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SOLVING FUZZY TIME-COST TRADE-OFF PROBLEM USING HAAR CRITICAL PATH METHOD OF OCTAGONAL FUZZY NUMBERS

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Abstract

The goal of this study is to present the octagonal fuzzy number (OCFN) in uncertain situations. The critical path method (CPM) is a significant role for project planning and control, especially for large and complex projects. The successful implementation of the CPM will be aided by a clear definition of time length. Time and cost are the most significant aspects to consider when designing a project. The goal of the project is to complete it on time, on budget, and to meet other objective of the project. However, in actual life, the length of time cannot be precisely specified. As a result, there is always a degree of ambiguity about the duration of activities, prompting the development of the fuzzy critical path technique. In this article, a wavelet-based ranking called Haar ranking for OCFN is used. The fuzzy parameters are first transformed to Haar tuples using the Haar wavelet approach, and then the critical path method is used to find the optimum cost and duration.

1. Introduction

When the source of vagueness and imprecision occurs, L. A. Zadeh [14] proposed fuzzy sets in 1965 to provide a logical manner of tackling problems.

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It can be used in a variety of domains, including control systems, expert systems, decision making, and artificial intelligence. Fuzzy numbers are defined by D. Dubois and H. Prade [4] as a fuzzy subset of the real line. Instead of being a single-valued quantity, a fuzzy number is a multi-valued quantity with a precise value. To deal with imprecision in real-life settings, the majority of studies have used triangular and trapezoidal fuzzy numbers [12, 13]. To determine the critical path, N. Ravishankar et al. [10] employed a new defuzzification formula. S. Elizebeth and L. Sujatha [5] proposed a new ranking algorithm for locating the fuzzy critical path. A project network is described as a collection of activities that must be completed in the order determined by precedence constraints, which specify which actions must begin after the completion of previous activities [8].

One of the routes from the starting node to the finishing node in a project network is called a path through the network. The length of a path, according to the critical path, is the sum of the durations of the activities on the path. Network problems involving fuzzy numbers are referred to as fuzzy network problems and the critical path corresponding to them is referred to as the fuzzy critical path. To determine the earliest start time of each activity, M. Hapke, R. Slowinski [6] employed fuzzy arithmetic techniques. D. Dubois et al. [3, 4] used a fuzzy arithmetic operations extension to compute the most recent starting time of each activity in a project network. A project network is defined as a set of activities that must be done in the sequence specified by precedence constraints, which define which actions must be performed after previous activities have been finished [8]. S. Dhanasekar, S. Hariharan et al. [2] proposed the Haar critical path technique, which involved converting triangular fuzzy integers into Haar tuples using the Haar wavelet principle. S. Karthik et al. [7] presented the Haar ranking system for solving assignment problems, which involves transforming Heptagonal fuzzy numbers into Haar tuples by using Haar wavelet principle. A path through the network is one of the ways in a project network from the start point to the completion point. According to the critical path, the length of a path is equal to the total of the durations of the activities on the path.

The fuzzy preliminaries are discussed in Section 2, and the recommended ranking algorithm is discussed in Section 3. Section 4 explains the suggested fuzzy critical path method and trade cost-trade off algorithm given in section

5. The execution of the method is discussed in Section 6, and the conclusion is discussed in Section 7.

2. Preliminaries

Some basic definitions are reviewed.

Definition 2.1. [**Mapping**] [11] A fuzzy set $\breve{0}$ in a universe of discourse X is defined as the following set of pair $\breve{0} \equiv \{(x, \mu_{\breve{0}}(x); x \in X)\}$. Here $\mu_{\breve{0}} : X \to [0, 1]$ is a mapping called the membership value $x \in X$ of in a fuzzy set $\breve{A}$.

Definition 2.2. [**α-cut**] [11] The α level set (or interval of confidence at level α (or α-cut) of the fuzzy set $\breve{0}$ of X is a crisp set 0_α that contains all the elements of X that have membership values in $\breve{0}$ greater than or equal to α.

$$\breve{0} \equiv \{(x, \mu_{\breve{0}}(x) \geq \alpha; x \in X, \alpha \in [0, 1])\}$$

Definition 2.3. [**Fuzzy Number**] [**9**] A fuzzy set $\breve{0}$, defined on the set of real numbers R is said to be a fuzzy number if its membership function has the following characteristics.

