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EXPLICIT RUNGE KUTTA METHOD IN SOLVING FUZZY INITIAL VALUE PROBLEM

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Abstract

In this paper, the explicit Runge-Kutta method of order four with Butcher [2] table is used to solve the fuzzy initial value problems, the co-efficient of the Runge Kutta method are taken from the Butcher's [2] table. The efficiency and accuracy of the proposed method is examined with a numerical example.

1. Introduction

In 1972, Chang and Zadeh [3] first presented the Fuzzy functions and its derivatives. In continuing, the principle approach was extended to solve the Fuzzy differential equations by Dubois and Prade [4]. Kaleva et al., [7] solve the fuzzy differential equation with initial values. The numerical method to solve the fuzzy initial value problems are introduced by various researchers like Ma et al. [8] studied classical Euler method and Abbas bandy et al., [1] introduced Taylor method. In this paper, the method of solving the fuzzy initial value problem through Explicit Runge Kutta method with Butcher's coefficients is studied.

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2. Preliminaries

Definition 2.1. Trapezoidal fuzzy number is a four tuples $u = (a, b, c, d)$ such that $a < b < c < d$, with base is the interval $[a, d]$ and vertex $x = b$, $x = c$, and its membership function is given by

$$u(x) = \begin{cases} 0, & x < 0 \\ \frac{x - a}{b - a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{c - x}{c - x}, & c \leq x \leq d. \end{cases}$$

And have,

(1) $u > 0$ if $a > 0$; (2) $u \geq 0$ if $b > 0$;

(3) $u > 0$ if $c > 0$; (4) $u > 0$ if $d > 0$.

Definition 2.2. A fuzzy number $\tilde{u} = \{u \mid u : R \to [0, 1]\}$ and satisfies the following

1. $\tilde{u}$ is upper semi-continuous.

2. $\tilde{u}$ is fuzzy convex, if $u(\lambda x + (1 - \lambda)y) \geq \min \{u(x), u(y)\} \; \forall x, y \in R$, $0 \leq \lambda \leq 1$.

3. $\tilde{u}$ is normal, $\exists x_0 \in R$ for which $u(x_0) = 1$

4. Closure of the set $\{x \in R, u(x) > 0\}$ is compact.

Definition 2.3. The parametric form of a fuzzy number $\tilde{u}$ is represented as a pair $(\underline{u}, \overline{u})$ of maps $(\underline{u}(\delta), \overline{u}(\delta))$, $0 \leq \delta \leq 1$, such that

1. $\underline{u}(\delta)$ is a left continuous, bounded and monotonic increasing map.

2. $\overline{u}(\delta)$ is a left continuous, bounded and monotonic decreasing map

3. $\underline{u}(\delta) \leq \overline{u}(\delta)$, for $\delta \in (0, 1]$.

Definition (Fuzzy Arithmetic) 2.4. Let $\tilde{u} = (\underline{u}(\delta), \overline{u}(\delta))$, $\tilde{v} = (\underline{v}(\delta), \overline{v}(\delta))$, $0 \leq \delta \leq 1$ be arbitrary Fuzzy numbers and let $k \in R$, the arithmetic operations on fuzzy numbers are defined by

$$\tilde{u} + \tilde{v} = (\underline{u}(\delta) + \underline{v}(\delta),\ \overline{u}(\delta) + \overline{v}(\delta))$$

$$\tilde{u} - \tilde{v} = (\underline{u}(\delta) + \overline{v}(\delta),\ \overline{u}(\delta) + \underline{v}(\delta))$$

$$\tilde{u} \cdot \tilde{v} = (\min\ \{\underline{u}(\delta)\underline{v}(\delta),\ \underline{u}(\delta)\overline{v}(\delta),\ \overline{u}(\delta)\underline{v}(\delta),\ \overline{u}(\delta)\overline{v}(\delta)\},$$

$$\max\ \{\underline{u}(\delta)\underline{v}(\delta),\ \underline{u}(\delta)\overline{v}(\delta),\ \overline{u}(\delta)\underline{v}(\delta),\ \overline{u}(\delta)\overline{v}(\delta)\})$$

$$C\tilde{u} = \begin{cases} (C\overline{u}(\delta),\ C\underline{u}(\delta)), & \text{if } C \geq 0 \\ (C\underline{u}(\delta),\ C\overline{u}(\delta)), & \text{if } C < 0. \end{cases}$$

Let $D : \tilde{u} \times \tilde{u} \to R^{+} \cup \{0\}$,

$D(u, v) = \sup_{\delta \in [0,1]} \max\ \{|\underline{u}(\delta) - \underline{v}(\delta)|, |\overline{u}(\delta) - \overline{v}(\delta)|\}$, be Hausdorff distance between fuzzy numbers, where $\tilde{u} = (\underline{u}(\delta) - \overline{u}(\delta)),\ \tilde{v} = (v(\delta) - \overline{v}(\delta))$.

