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POWER MEAN LABELING OF THE GRAPHS $P_n \Theta S_3$ AND $P_n + S_4$

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Abstract

The graph structures admit the enormous function that assign the real numbers to the vertices and edges if some of the given conditions are met. Meanwhile the numbered undirected graph is progressive and plays vital role in family of mathematical models for a vast range of application now a days. If a positive or non-negative integer $f(e)$ is assigned to each edges e, as a result the edges of G are said to be numbered.

A graph $G = (V, E)$ is referred as Power mean graph with (p, q), if it is feasible to label the vertices $x \in V$ with different elements $f(x)$ from $1, 2, 3, \ldots q + 1$ in such a way that when each edge $e = uv$ is labeled with

$$f(e = uv) = \left\lceil (f(u)^{f(v)} f(v)^{f(u)})^{\frac{1}{f(u)+f(v)}} \right\rceil$$

or

$$f(e = uv) = \left\lfloor (f(u)^{f(v)} f(v)^{f(u)})^{\frac{1}{f(u)+f(v)}} \right\rfloor.$$

Therefore ensuring edge labels are different. Here f is denoted as Power Mean Labeling of G. We further admit Power Mean Labeling for various instances of graphs. In this work, the Power Mean Labeling of the connected graphs $P_N \Theta S_3$ and $P_N + S_4$ is obtained.

1. Introduction

Consider the importance of graph theory holds the major part in discrete mathematics which deal with graph studies. Graph labeling is a practice of

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assigning integers to the vertices or Edges, or both with respect to certain condition. The Concept of Graph labeling was first introduced in late 1960s in mathematics world. In the consecutive years dozens of Graph labeling techniques had been studied in more than 800 paper works.

Rosa Alexandra, introduced the techniques of Graph Labeling methods in his work. He defined an evaluation of graph G with q edges as an injection from the vertices of G to the integer set $\{0, 1, 2, \ldots, q\}$ such that when each edge of xy is assigned the label $|f(x) - f(y)|$, and found resulting edge labels are in the form of distinct.

We analyzed various graph labeling techniques from the work dynamic survey of graph labeling by Gallian Ref [4], with the references and diagnosis, we came up with Power Mean Labelling and investigate some connected graphs for $P_N \Theta S_3$ and $P_N + S_4$ Ref [5].

We initiated by simple, finite, connected and undirected graph $G = (V, E)$ with (p, q). For all other definitive results and conceptions we preceded with the work of Harary [2] connected and undirected graph $G = (V, E)$ with (p, q). For all other definitive results and conceptions we follow Harary [2].

Graph labeling has repeatedly motivated scope for realistic problems and gives interesting areas to do research work. A Systematic study of versatile application of graph labelling was carried out in the work of Bloom and Golomb.

2. Basic Definitions

2.1. Graph. Let $G(V, E)$ be a graph with the vertex set V and the edge set E, correspondingly. By a graph $G(V, E)$ we mean a finite, undirected graph with neither loops nor multiple edges. The number of vertices of G is called order of G and it is denoted by p. The number of edges of g is called size of G and it is denoted by q. A (p, q) graph is a graph G with p vertices and q edges.

2.2. Graph Labeling. A graph labeling is an assignment of integers to the vertices or edges, or both, subject to few conditions. Graph labeling is one of the captivating areas of graph theory with broad range of applications. If area is the set of vertices we speak about the vertex labeling and if area is the set of edges, then the labeling is called the edge labeling.

A huge body of literature has grown around graph labeling in the last four decades. Labeled graphs provide mathematical models for a wide range of applications. The qualitative labeling of a graph elements have been used in diverse fields such as conflict resolutions in social psychology, energy crises etc. Quantitative labeling of graph elements have been used in coding theory, missile guidance codes, radar location codes, astronomy, circuit design, x-ray crystallography, communication network etc.

2.3. Mean Labeling. A graph G with p vertices and q edges is a mean graph if there is an injective function f from the vertices of G to $\{0, 1, 2, \ldots, q\}$ such that when each edge uv is labeled with $\frac{f(u)+f(v)}{2}$ if $(f(u)+f(v))$ is even and if $\frac{f(u)+f(v)+1}{2}$ if $(f(u)+f(v))$ is odd then the resulting edges are distinct.

2.4. Power Mean Labeling. A graph $G = (V, E)$ is called a Power Mean Graph with p vertices and q edges, if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1, 2, 3, \ldots, (q+1)$ in such way that when each edge $e = uv$ is labeled with

$$f(e = uv) = \left\lceil \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rceil$$

or

$$f(e = uv) = \left\lfloor \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rfloor$$

so that the resulting edge labels are distinct.

Remark 2.4.1. If G is a power mean labeling graph, either the vertices or edges must be labeled with number 1.

Remark 2.4.2. $G(p, q)$ will not consider to be a power mean graph, if it has $p > q+1$.

3. Main Result

In this section the power mean labeling of the connected graph $P_n \Theta S_3$ and $P_n + S_4$ is presented.

Theorem 3.1. *The connected graph $P_n \Theta S_3$ is a power mean graph.*

Proof. The following graph is $P_n \Theta S_3$ with $4n$ vertices and $4n - 1$ edges.