(i) $\breve{A}$ is convex, i.e., $\breve{0}(x_1 + (1 - \lambda)x_2) \geq \min(\mu_{\breve{0}}(x_1), \mu_{\breve{0}}(x_2)) \forall x_1, x_2 \in R$ and $\lambda \in [0, 1]$

(ii) $\breve{A}$ is normal if $\max \mu_{\breve{0}}(x) = 1$

(iii) $\mu_{\breve{0}}(x)$ is piecewise continuous.

Figure 1. Membership Function of OCFN.

Definition 2.4. [Octagonal fuzzy number] [9] A fuzzy number $\breve{0}$ is a normal octagonal fuzzy number denoted by $(o_1, o_2, o_3, o_4, o_6, o_7, o_8)$, whose membership function is

$$\mu_{\breve{0}}(x) = \begin{cases} 0, & x \le o_1 \\ \lambda\left(\dfrac{x - o_1}{o_2 - o_1}\right), & o_1 \le x \le o_2 \\ \lambda & o_2 \le x \le o_3 \\ \lambda + (1 - \lambda)\left(\dfrac{x - o_3}{o_4 - o_3}\right), & o_3 \le x \le o_4 \\ 1 & o_4 \le x \le o_5, \text{ where } 0 < \lambda < 1 \\ \lambda + (1 - \lambda)\left(\dfrac{o_6 - x}{o_6 - o_5}\right), & o_5 \le x \le o_6 \\ \lambda & o_6 \le x \le o_7 \\ \lambda\left(\dfrac{o_8 - x}{o_8 - o_7}\right), & o_7 \le x \le o_8 \\ 0, & x \ge o_8 \end{cases}$$

3. HAAR Ranking Method for Octagonal Fuzzy Number

Let $\breve{0}(o_1, o_2, o_3, o_4, o_6, o_7, o_8)$, be an octagonal fuzzy number. The formulas for determining the average and detailed coefficients of the OCFN, namely the scaling and wavelet coefficients are listed below.

(i) Pair up the given OCFNs. i.e., $[o_1, o_2], [o_3, o_4], [o_5, o_6], [o_7, o_8]$

(ii) Replace the first four elements of $\breve{0}$ with half of the difference between these pairs (approximation coefficients) and the last four elements of $\breve{0}$ with the average of these pairs (detailed coefficients).

$\therefore$ The can be rewritten as $\breve{0}_1(\delta_1, \delta_2, \delta_3, \delta_4, \gamma_1, \gamma_2, \gamma_3, \gamma_4)$, Where

$$\delta_1 = \left(\frac{o_1 + o_2}{2}\right), \delta_2 = \left(\frac{o_3 + o_4}{2}\right), \delta_3 = \left(\frac{o_5 + o_6}{2}\right), \delta_4 = \left(\frac{o_7 + o_8}{2}\right)$$

$$\gamma_1 = \left(\frac{o_1 + o_2}{2}\right), \gamma_2 = \left(\frac{o_3 + o_4}{2}\right), \gamma_3 = \left(\frac{o_5 + o_6}{2}\right), \gamma_4 = \left(\frac{o_7 + o_8}{2}\right)$$

(iii) The pair of $\breve{0}$ approximation coefficients should be grouped together.

Then, for the pair of $\breve{0}$ approximation coefficients $[\delta_1, \delta_2], [\delta_3, \delta_4]$ find the new approximation coefficients and detailed coefficients.

$$\alpha_1 = \left(\frac{\delta_1 + \delta_2}{2}\right), \alpha_2 = \left(\frac{\delta_3 + \delta_4}{2}\right), \beta_1 = \left(\frac{\delta_1 + \delta_2}{2}\right), \beta_2 = \left(\frac{\delta_3 + \delta_4}{2}\right)$$

after then, $\breve{0}_1$ became $\breve{0}_2 = (\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2, \gamma_3, \gamma_4)$

(iv) In $\breve{0}_2$ locate the pair of approximation coefficients. Then, for the pair of $\breve{0}_2$ approximation coefficients $[\alpha_1, \alpha_2]$, get the new approximation and detailed coefficient $\omega_1 = \left(\frac{\alpha_1 + \alpha_2}{2}\right), \omega_2 = \left(\frac{\alpha_1 - \alpha_2}{2}\right)$. After then, $\breve{0}_2$ became $H(\breve{0}) = (\omega_1, \omega_2, \beta_1, \beta_2, \gamma_1, \gamma_2, \gamma_3, \gamma_4)$

(v) Find the ranking.