The following properties are well known:

$$D(u + w, v + w) = D(u, v),\ \forall u, v, w \in \tilde{u},$$

$$D(ku, kv) = |k| D(u, v),\ \forall k \in R,\ u, v \in \tilde{u},$$

$$D(u + v, w + e) = D(u, w) + D(v, e),\ \forall u, v, w, e \in \tilde{u}.$$

And $(\tilde{u}, D)$ is a complete metric space.

Definition 2.5. Let F be the set of all fuzzy numbers, the δ-level set of fuzzy number $\tilde{u} \in F,\ 0 \leq \delta \leq 1$, is defined by $[u]_{\delta} = \{x \in R / u(x) \geq \delta\ if\ 0 \leq \delta, 1\}$. The δ-level set $[u]_{\delta} = (\underline{u}(\delta), \overline{u}(\delta))$ is closed and bounded.

Lemma 2.1 [8]. *If the sequence of positive numbers* $\{W_n\}_{n=0}^{N}$ *satisfy*

$$|W_{n+1}| \leq A|W_n| + B,\ 0 \leq n \leq N - 1,$$

for the given $A, B \in Z^{+}$, $|W_n| \leq A^n |W_0| + B \dfrac{A^n - 1}{A - 1},\ 0 \leq n \leq N - 1.$

Lemma 2.2 [8]. *If the sequence of positive numbers* $\{W_n\}_{n=0}^{N}, \{V_n\}_{n=0}^{N}$ *satisfy*

$$|W_{n+1}| \leq |W_n| + A \max\ \{|W_n|, |V_n|\} + B,$$

$$|V_{n+1}| \leq |V_n| + A \max\ \{|W_n|, |V_n|\} + B,$$

for the given $A, B \in Z^+, U_n = |W_n| + |V_n|, 0 \leq n \leq N,$ *then* $U_n \leq \overline{A^n} U_0 + \overline{B} \frac{\overline{A^n} - 1}{\overline{A} - 1}, 0 \leq n \leq N,$ *where* $\overline{A} = 1 + 2A$ *and* $\overline{B} = 2B.$

Theorem 2.1 [8]. *Let* $F(t, u, v)$ *and* $G(t, u, v)$ *be in* $C^l(K)$ *and its partial derivatives are bounded above K then, for random fixed* $\delta, 0 \leq \delta \leq 1,$ *the approximate solutions* $\underline{y}(t_{n+1}; \delta)$ *and* $\overline{y}(t_{n+1}; \delta)$ *meet the exact solutions* $\underline{Y}(t; \delta)$ *and* $\overline{Y}(t; \delta)$ *regularly in t.*

Theorem 2.2 [8]. *Let* $F(t, u, v)$ *and* $G(t, u, v)$ *be in* $C^l(K)$ *and its partial derivatives are bounded above* $K, 2Lh < l,$ *then, for random fixed* $\delta, 0 \leq \delta \leq 1,$ *the solutions* $\underline{y}^i(t_n; \delta)$ *and* $\overline{y}^i(t_n; \delta), i = 1, 2, \ldots$ *not diverge to the algebraic solutions* $\underline{y}(t_n; \delta)$ *and* $\overline{y}(t_n; \delta)$ *in* $t_0 \leq t_n \leq t_N,$ *when* $i \to \infty.$

3. Fuzzy Initial Value Problems (FIVP)

Consider the fuzzy initial value differential equation has the form:

$$\begin{cases} y'(t) = f(t, y(t)); \; t \in [t_0, l] \\ y(t_0) = y_0, \end{cases} \tag{1}$$

here y is a fuzzy map in t, $f(t, y)$ is a fuzzy map of t and fuzzy variable y, the derivative of y is denoted by y' and $y(t_0) = y_0$ is a fuzzy number (in triangular shaped).

