}\log(1 + \theta(T - t_1))\right) + C_3 D_0\left(T - t_1 - \frac{1}{\theta}\log(1 + \theta(T - t_1))\right)]$$

$$+[\widetilde{A_3} + \widetilde{h_3}\left(\frac{\widetilde{a_3}\widetilde{t_1^2}}{2} + \frac{\widetilde{a_3}(\widetilde{b_3} + \widetilde{a_3})}{3}t_1^3 - \frac{\widetilde{a_3}\widetilde{b_3}\widetilde{a_3}}{8}t_1^4 + \frac{\widetilde{a_3}\widetilde{\beta_3}}{60}t_1^5 + \frac{\widetilde{a_3}\widetilde{b_3}\widetilde{\alpha_3}}{72}t_1^6\right)$$

$$+C_1\left(\widetilde{a_3}t_1 + \frac{\widetilde{a_3}(\widetilde{b_3} + \widetilde{a_3})}{2}t_1^2 + \frac{\widetilde{a_3}\widetilde{b_3}\widetilde{\alpha_3}}{3}t_1^3 + \frac{\widetilde{a_3}\widetilde{\beta_3}}{12}t_1^4 + \frac{\widetilde{a_3}\widetilde{b_3}\widetilde{\beta_3}}{15}t_1^5 - \frac{\widetilde{a_3}e^{\widetilde{b_3}t_3}}{\widetilde{b_2}}\right)$$

$$\frac{C_2 D_0}{\theta}\left(T - t_1 - \frac{1}{\theta}\log(1 + \theta(T - t_1))\right) + C_3 D_0\left(T - t_1 - \frac{1}{\theta}\log(1 + \theta(T - t_1))\right)].$$

To obtain the optimum values of $t_1$ and $T$ to be minimum is $\frac{\partial TC_{dc}}{\partial t_1} = 0$, $\frac{\partial \tau C_{dc}}{\partial T} = 0$

$$\frac{1}{3T}[[\widetilde{h_1}(\widetilde{a_1}t_1 + \widetilde{a_1}\widetilde{b_1}\widetilde{t_1^2} + \frac{\widetilde{a_1}\widetilde{\alpha_1}}{2}t_1^2 - \frac{\widetilde{a_1}\widetilde{b_1}\widetilde{\alpha_1}}{2}t_1^3 + \frac{\widetilde{a_1}\widetilde{\beta_1}}{12}t_1^4 + \frac{\widetilde{a_1}\widetilde{\beta_1}\widetilde{b_1}}{12}t_1^5)$$

$$+C_1(\widetilde{a_1}+\widetilde{a_1}(\widetilde{b_1}+\widetilde{a_1})t_1+\widetilde{a_1}\widetilde{b_1}\widetilde{a_1}\,t_1^2+\frac{\widetilde{a_1}\widetilde{\beta_1}}{3}(t_1^3+\widetilde{b_1}t_1^4)+\widetilde{a_1}e^{\widetilde{b_1}t_1})$$

$$-\frac{(C_2+\partial C_3)(T-t_1)D_0}{1+\theta(T-t_1)}]+[\widetilde{h_2}(\widetilde{a_2}t_1+\widetilde{a_2}\widetilde{b_2}t_1^2+\frac{\widetilde{a_2}\widetilde{\alpha_2}}{2}t_1^2$$

$$-\frac{\widetilde{a_2}\widetilde{b_2}\widetilde{\alpha_2}}{2}t_1^3+\frac{\widetilde{a_2}\widetilde{\beta_3}}{12}t_1^4+\frac{\widetilde{a_2}\widetilde{\beta_2}\widetilde{b_2}}{12}t_1^5)+C_1(\widetilde{a_2}+\widetilde{a_2}$$

$$(\widetilde{b_2}+\widetilde{\alpha_2})t_1+\widetilde{a_2}\widetilde{b_2}\widetilde{\alpha_2}t_1^2+\frac{\widetilde{a_2}\widetilde{\beta_3}}{3}(t_1^3+\widetilde{b_2}t_1^4)+\widetilde{a_2}e^{\widetilde{b_2}t_1})$$

$$-\frac{(C_2+\partial C_3)(T-t_1)D_0}{1+\theta(T-t_1)}]+[\widetilde{h_3}(\widetilde{a_3}t_1+\widetilde{a_3}\widetilde{b_3}t_1^2+\frac{\widetilde{a_3}\widetilde{\alpha_3}}{2}t_1^2$$

$$-\frac{\widetilde{a_3}\widetilde{b_3}\widetilde{\alpha_3}}{2}t_1^3+\frac{\widetilde{a_3}\widetilde{\beta_3}}{12}t_1^4+\frac{\widetilde{a_3}\widetilde{\beta_3}\widetilde{b_3}}{12}t_1^5)+C_1(\widetilde{a_3}+\widetilde{a_3}$$

$$(\widetilde{b_3}+\widetilde{\alpha_3})t_1+\widetilde{a_3}\widetilde{b_3}\widetilde{\alpha_3}t_1^2+\frac{\widetilde{a_3}\widetilde{\beta_3}}{3}(t_1^3+\widetilde{b_3}t_1^4)+\widetilde{a_3}e^{\widetilde{b_3}t_1})$$

$$-\frac{(C_2+\theta C_3)(T-t_1)D_0}{1+\theta(T-t_1)}]]=0.$$

## 5. Numerical Example

**Crisp Model:** Let $A = 20$, $a = 16$, $b = 6$, $C_1 = 1.5$, $C_2 = 2.5$, $C_3 = 2$, $D_0 = 20$, $\alpha = 2$, $\beta = 0.2$, $\theta = 2$, $h = 2$ we get $t_1 = 0.2233$, $T = 1.0939$, $TC = 41.2879$.

**Centroid Method:**

Let $\widetilde{A} = (10, 20, 30)$, $\widetilde{a} = (8, 16, 24)$, $\widetilde{b} = (3, 6, 9)$ $\widetilde{\alpha} = (1, 2, 3)$, $\widetilde{\beta} = (0.1, 0.2, 0.3)$, $C_1 = 1.5$, $C_2 = 2$, $D_0 = 20$, $\theta = 2$, $h = (1, 2, 3)$ we get $t_1 = 0.1815$, $T = 0.9767$, $TC = 39.9057$.

## 6. Conclusion

This fuzzy model was created for products that deteriorate at a parabolic rate and have an exponential demand rate. The rate of deterioration is time dependent in this case. With partial backlog, there are no shortages. Making use of centroid approach the model is defuzzified by using triangular fuzzy numbers. A numerical illustration is given to back up this model. This model can be enhanced by adding a payment delay, a stochastic demand rate, and other variables.

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