- If the first element of $H(\breve{0})'s$ tuple is less than the first element of $H(\breve{P})'s$ tuple, then $\breve{0}, \breve{P}$.
- If the first element in both $H(\breve{0})$ and $H(\breve{P})$ is the same, then compare the second element in both $H(\breve{0})$ and $H(\breve{P})$, and so on until the last element. $\breve{0} = \breve{P}$ in such conditions.

4. Algorithmic Rule (Fuzzy Critical Path)

Step 1. Determine fuzzy activities in a fuzzy project.

Step 2. As in fuzzy network diagram, provide octagonal fuzzy numbers to each $OCTFA_{ij}$ period.

Step 3. Determine the order of importance into all fuzzy operations using a fuzzy ranking function.

Step 4. As in fuzzy network diagram, provide octagonal fuzzy numbers to each fuzzy activity period. Convert all the fuzzy durations into Haar tuples using Haar ranking.

Step 5. Initial step: Assume the initial node is 1 and the terminal node is

n. Form the duration of Haar tuple Octagonal Fuzzy Earliest Starting time (HOCTFES) and Haar tuple Octagonal Fuzzy Latest Starting time (HOCTFLS) initially '0' at the starting node 1 are

$$\langle HOCTFES\rangle_1 = \langle 0, 0, 0, 0, 0, 0, 0, 0\rangle \text{ and } \langle HOCTFLS\rangle_1 = \langle 0, 0, 0, 0, 0, 0, 0, 0\rangle$$

Step 6. Compute the Haar tuple Octagonal Earliest Starting time (HOCTES)

$$\langle HOCTFES\rangle_i = \max\left\langle \begin{matrix} HOCTFES_{i1} + HOCTFD_{I1j},\ HOCTFES_{i2} \\ + HOCTFD_{i2j},\ \ldots,\ HOCTFES_{in} + HOCTFD_{inj} \end{matrix} \right\rangle$$

Where, $HOCTFD \to$ Haar tuple Octagonal Fuzzy duration

Step 7. Calculate the Haar tuple Octagonal Latest Finish time (HOCTFLF)

$$\langle HOCTFL\rangle F_j = Min\left\langle \begin{matrix} HOCTFLF_{i1} - HOCTFD_{i1j},\ HOCTFLF \\ - HOCTFD_{i2j},\ \ldots,\ HOCTFLF_{in} - HOCTFD_{inj} \end{matrix} \right\rangle$$

Step 8. Find the Haar floats of each fuzzy activities using the following conditions

$$\langle HOCTFTF\rangle_{ij} = HOCTFLF_j - HOCTFES_i - HOCTFD_{ij} \text{ (Total)}$$

$$\langle HOCTFFF\rangle_{ij} = HOCTFTF_j - HOCTFLF_i - HOCTFFF_{ij} \text{ (Free)}$$

$$\langle HOCTFIF\rangle_{ij} = HOCTFFF_{ij} - HOCTFLE_i - HOCTFEF_j$$

(Independent)

Step 9. If total float $\langle HOCTFTF\rangle_{ij} = 0,$ the activity $OCTFA_{ij}$ is a Fuzzy critical activity. From the other side, Fuzzy critical activities have no total float and are always found with one or more Fuzzy critical paths.

Step 10. The minimal time required to accomplish the fuzzy project is the length of the longest fuzzy critical path from the start to the completion. The minimum fuzzy project duration is determined by this (or these) fuzzy critical path(s).

5. The Time-cost-Trade-off method

Step 1. From the project network, identify the critical path and critical activities.

Step 2. To crash with a typical time critical path, calculate the cost slope and rank activities according to the cost slope (HCC-HNC)/(HNT-HCT) in the increasing order. Here, HCC, HNC refer to Haar crash cost, Haar normal cost and HNT, HCT refer to Haar normal time, Haar crash time, respectively.

Step 3. Find an activity period on the critical path with the lowest cost slope and crash it.

Step 4. If other paths in the network become critical as a result of crashing an activity in step 3 then identify and crash the activities on the critical paths with the lowest joint cost slope.

Step 5. When each activity has been slowed to the shortest possible time, call the procedure to a close. Calculate the overall project cost for various project periods.

6. Illustration

The following table shows the normal and crash durations, as well as costs, for different functions included in a repair job. The direct cost of job supervision is also included.

Table 1. List of Job Description.