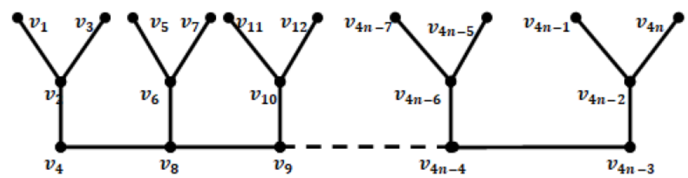

Figure 1. Power Mean Labeling of the Graph $P_n \Theta S_3$ for odd 'n'.

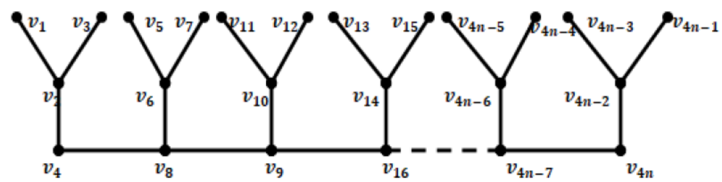

Figure 2. Power Mean Labeling of the Graph $P_n \Theta S_3$ for even 'n'.

To find Power Mean Labeling, we define $f : V(G) \to \{1, 2, \ldots, q + 1\}$ by $f(v_i) = i$, for $1 \le i \le 4n$.

Hence the edges are labeled as per the definition, $E(G) \to \{1, 2, \ldots, q\}$ by

$$f(e_i = u_i v_i) = \left\lceil (f(u_i)^{f(v_i)} f(v_i)^{f(v_i)})^{\frac{1}{f(u_i)+f(v_i)}} \right\rceil$$

or

$$f(e_i = u_i v_i) = \left\lfloor (f(u_i)^{f(v_i)} f(v_i)^{f(v_i)})^{\frac{1}{f(u_i)+f(v_i)}} \right\rfloor.$$

Since the graph admits power mean labeling and the edge labelings are distinct, the graph $P_n \Theta S_3$ is power mean graph.

Example 3.1. The connected graph $P_3 \Theta S_3$ is a power mean graph.

It has 12 vertices and 11 edges. The graph is labeled as per figure 1 and is given below:

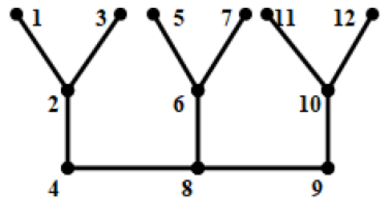

Figure 3. Power Mean graph $P_3 \Theta S_3$.

Example 3.2. The connected graph $P_4 + S_3$ is a power mean graph.

It has 16 vertices and 15 edges. The graph is labeled as per figure 2 and is given below:

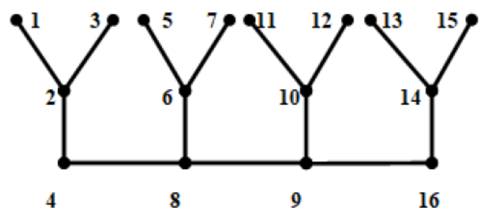

Figure 4. Power Mean graph $P_4 \Theta S_3$.

Theorem 3.2. *The connected graph* $P_n + S_4$ *is a power mean graph.*

Proof. The following graph is $P_n + S_4$ with $5n$ vertices and $5n - 1$ edges.

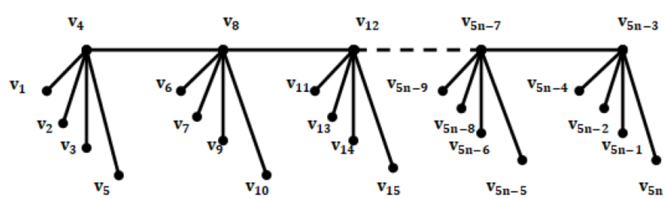

Figure 5. Power Mean Labeling of the Graph $P_n + S_4$.

To find Power Mean labeling we define $V(G) \to \{1, 2, \ldots, q+1\}$ for $P_n + S_4$ as $f(v_i) = i$, for $1 \leq i \leq 5n$.

The edges are labeled as per the definition, $E(G) \to \{1, 2, \ldots, q\}$ by

$$f(u_i = ev_i) = \left\lceil (f(u_i)^{f(v_i)} f(v_i)^{f(v_i)})^{\frac{1}{f(u_i)+f(v_i)}} \right\rceil$$

or

$$f(u_i = ev_i) = \left\lfloor (f(u_i)^{f(v_i)} f(v_i)^{f(v_i)})^{\frac{1}{f(u_i)+f(v_i)}} \right\rfloor.$$

Therefore, the graph holds the definition of Power Mean Labeling and the edge labelings are distinct. Hence the graph $P_n + S_4$ is Power Mean graph.

Example 3.3. The connected graph $P_6 + S_4$ is a power mean graph. This graph $P_6 + S_4$ has 30 vertices and 29 edges. The graph is labeled as per figure 4 and is given below:

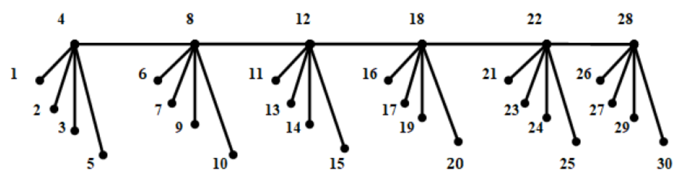

Figure 6. Power Mean graph $P_6 + S_4$.

Conclusion

In this paper, we have investigated the Power Mean labeling for the graphs $P_n \Theta S_3$ and $P_n + S_4$.

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