The exact solution of the problem in (1) $[Y(t)]_\delta = [\underline{Y}(t; \delta), \overline{Y}(t; \delta)]$ be approximated by some $[y(t)]_\delta = [\underline{y}(t; \delta), \overline{y}(t; \delta)].$

$$[y(t_0)]_\delta = [\underline{y}(t_0; \delta), \overline{y}(t_0; \delta)], \; \delta \in (0, 1]$$

we write $f(t, y) = [\underline{f}(t, y), \overline{f}(t, y)]$ and $\underline{f}(t, y) = F[t, \underline{y}, \overline{y}], \overline{f}(t, y) = G[t, \underline{y}, \overline{y}].$

Because of $y' = f(t, y)$ we have

$$\underline{f}(t, y(t); \delta) = F[t, \underline{y}(t; \delta), \overline{y}(t, \delta)]$$

$$\bar{f}(t, y(t); \delta) = G[t, \underline{y}(t; \delta), \bar{y}(t, \delta)]$$

The extension principle gives the membership map as

$$f(t, y(t))(s) = \sup \{y(t)(\tau) \mid s = f(t, \tau)\}, s \in R$$

so fuzzy number $f(t, y(t))$. From this it follows that

$$[f(t, y(t))]_r = [\underline{f}(t, y(t); \delta), \bar{f}(t, y(t); \delta)], \delta \in (0, 1],$$

where

$$\underline{f}(t, y(t); \delta) = \min \{f(t, u) \mid u \in [y(t)]_\delta\}$$

$$\bar{f}(t, y(t); \delta) = \max \{f(t, u) \mid u \in [y(t)]_\delta\}.$$

Theorem 3.1 [8]. *If a function f satisfy the following*

$$|f(t, u) - f(t, u')| \leq g(t, |u - u'|), t \geq 0, u, u' \in R,$$

where $g : R^+ \to R^+$ *is a continuous function and* $\delta \to g(t, \delta)$ *is increasing, the initial value problem* $u'(t) = g(t, u(t)), u(0) = u_0$, *has a solution on* R^+ *for* $u_0 = 0$. *Then the FIVP* (1) *has a unique fuzzy solution.*

4. Explicit Runge-Kutta Method

The family of explicit Runge-Kutta methods is a generalization of the Runge Kutta method. It is given by

$$y_{n+1} = y_n + h\sum_{i=1}^{s} b_i k_i$$

where,

$$k_1 = f(t_n, y_n),$$

$$k_2 = f(t_n + hc_2, y_n + h(a_{21} k_1)),$$

$$k_3 = f(t_n + hc_3, y_n + h(a_{31} k_1 + a_{32} k_2)),$$

$$\vdots$$

$$k_s = f(t_n + hc_s, y_n + h(a_{s1} k_1 + a_{s2} k_2 + \ldots + a_{s\,s-1} k_{s-1})).$$

The integer s (the number of stages), and the coefficients $a_{ij}(1 \leq j < i \leq s)$, $b_i(i = 1, 2, \ldots, s)$ and $c_i(i = 2, 3, \ldots, s)$. The matrix $[a_{ij}]$ is called the Runge Kutta matrix, while the b_i and c_i are known as the weights and the nodes. These data are usually arranged in a mnemonic device, known as a Butcher tableau [2]:

$$
\begin{array}{c|ccccc}
0 & & & & & \\
c_2 & a_{21} & & & & \\
c_3 & a_{31} & a_{32} & & & \\
\vdots & \vdots & & & & \\
\vdots & \vdots & & & & \\
c_s & a_{s1} & a_{s2} & \cdots & a_{s,s-1} & \\
\hline
 & b_1 & b_2 & \cdots & b_{s-1} & b_s
\end{array}
$$

The Runge-Kutta method is consistent if $\sum_{j=1}^{i-1} a_{ij} = c_i,\ i = 2, 3, \ldots, s.$

There are also accompanying requirements if one requires the method to have a certain order p, meaning that the local truncation error is $O(h^{p+1})$, these can be derived from the definition of the truncation error itself.

In general, if an explicit s-stage Runge-Kutta method has order p, then it can be proven that the number of stages must satisfy $s \geq p$, and if $p \geq 5$, then $s \geq p + 1$. However, it is not known whether these bounds are sharp in all cases; for example, all known methods of order 8 have at least 11 stages, though it is possible that there are methods with fewer stages. Indeed, it is an open problem what the precise minimum number of stages 8 is for an explicit Runge-Kutta method to have order p in those cases where no methods have yet been discovered that satisfy the bounds above with equality. Some values which are known are:

p	1	2	3	4	5	6	7	8
mins	1	2	3	4	6	7	9	11

The provable bounds above then imply that we cannot find methods of orders $p = 1, 2, \ldots, 6$ that require fewer stages than the methods we already know for these orders. However, it is conceivable that we might find a method of order $p = 7$ that has only 8 stages, whereas the only ones known today have at least 9 stages as shown in the table.