Activity	Description	Immediate Predecessor	Expenditure Cost/Day (Rs.)
P	Examine the products and Find the estimated volumes	--	Approx. Rs.20/- (13,15,17,19,21,23,25,27)
Q	Define quality standards and identify essential vendors	--	Approx. Rs.60/- (46,50,54,58,62,66,70,74)
R	Identify required additional man power	P	Approx. Rs.40/- (33,35,37,39,41,43,45,47)

S	AS form of pre-testing, roll out the basic product	Q	Approx. Rs.50/- (36,40,44,48,52,56,60,64)
T	Inspection and dismantling	Q	Approx. Rs.15/- (8,10,12,14,16,18,20,22)
U	Sub-contract repairs	R, S	Approx. Rs.7/- (0,2,4,6,8,10,12,14)
V	Repair/Rebuilt	T	Approx. Rs.8/- (1,3,5,7,9,11,13,15)
W	Purchase spares	T	Approx. Rs.9/- (2,4,6,8,10,12,14,16)
X	Assembling	U, V	Approx. Rs.11/- (4,6,8,10,12,14,16,18)

Table 2. Normal and Crash Duration.

Activity	Time duration	
	Normal	Crash
P	Approx. 6 days (3.6,4.5,4.7,6,6.5,7.2,7.5,8,9)	Approx. 2 days (1.5,1.7,1.8,2,2.1,2.2,2.3,2.4)
Q	Approx. 8 days (7.5,7.7,7.8,8,8.1,8.2,8.3,8.4)	Approx. 3 days (2.5,2.7,2.8,3,3.1,3.2,3.3,3.4)
R	Approx. 7 days (6.5,6.7,6.8,7,7.1,7.2,7.3,7.4)	Approx. 4 days (1,2,3,4,4,5,6,7)
S	Approx. 12 days (11.4,11.6,11.7,12,12.1, 12.2,12.3,12.4)	Approx. 8 days (7.6,7.6,7.8,8,8.1,8.2,8.3,8.4)
T	Approx. 4 days (3.5,3.7,3.8,4,4.1,4.2,4.3,4.4)	Approx. 1 day (0.5,0.7,0.8,1,1.1,1.2,1.3,1.4)
U	Approx. 5 days	Approx. 2 days

	(2.6,3.5,3.7,3.8,4,4.1,4.2,4.3,4.4)	(1.8,1.6,2,2,2,2.1,2.2,2.3)
V	Approx. 7 days (4.6,5.5,5.7,7,7.5,8.2,8.5,9)	Approx. 3 days (2.8,2.8,2.8,3,3,3.1,3.2,3.3)
W	Approx. 11 days (8.6,9.5,9.7,11,11.5,12.2,12.5,13)	Approx. 5 days (4.5,4.7,4.8,5,5.1,5.1,5.3,5.5)
X	Approx.10 days (9.5,9.7,9.8,10,10.1,10.2,10.3, 10.4)	Approx. 6 days (3.6,4.5,4.7,6,6.5,7.2,7.5,8)

Table 3. Normal and Crash Cost.

Activity	Time duration	
	Normal	Crash
P	Approx. Rs.39/- (32,34,36,38,40,42,44,46)	Approx. Rs.120/- (100,105,110,115,120,125,130, 135)
Q	Q Approx. Rs.29/- (22,24,26,28,30,32,34,36)	Approx. Rs.60/- (20,30,40,50,60,70,80,90)
R	R Approx. Rs.26/- (12,16,20,24,28,32,36,40)	Approx. Rs.40/- (0,10,20,30,40,50,60,70)
S	S Approx. Rs.85/- (50,60,70,80,90,100,110,120)	Approx. Rs.110/- (70,80,90,100,110,120,130,140)
T	T Approx. Rs.95/- (60,70,80,90,100,110,120, 130)	Approx. Rs.130/- (110,115,120,125,130,135,140, 145)
U	U Approx. Rs.48/- (41,43,45,47,49,51,53,55)	Approx. Rs.70/- (30,40,50,60,70,80,90,100)
V	V Approx. Rs.18/- (11,13,15,17,19,21,23,25)	Approx. Rs.50/- (10,20,30,40,50,60,70,80)
W	W Approx. Rs.65/- (58,60,62,64,66,68,70,72)	Approx. Rs.120/- (80,90,100,110,120,130,140,15

		0)
X	X Approx. Rs.35/- (0,10,20,30,40,50,60,70)	Approx. Rs.84/- (44,54,64,74,84,94,104,114)

The indirect cost of the project is approx. Rs.20/day (13, 15, 17, 19, 21, 23, 25, 27).

