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SIGMA INDEX IN SOME SPECIAL GRAPHS

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Abstract

A topological index, also identified by way of connective index, is a molecular structure descriptor projected from a molecular graph of a chemical composite which represents its topology. Various topological indices remain exclusive established on their degree, spectrum and distance. In this paper we intended and examined the degree oriented topological directories such as sigma index $\sigma(G)$. Further investigated the $\sigma(G)$ index in regular graph, complete graph, complete bipartite graph, Ladder graph, brush graph and join of graphs are derived. Further explain the results by examples.

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1. Introduction

The sigma index of a graph is formulated $\sigma(G) = \sum_{uvE(G)} (d_G(u) - d_G(v))^2$ as where d_i are d_j the degrees of the adjacent nodes. It is also know as connectivity index of a graph. Here we intended and examined the degree oriented topological directories such as sigma index $\sigma(G)$. Further investigated the $\sigma(G)$ index in regular graph, complete graph, complete bipartite graph, ladder graph, brush graph and join of graphs are derived. Further explain the results by examples.

2. Sigma index in Various Graphs

In this section we intended and examined the degree oriented topological directories such as sigma index $\sigma(G)$. Further investigated the $\sigma(G)$ index in regular graph, complete graph, complete bipartite graph, ladder graph, brush graph and join of graphs are derived. Further explain the results by examples.

Theorem 2.1. *The sigma index of an r-regular graph G with n nodes is a constant that is equal to* $\sigma(G) = 0$.

Proof. Let G be r-regular graph with n nodes. This implies $d(v_i) = r$, $\forall v_i \in G$. In an r-regular graph there $\left(\frac{nr}{2}\right)$ is edges in regular graph. The sigma index of a graph is $\sigma(G) = \sum_{uvE(G)} (d_G(u) - d_G(v))^2$ where $d_G(u)$ and $d_G(v)$ the degrees of the adjacent nodes.

$$\sigma(G) = \sum_{uvE(G)} (d_G(u) - d_G(v))^2$$

$$= \sum_{uvE(G)} (r - r)^2 \therefore d_G(u) = r \; \forall \, u \in G$$

$$\sigma(G) = 0$$

Hence 47. The sigma index of an r-regular graph G with n nodes is zero, then $\sigma(G) = 0$.

Example 2.1.

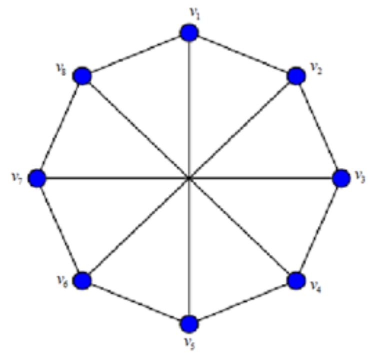


Figure 2.1. 3-Regular graph.

In the above example $O(G) = n = 8,$ and $E(G) = 12\, d(u_i) = 3, \forall\, u_i \in V.$ The sigma index $\sigma(G) = 0.$

Remarks.

i. The sigma index of a complete K_n is zero. Since every complete graph is a $(n-1)$-regular graph.

ii. The sigma index of a cycle C_n is zero. Since every complete graph is a 2-regular graph.

Theorem 2.2. *The sigma index of a complete bipartite graph is* $K_{m,n}$ *is* $\sigma(G) = (mn)(n-m)^2.$

Proof. Let $K_{m,n}$ be complete bipartite graph with vertex set V_1 and V_2. This implies the vertex set V_1 and V_2 having V_1 and mn nodes respectively. Therefore the degree of each vertex in V_1 and V_2 are V_1 and $d(v_i) = n, \forall\, v_i \in V_1$ and $d(v_j) = m, \forall\, v_j \in V_2$. There are number of edges in a complete bipartite graph $K_{m,n}$. The sigma index of a graph is $\sigma(G) = \sum_{uv \in E(G)} (d_G(u) - d_G(v))^2$ where $d_G(u)$ and $d_G(v)$ the degrees of the adjacent nodes.

$$\sigma(G) = \sum_{uv \in E(G)} (d_{V_1}(u) - d_{V_2}(v))^2$$

$$= \sum_{uv \in E(G)} (n - m)^2 \;\therefore d_{V_1}(u) = n \;\forall\; u \in V_1 \text{ and } d_{V_2}(v) = m \;\forall\; v \in V_2$$

$$\sigma(G) = (n - m)^2 + (n - m)^2 + (n - m)^2 + \ldots (mn)\, times$$

$$\sigma(G) = (mn)(n - m)^2$$

Hence the sigma index of complete bipartite graph $K_{m,n}$ is $(mn)(n - m)^2$.