A slight variation of "the" Runge-Kutta method is also due to Kutta in 1901 and is called the 3/8-rule. The primary advantage this method has is that almost all of the error coefficients are smaller than in the popular method, but it requires slightly more FLOPs (floating-point operations) per time step. Its Butcher [3] tableau is

$$\begin{array}{c|cccc} 0 & & & & \\ \frac{1}{3} & \frac{1}{3} & & & \\ \frac{2}{3} & \frac{-1}{3} & 1 & & \\ 1 & 1 & -1 & 1 & \\ \hline & \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \end{array}$$

5. Explicit Runge-Kutta Fourth Order

The explicit Runge Kutta methods is to prompt the variance among the standards of y at t_{n+1} and t_n as $y_{n+1} - y_n = h \sum_{i=0}^{s} b_i k_i$. Where $b_i's$ are persistent for all i and $k_i = f(t_n + hc_i, y_n + h \sum_{j=1}^{i-1} \lambda_{ij} k_j)$ with $h = t_{n+1} - t_n$ and $y_0 = t_0$,

$$k_1 = f(t_i, y_i)$$

$$k_2 = f(t_i + hc_2, y_i + h(\lambda_{21} k_1))$$

$$k_3 = f(t_i + hc_3, y_i + h(\lambda_{31} k_1 + \lambda_{32} k_2))$$

$$k_4 = f(t_i + hc_4, y_i + h(\lambda_{41} k_1 + \lambda_{42} k_2 + \lambda_{43} k_3)).$$

From Butcher table,

$c_1 = 0$	$c_2 = \frac{1}{3}$	$c_3 = \frac{2}{3}$	$c_4 = 1$	$\lambda_1 = 0$
$b_1 = \frac{1}{8}$	$b_2 = \frac{3}{8}$	$b_3 = \frac{3}{8}$	$b_4 = \frac{1}{8}$	$\lambda_{43} = 1$
$\lambda_{21} = \frac{1}{3}$	$\lambda_{31} = \frac{-1}{3}$	$\lambda_{32} = 1$	$\lambda_{41} = 1$	$\lambda_{42} = -1$

Hence,

$$y_{n+1} = y_n + h\left[\frac{1}{8}(k_1 + k_4) + \frac{3}{8}(k_2 + k_3)\right],$$

$$k_1 = f(t_i, y_i),$$

$$k_2 = f\left(t_i + \frac{h}{3}, y_i + \frac{hk_1}{3}\right)$$

$$k_3 = f\left(t_i + \frac{2h}{3}, y_i + h\left(\frac{-1}{3}k_1 + k_2\right)\right)$$

$$k_4 = f(t_i + h, y_i + h(k_1 - k_2 + k_3)).$$

6. Explicit Runge-Kutta Method for Solving Fuzzy Initial Value Problem

The exact solution of the problem in (1) $[Y(t)]_\delta = [\underline{Y}(t; \delta), \overline{Y}(t; \delta)]$ be estimated by some $[y(t)]_\delta = [\underline{y}(t; \delta), \overline{y}(t; \delta)]$. The grating points are $h = \frac{T - t_0}{N}, t_1 = t_0 + i\,h;\ 0 \le i \le N$.

Now, by the equations (3) and (4), we define

$$y_{n+1} = y_n + h\left[\frac{1}{8}(k_1 + k_4) + \frac{3}{8}(k_2 + k_3)\right]$$

$$\underline{Y}(t_{n+1}; \delta) = \underline{Y}(t_n; \delta) + \left[\frac{h}{8}(\underline{k_1}(t, y(t; \delta)) + \underline{k_4}(t, y(t; \delta)))\right.$$

$$\left. + \frac{3h}{8}(\underline{k_2}(t, y(t; \delta)) + \underline{k_3}(t, y(t; \delta)))\right]$$

where

$$k_1 = F(t_n, \underline{Y}(t; \delta), \overline{Y}(t; \delta))$$

$$k_2 = F(t_n + \frac{h}{3}, \underline{Y}(t; \delta) + \frac{hk_1}{3}, \overline{Y}(t; \delta) + \frac{hk_1}{3})$$

$$k_3 = F\left(t_n + \frac{2h}{3}, \underline{Y}(t; \delta) + h\left(\frac{-k_1}{3} + k_2\right), \overline{Y}(t; \delta) + \left(\frac{-k_1}{3} + k_2\right)\right)$$

$$k_4 = F(t_n + h, \underline{Y}(t; \delta) + h(k_1 - k_2 + k_3), \overline{Y}(t; \delta) + h(k_1 - k_2 + k_3))$$