Calculation:

Table 4. Earliest Start and Finish.

Activity	HOCTFD	HOCTFES	HOCTFEF
P 1-2	(6,-1.3,-1,-1,-0.5, -0.5,-0.5,-0.5)	(0,0,0,0,0,0,0,0)	(6,-1.3,-1,-1,-0.5, -0.5,-0.5,-0.5)
Q 1-3	(8,-0.25,-1,-1,-0.5, -0.5,-0.5,-0.5)	(0,0,0,0,0,0,0,0)	(8,-0.25,-1,-1,-0.5, -0.5, -0.5,-0.5)
R 2-4	(7,-0.25,-1,-1,-0.5, -0.5,-0.5,-0.5)	(6,-1.3,-1,-1,-0.5, -0.5,-0.5,-0.5)	(13,-1.55,-2,-2,-1, -1,-1,-1)
S 3-4	(12,-0.29,-1,-1,-0.5, -0.5,-0.5,-0.5)	(8,-0.25,-1,-1,-0.5, -0.5, -0.5,-0.5)	(20,-0.54,-2,-2,-1, -1,-1,-1;0)
T 3-5	(7,-1.3,-1,-1,-0.5, -0.5,-0.5-0.5)	(8,-0.25,-1,-1,-0.5, -0.5, -0.5,-0.5)	(15,-1.55,-2,-2,-1, -1,-1,-1)
U 4-6	(3,-1.3,-0.5,-0.5, -0.25,-0.25,-0.25, -0.25)	(20,-0.54,-2,-2,-1, -1,-1,-1)	(23,-1.84,-2.5, -2.5,-1.25,-1.25, -1.25,-1.25)
V 5-6	(5,-1.3,-1.5,-1.5, -0.75,-0.75,-0.75, -0.75)	(15,-1.55,-2,-2,-1, -1,-1,-1)	(20,-2.85,-3.5, -3.5,-1.75,-1.75, -1.75,-1.7)
W 5-7	(11,-1.3,-2,-2,-1,-1, -1,-1)	(15,-1.55,-2,-2,-1, -1,-1,-1)	(26,-2.85,-4,-4,-2, -2,-2,-2)
X	(10-0.25,-2,-2-1,-1,1 -1)	(23,-1.84,-2.5,-2.5, -1.25,-1.25,-1.25, -1.25)	(33,-2.09,-4.5, -4.5,- 2.25,-2.25, -2.25,-2.25)

The duration of the project is (33,-2.09,-4.5,-4.5,-2.25,-2.25,-2.25,-2.25) days

The Haar indirect cost of the project is (20, -4, -2, -2, -1, -1, -1, -1)

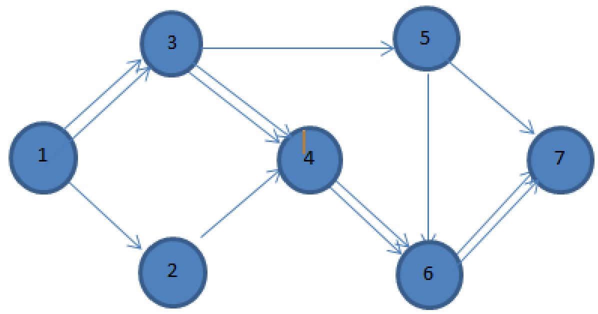

Figure 2. Project Network.

Table 5. Latest start and Finish.