Example 2.2.

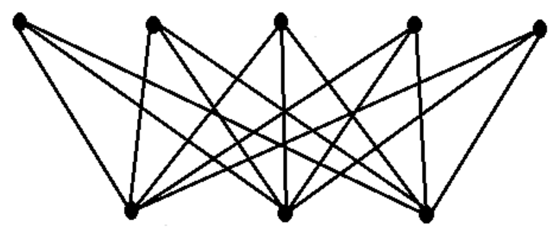


Figure 2.2. Complete bipartite graph $K_{5,3}$.

The graph G is a complete bipartite graph of V_2 and V_3 node set. Therefore $d(u_i) = 3, \forall u_i \in V_5$ and $d(u_j) = 5, \forall u_j \in V_3$. The sigma index $\sigma(G) = (5.3)(3 - 5)^2 = 60$.

Theorem 2.3. *The sigma index of a path* P_n *of any number of vertices is a constant that is equal to* $\sigma(G) = 2$.

Proof. Let P_n a path of any number of vertices. This implies the degree of an end vertices are 2, i.e. $d(v_i) = 2$, for $i = 1, n$. There are $(n - 1)$ number of edges in a path P_n. The sigma index of a graph is $\sigma(G) = \sum_{uv \in E} (d_G(u) - d_G(v))^2$ where $d_G(u)$ and $d_G(v)$ the degrees of the adjacent nodes.

$$\sigma(G) = \sum_{v_i v_j E(G)} (d_G(v_i) - d(v_{i+1}))^2$$

$$= (d(v_1) - d(v_2))^2 + \sum_{v_i v_j E(G)} (d(v_i) - d(v_{i+1}))^2 + (d(v_{n-1}) - d(v_n))^2$$

$$= (1-2)^2 + \sum_{v_i v_j E(G)} (d_G(v_i) - d(v_{i+1}))^2 (2-1)^2$$

$$\therefore\ d(v_1) = 1 \text{ and } d(v_n) = 1,\ d(v_n) = 1,\ d(v_i) = 2 \text{ for } i \neq 1,\ n$$

$$\sigma(G) = 2 + \sum_{v_i v_j E(G)} (d(v_i) - d(v_{i+1}))^2$$

$$\sigma(G) = 2 \ \therefore \sum_{v_i v_j E(G)} (d(v_i) - d(v_{i+1}))^2 = 0$$

Hence the sigma index of a path P_n of any number of vertices is $\sigma(G) = 2$.

Example 2.3.

Figure 2.3. Path P_6.

The Path P_6 is a path of 6 vertices. Therefore $d(u_i) = 2, \forall u_i \in V_I$ and $d(u_j) = 1, \forall u_j \notin V_3$. The sigma index of a path P_n is $\sigma(P_n) = 2$.

Theorem 2.4. *The sigma index of a ladder graph L_n with n nodes is a constant that is equal to $\sigma(G) = 4$.*

Proof. The node set of the ladder graph $L_n, V(L_n) = \{x_i, 1 \leq i \leq n\} \cup \{y_i, 1 \leq i \leq n\}$. Note that there is $(2n)$ nodes in ladder graph L_n. Therefore edges set $E(L_n) = \{(x_i, y_i) \mid 1 \leq i \leq (n-1)\} \cup \{(x_i, x_{i+1}) \mid 1 \leq i \leq (n-1)\} \cup \{(y_i, y_{i+1}) \mid 1 \leq i \leq (n-1)\}$. This implies size of L_n are $3n - 2$. This implies the degree of the vertices in L_n are $d(u_i) = 2$, for $i = 1, n, d(v_i) = 2$, for $i = 1, n$ and $d(u_i) = 3$, for $i \neq 1, n, d(v_i) = 3$, for

$i \neq 1, n$. The sigma index of a graph is $\sigma(G) = \sum_{uv \in E(L_n)} (d_{L_n}(u) - d_{L_2}(v))^2$ where $d_G(u)$ and $d_G(v)$ the degrees of the adjacent nodes.