And,

$$\overline{Y}(t_{n+1}; \delta) = \overline{Y}(t_n; \delta) + \left[\frac{h}{8}(\overline{k_1}(t, y(t; \delta)) + \overline{k_4}(t, y(t; \delta)))\right.$$

$$\left. + \frac{3h}{8}(\overline{k_2}(t, y(t; \delta)) + \overline{k_3}(t, y(t; \delta)))\right]$$

where

$$k_1 = G(t_n, \underline{Y}(t; \delta), \overline{Y}(t; \delta))$$

$$k_2 = G\left(t_n + \frac{h}{3}, \underline{Y}(t; \delta) + \frac{hk_1}{3}, \overline{Y}(t; \delta) + \frac{hk_1}{3}\right)$$

$$k_3 = G\left(t_n + \frac{2h}{3}, \underline{Y}(t; \delta) + h\left(\frac{-k_1}{3} + k_2\right), \overline{Y}(t; \delta) + h\left(\frac{-k_1}{3} + k_2\right)\right)$$

$$k_4 = G(t_n + h, \underline{Y}(t; \delta) + h(k_1 - k_2 + k_3), \overline{Y}(t; \delta) + h(k_1 - k_2 + k_3)).$$

Also we have

$$\underline{y}(t_{n+1}; \delta) = \underline{y}(t_n; \delta) + \left[\frac{h}{8}(\underline{k_1}(t, y(t; \delta)) + \underline{k_4}(t, y(t; \delta)))\right.$$

$$\left. + \frac{3h}{8}(\underline{k_2}(t, y(t; \delta)) + \underline{k_3}(t, y(t; \delta)))\right]$$

where

$$k_1 = F(t_n, \underline{y}(t; \delta), \overline{y}(t; \delta))$$

$$k_2 = F\left(t_n + \frac{h}{3}, \underline{y}(t; \delta) + \frac{hk_1}{3}, \overline{y}(t; \delta) + \frac{hk_1}{3}\right)$$

$$k_3 = F\left(t_n + \frac{2h}{3},\ \underline{y}(t;\ \delta) + h\left(\frac{-k_1}{3} + k_2\right),\ \overline{y}(t;\ \delta) + h\left(\frac{-k_1}{3} + k_2\right)\right)$$

$$k_4 = F(t_n + h,\ \underline{y}(t;\ \delta) + h(k_1 - k_2 + k_3),\ \overline{y}(t;\ \delta) + h(k_1 - k_2 + k_3))$$

and,

$$\overline{y}(t_{n+1};\ \delta) = \overline{y}(t_n;\ \delta) + \left[\frac{h}{8}(\overline{k_1}(t,\ y(t;\ \delta)) + \overline{k_4}(t,\ y(t;\ \delta)))\right.$$

$$\left. + \frac{3h}{8}(\overline{k_2}(t,\ y(t;\ \delta)) + \overline{k_3}(t,\ y(t;\ \delta)))\right]$$

Where

$$k_1 = G(t_n,\ \underline{y}(t;\ \delta),\ \overline{y}(t;\ \delta))$$

$$k_2 = G(t_n + \frac{h}{3},\ \underline{y}(t;\ \delta) + \frac{hk_1}{3},\ \overline{y}(t;\ \delta) + \frac{hk_1}{3})$$

$$k_3 = G\left(t_n + \frac{2h}{3},\ \underline{y}(t;\ \delta) + h\left(\frac{-k_1}{3} + k_2\right),\ \overline{y}(t;\ \delta) + h\left(\frac{-k_1}{3} + k_2\right)\right)$$

$$k_4 = G(t_n + h,\ \underline{y}(t;\ \delta) + h(k_1 - k_2 + k_3),\ \overline{y}(t;\ \delta) + h(k_1 - k_2 + k_3)).$$