Activity	HOCTFD	HOCTFLS	HOCTFLF
P 1-2	(6,-1.3,-1,-1,-0.5, -0.5, -0.5,-0.5)	(7,1.01,0,0,0,0,0,0,0)	(13,-0.29,-1,-1, -0.5,-0.5,-0.5,-0.5)
Q 1-3	(8,-0.25,-1,-1,-0.5, -0.5, -0.5,-0.5)	(0,0,0,0,0,0,0,0)	(8,-0.25,-1,-1,-0.5, -0.5,-0.5,-0.5)
R 2-4	(7,-0.25,-1,-1,-0.5, -0.5, -0.5,-0.5)	(13,-0.29,-1,-1,-0.5, -0.5,-0.5,-0.5)	(20,-0.54,-2,-2,-1, -1,-1,-1)
S 3-4	(12,-0.29,-1,-1, -0.5,-0.5,-0.5,-0.5)	(8,-0.25,-1,-1,-0.5, -0.5,-0.5,-0.5)	(20,-0.54,-2,-2,-1, -1,-1,-1)
T 3-5	(7,-1.3,-1,-1,-0.5, -0.5,-0.5-0.5)	(11,-0.29,0,0,0,0,0,0)	(18,-0.54,-1,-1, -0.5,-0.5,-0.5,-0.5)
U 4-6	(3,-1.3,-0.5,-0.5, -0.25,-0.25,-0.25,- 0.25)	(20,-0.54,-2,-2,-1,-1, -1,-1)	(23,-1.84,-2.5,-2.5, -1.25,-1.25,-1.25, -1.25)
V 5-6	(5,-1.3,-1.5,-1.5, -0.75,-0.75,-0.75, -0.75)	(18,-0.54,-1,-1,-0.5, -0.5,-0.5,-0.5)	(23,-1.84,-2.5,-2.5, -1.25,-1.25,-1.25, -1.25)

W5-7	(11,-1.3,-2,-2,-1,-1,-1,-1;0.8,1)	(22,-0.79,-2.5,-2.5,-1.25,-1.25,-1.25,-1.25)	(33,-2.09,-4.5,-4.5,-2.25,-2.25,-2.25,-2.25)
X 6-7	(10,-0.25,-2,-2,-1,-1,-1,-1)	(23,-1.84,-2.5,-2.5,-1.25,-1.25,-1.25,-1.25)	(33,-2.09,-4.5,-4.5,-2.25,-2.25,-2.25,-2.25)

Table 6. Total Float.

Activity	HOCTFLS	HOCTFES	HOCTFTF
P 1-2	(7,1.01,0,0,0,0,0,0,0)	(0,0,0,0,0,0,0,0;0.8,1)	(7,1.01,0,0,0,0,0,0)
Q 1-3	(0,0,0,0,0,0,0,0)	(0,0,0,0,0,0,0,0;0.8,1)	(0,0,0,0,0,0,0,0)
R 2-4	(13,-0.29,-1,-1,-0.5,-0.5,-0.5,-0.5)	(6,-1.3,-1,-1,-0.5,-0.5,-0.5,-0.5)	(7,1.01,0,0,0,0,0,0)
S 3-4	(8,-0.25,-1,-1,-0.5,-0.5,-0.5,-0.5)	(8,-0.25,-1,-1,-0.5,-0.5, -0.5,-0.5)	(0,0,0,0,0,0,0,0)
T 3-5	(11,-0.29,0,0,0,0,0,0)	(8,-0.25,-1,-1,-0.5,-0.5, -0.5,-0.5)	(3,-.04,1,1,0.5,0.5,0.5,0.5)
U 4-6	(20,-0.54,-2,-2,-1,-1,-1,-1)	(20,-0.54,-2,-2,-1,-1,-1,-1)	(0,0,0,0,0,0,0,0)
V 5-6	(18,-0.54,-1,-1,-0.5,-0.5,-0.5,-0.5)	(15,-1.55,-2,-2,-1,-1,-1,-1)	(3,1.01,1,1,0.5,0.5,0.5,0.5)
W 5-7	(22,-0.79,-2.5,-2.5,-1.25,-1.25,-1.25,-1.25)	(22,-0.79,-2.5,-2.5,-1.25,-1.25,-1.25,-1.25)	(9,0.76,-0.5,-0.5 -0.25,-0.25,-0.25,-0.25)
X 6-7	(23,-1.84,-2.5,-2.5,-1.25,-1.25,-1.25,-1.25)	(23,-1.84,-2.5,-2.5,-1.25,-1.25,-1.25,-1.25)	(0,0,0,0,0,0,0,0)

The Critical path is 1-3-4-6-7.

Table 7. Cost slope.