$$\sigma(G) = \sum_{uv \in E(L_n)} (d_{L_n}(u) - d_{L_2}(v))^2$$

$$= \sum_{i=1}^{n-1} (d_{L_n}(u_i) - d_{L_n}(u_{i+1}))^2 + \sum_{i=1}^{n-1} (d_{L_n}(v_i) - d_{L_n}(v_{i+1}))^2$$

$$+ \sum_{i=1}^{n} (d_{L_n}(u_i) - d_{L_n}(v_i))^2$$

$$= (d_{L_n}(u_1) - d_{L_n}(u_2))^2 + (d_{L_n}(u_{n-1}) - d_{L_n}(u_n))^2$$

$$+ \sum_{i=2}^{n-2} (d_{L_n}(u_i) - d_{L_n}(u_{i+1}))^2$$

$$= (d_{L_n}(v_1) - d_{L_n}(v_2))^2 + (d_{L_n}(v_{n-1}) - d_{L_n}(v_n))^2$$

$$+ \sum_{i=2}^{n} (d_{L_n}(v_i) - d_{L_n}(v_{i+1}))^2$$

$$+ \sum_{i=2}^{n} (d_{L_n}(u_i) - d_{L_n}(v_i))^2$$

$$= (2-3)^2 + (3-2)^2 + \sum_{i=2}^{n-2} (d_{L_n}(u_i) - d_{L_n}(u_{i+1}))^2$$

$$+ (2-3)^2 + (3-2)^2 + \sum_{i=2}^{n-2} (d_{L_n}(v_i) - d_{L_n}(v_{i+1}))^2$$

$$[\because d_{L_n}(u_i) = d_{L_n}(v_i) = \forall i]$$

$$= (2-3)^2 + (3-2)^2 + (2-3)^2 + (3-2)^2$$

$[\because d_{L_n}(u_i) = d_{L_n}(u_{i+1}) = d_{L_n}(v_i) = d_{L_n}(v_{i+1}) = 0 \text{ for } i \neq 1, n]$

$\sigma(G) = 4$

Hence the sigma index of a ladder graph L_n with n nodes is $\sigma(G) = 4$.

Example 2.4.

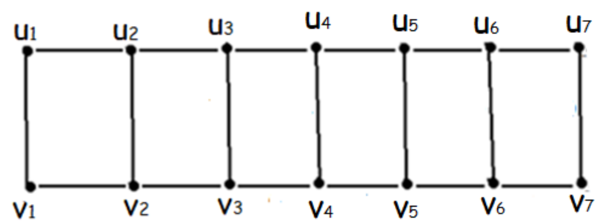


Figure 2.4. Ladder Graph L_7.

In ladder graph L_7 the degree of the vertices are $d(u_i) = 2$ and $d(v_i) = 2$ for $i = 1, 7$, $d(v_i) = 3$ for $i \neq 1, 7$. The sigma index of a ladder graph is L_n and $\sigma(L_n) = 4$.

Theorem 2.5. *The sigma index of a brush graph B_n with n nodes is $\sigma(B_n) = (n-2)(4)$.*

Proof. The node set of the brush graph $V(B_n) = \{x_i, 1 \leq i \leq n\} \cup \{y_i, 1 \leq i \leq n\}$. Note that there is $(2n)$ nodes in ladder graph B_n. Therefore edges set $E(B_n) = \{(x_i y_i) \mid 1 \leq i \leq (n)\} \cup \{(y_i y_{i+1}) \mid 1 \leq i \leq (n-1)\}$. This implies size of B_n are $((n-1)+n) = 2n-1$. The degree of the nodes in the set $\{d(u_i) = 1, 1 \leq i \leq n\}$ are pendent nodes, the degree of every nodes in the set are $\{d(v_i) = 3 \mid 2 \leq i \leq n-1\}$ and the degree of nodes $d(v_i) = 2$, $i = 1, n$. Therefore sigma index

$$\sigma(B_n) = \sum_{uv \in E(B_n)} (d(u) - d(v))^2$$

$$= \sum_{\substack{u_i \in U \\ v_i \in V}} (d(u) - d(v))^2 + \sum_{uv_i \in V} (d(u) - d(v))^2$$

$$= (d(u_1) - d(v_1))^2 + \sum_{i=2}^{n-2} (d(u_i) - d(v_i))^2 + (d(u_n) - d(v_n))^2$$

$$+ (d(v_1) - d(v_2))^2 + \sum_{i=2}^{n-2} (d(v_i) - d(v_{i+1}))^2 + (d(v_{n-1}) - d(v_n))^2$$

$$= (1-2)^2 + \sum_{i=2}^{n-1} (1-3)^2 + (1-2)^2 + (2-3)^2 + \sum_{i=2}^{n-2} (3-3)^2 + (2-3)^2$$

$$\sigma(B_n) = 4 + (n-3)(4)$$

$$\sigma(B_n) = (n-2)(4)$$

Hence the sigma index of a brush graph B_n with n nodes is $\sigma(B_n) = (n-2)(4)$.