Define,

$$F(t,\ y(t,\ \delta)) = \left[\frac{h}{8}(\underline{k_1}(t,\ y(t;\ \delta)) + \underline{k_4}(t,\ y(t;\ \delta))) + \frac{h}{8}(\underline{k_2}(t,\ y(t;\ \delta)) + \underline{k_3}(t,\ y(t;\ \delta)))\right]$$

$$G(t,\ y(t,\ \delta)) = \left[\frac{h}{8}(\overline{k_1}(t,\ y(t;\ \delta)) + \overline{k_4}(t,\ y(t;\ \delta))) + \frac{3h}{8}(\overline{k_2}(t,\ y(t;\ \delta)) + \overline{k_3}(t,\ y(t;\ \delta)))\right]$$

Thus, $[Y(t_n)]_\delta = [\underline{Y}(t_n;\ \delta),\ \overline{Y}(t_n;\ \delta)]$ and $[y(t_n)]_\delta = [\underline{y}(t_n;\ \delta),\ \overline{y}(t_n;\ \delta)]$ are the exact and approximate solutions at $t_n,\ 0 \le n \le N$. The solution at grid points, $l = t_0 \le t_1 \le t_2 \le \ldots \le t_N = m$ and $h = \frac{m - l}{N} = t_{n+1} - t_n$. By above Equations, let $\underline{Y}(t_{n+1};\ \delta) = \underline{Y}(t_n;\ \delta) + F[t_n,\ Y(t_n;\ \delta)]$ and $\overline{Y}(t_{n+1};\ \delta) = \overline{Y}(t_n;\ \delta) + G[t_n,\ Y(t_n;\ \delta)]$ and $\underline{y}(t_{n+1};\ \delta) = \underline{y}(t_n;\ \delta) + F[t_n,\ y(t_n;\ \delta)]$ $\overline{y}(t_{n+1};\ \delta) = \overline{y}(t_n;\ \delta) + G[t_n,\ y(t_n;\ \delta)]$.

By Lemma 2.1 and Lemma 2.2,

$\lim_{n \to \infty} \underline{y}(t, \delta)$ and $\lim_{h \to \infty} \overline{y}(t, \delta) = \overline{Y}(t, \delta)$. Let $F(t, u, v)$ and $G(t, u, v)$ be found by replacing $[y(t)]_\delta = [u, v]$

$$F(t, u, v) = \left[\frac{h}{8}(\underline{k_1}(t, u, v) + \underline{k_4}(t, u, v)) + \frac{3h}{8}(\underline{k_2}(t, u, v) + \underline{k_3}(t, u, v))\right]$$

$$G(t, u, v) = \left[\frac{h}{8}(\overline{k_1}(t, u, v) + \overline{k_4}(t, u, v)) + \frac{3h}{8}(\overline{k_2}(t, u, v) + \overline{k_3}(t, u, v))\right].$$

The territory where F and G are well-defined, therefore

$$K = \{(t, u, v) \mid 0 \leq t \leq T, -\infty < v < \infty, -\infty < u \leq v\}.$$

By Theorem 2.1, the approximate solutions $\underline{y}(t; r)$ and $\overline{y}(t; r)$ converges to the precise solution $\underline{Y}(t; r)$ and $\overline{Y}(t; r)$ consistently in t.

7. Numerical Example

Consider the FIVP,

$$y'(t) = y(t),\ t \in [0, 1],\ y(0) = (0.8 + 0.13\delta, 1.1 - 0.1\delta),\ 0 < \delta \leq 1.$$

Now, $\underline{Y}(t; \delta) = \underline{y}(t; \delta)e^t$, $\overline{Y}(t; \delta) = \overline{y}(t; \delta)e^t$ are the exact solutions, i.e., $Y(t, \delta) = [\underline{Y}(t; \delta), \overline{Y}(t; \delta)] = [\underline{y}(t; \delta)e^t, \overline{y}(t; \delta)e^t]$

$$Y(t, \delta) = [(0.8 + 0.13\delta)e^t, (1.1 - 0.1\delta)e^t],\ 0 < \delta \leq 1.$$

At $t = 1$, we get $Y(1, \delta) = [(0.8 + 0.13\delta)e, (1.1 - 0.1\delta)e],\ 0 < \delta \leq 1.$

δ		0	0.2	0.4	0.6	0.8	1
Exact Solution	$\underline{Y}(1; \delta)$	2.175	2.245	2.315	2.387	2.457	2.528
	$\overline{Y}(1; \delta)$	2.990	2.936	2.881	2.827	2.773	2.718

8. Conclusion

In this paper, the explicit Runge-Kutta method is used to solve the fuzzy initial valued problem, and used the Butcher table for the coefficient of the Runge Kutta formula. A numerical example is also discussed.

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