Critical Activity	Haar Cost		Cost Slope
	Normal	Crash	
-2	(39,-4,-2,-2,-1,-1,-1,-1)	(118,-10,-5,-5,-2.5,-2.5,-2.5,-2.5)	--
-3	(29,-4,-1,-1,-0.5,-0.5, -0.5,-0.5)	(55,-20,-10,-10,-5,-5,-5,-5)	5.2
-4	(26,-8,-1,-1,-0.5,-0.5, -0.5,-0.5)	(35,-20,-5,-5,-2.5,-2.5,- 2.5,-2.5)	--
-4	(85,-20,-5,-5,-2.5,-2.5, -2.5,2.5)	(105,-20,-10,-10,-5,-5,-5,-5)	5*
-5	(95,-20,-10,-10,-5,-5, -5,-5)	(128,-10,-5,-5,-2.5,-2.5,-2.5,-2.5)	--
-6	(48,-4,-1.-1,-0.5,-0.5, -0.5,-0.5)	(65,-20,-5,-5,-2.5,-2.5,-2.5,-2.5)	17
-7	(65,-4,-2,-2,-1,-1,-1,-1)	(115,-20,-10,-10,-5,-5,-5,-5)	--
-7	(35,-20,-5,-5,-2.5,-2.5, -2.5,2.5)	(79,-20,-10,-10,-5,-5,-5,-5)	11

The normal and its cost is Rs.(440,-88,-28,-28,-14,-14,-14,-14)

Table 8. Crashing.

Crashing		Length of Project	Critical Paths
Order	Processing Activity		
I	3-4 /3	30	1-3-4-6-7; 1-3-5-6-7
II	1-3/ 4	26	1-3-4-6-7; 1-3-5-6-7; 1-2 -4-6-7
III	2-4,1-3/ 1	25	1-3-4-6-7; 1-3-5-6-7; 1-2 -4-6-7

IV	6-7 /4	21	1-3-4-6-7; 1-3-5-6-7; 1-2 -4-6-7
V	2-4,3-4, 3-5 /1	20	1-3-4-6-7; 1-3-5-6-7; 1-2 -4-6-7; 1-3-5-7
VI	3-5,4-6/2	18	1-3-4-6-7; 1-3-5-6-7; 1-2 -4-6-7 1-3-5-7

Table 9. Total Cost.

Haar Total Cost (Rs.)	
Direct (Normal+Crashing)	Total (Direct+Indirect)
(590,-112,-43,-43,-21.5,-21.5, -21.5,-21.5)	(1190,-232,-103,-51.5,-51.5, -51.5,-51.5,-51.5)
(830,-136,-47,-47,-23.5,-23.5, -23.5,-23.5)	(2390,-240,-99,-99,-49.5, -49.5,-49.5,-49.5)
(930,-148,-49,-49,-24.5,-24.5, -24.5,-24.5)	(1430,-248,-99,-99,-49.5, -49.5,-49.5,-49.5)
(974,-164,-69,-69,-25.5,-25.5, -25.5,-25.5)	(1394,-248,-111,-111,-46.5, -46.5,-46.5,46.5)
(1079,-180,-85,-85,-33.5, -33.5,-33.5,-33.5)	(1479,-260,-125,-125,-53.5, -53.5,-53.5,-53.5)
(1101,-188,-96,-96,-39,-39, -39,-39)	(1461,-260,-132,-132, -57,-57, -57,-57)

5. Observations

(i) The Critical path is 1-3-4-6-7

(ii) The critical activities are 1-3, 3-4, 4-6, and 6-7

(iii) The project duration is (33,-2.09,-4.5,-4.5,-2.25,-2.25,-2.25,-2.25)

(iv) The normal cost of the project is Rs. (440,-88,-28,-28,-14,-14,-14,-14)

(v) The total cost is (1100,-220.-94,-94,-47,-47,-47,-47)

(vi) The total direct cost of the project increases when the crashing begins, while the total indirect cost of the project falls.

Result

From the above example, we observe that crashing should have been done for at least 26 days to acquire the best price of Rs.2390/-on the 26th day. Further crashes, on the other hand, shortened the project time to the 20th day at a cost of Rs.1479/-, after which the cost would increase. As a result, the cost on the 20th day is the most cost-effective project cost as well as 13 days were saved.

7. Conclusion

The proposed approach is easy to explain and execute since it reflects the usual aspects of the standard critical path technique by using the Haar ranking technique. The outcome clearly demonstrates that the proposed method gives us the fuzzy critical path and also identifies the critical activities, as demonstrated in a specific example. To find the fuzzy critical path, the proposed method is more effective, simple to understand, and execute. The proposed method can be used to identify critical activities as well as determine the project cost for a time-cost trade-off problem with octagonal fuzzy numbers. The decision maker may decide the total cost and the respective duration based on the calculation table. This type of procedure may applicable in some other ranking a method for future research.

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