Example 2.5.

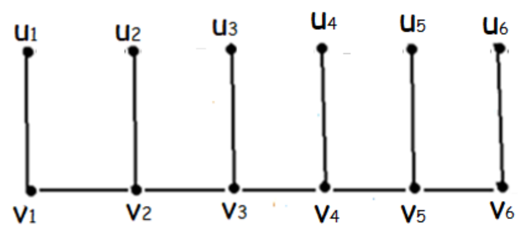


Figure 2.5. Brush Graph B_6.

In a brush graph B_6 the degree of the vertices are $d(u_i) = 1 \,\forall\, i$ and $d(v_i) = 2$ for $i = 1, 6$, $d(v_i) = 3$ for $i \neq 1, 6$. The sigma index of a brush graph is B_6 and $\sigma(B_n) = (6-2)(4) = 16$.

Theorem 2.6. *For a join of two k_1 regular and k_2 regular graphs G_1 and G_2 with m and n vertices respectively, then the sigma index* $\sigma(G_1 + G_2) = (mn)((k_1 - k_2) + (n - m))^2$.

Proof. Consider a join of two k_1 regular and k_2 regular graphs G_1 and G_2 with m and n vertices respectively. By the definition of join of two graphs

G_1 and G_2 there is an edge between every vertices between G_1 and G_2. This implies the degree of nodes in $G_1 + G_2$ are $(k_1 + n), \forall v_i \in V_1$ and $(k_2 + m), \forall v_j \in V_2$. Therefore sigma index

$$\sigma(G_1 + G_2) = \sum_{uv \in E(G_1 + G_2)} (d(u) - d(v))^2$$

$$= \sum_{uv \in G_1} (d(u) - d(v))^2 + \sum_{uv \in G_2} (d(u) - d(v))^2 + \sum_{\substack{u \in G_1 \\ v \in G_2}} (d(u) - d(v))^2$$

$$= 0 + 0 + \sum_{uv \in G_1} (d(u) - d(v))^2 \because G_1 \text{ and } G_2 \text{ graphs regular are}$$

$$= 0 + 0 + \sum_{\substack{uv \in G_1 \\ v \in G_2}} ((k_1 + n) - (k_2 + m))^2$$

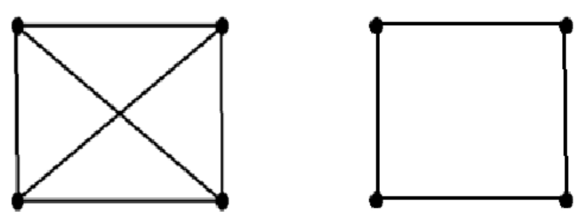
$$= 0 + 0 + ((k_1 + n) - (k_2 + m))^2 + ((k_1 + n) - (k_2 + m))^2 + (mn)\,times$$

$$\sigma(G_1 + G_2) = (mn)((k_1 + n) + (k_2 + m))^2$$

$$\sigma(G_1 + G_2) = (mn)((k_1 + k_2) + (n - m))^2$$

Hence the sigma index of a join of two k_2 regular and k_2 regular graphs G_1 and G_2 with m and n vertices respectively is $\sigma(G_1 + G_2) = (mn)((k_i + k_2) + (n - m))^2$

Example 2.5.



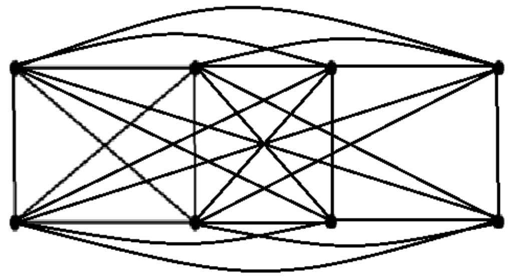


Figure 2.5. Join of graphs 3-regular and 2- regular graphs G_1 and G_2.

The join of the 3-regular and 2- regular graphs G_1 and G_2 with $O(G_1) = 4$ and $O(G_2) = 4$ the sigma index is $\sigma(G_1, G_2)$ $= (4 * 4)((3-2)+(4-4))^2 = 16^*(1)^2 = 16.$

3. Conclusion

In this work, we intended and examined the degree oriented topological directories such as sigma index $\sigma(G)$. Further investigated the $\sigma(G)$ index in regular graph, complete graph, complete bipartite graph, Ladder graph, brush graph and join of graphs are derived. Further explain the results by examples. In future we will investigate on some more topological indices.

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