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OPERATIONS ON MULTI FUZZY GRAPH

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Abstract

In this paper, we examined some operations on Multi fuzzy graph such as like union, intersection and join which is related to the view of operations on fuzzy graph were defined and also prove some theorems related to them.

1. Introduction

In 1975, Rosenfeld [9] proposed the concepts of fuzzy graphs. Fuzzy graph theory is evolved with plenty of sections. Thereafter in 1987, Bhattacharya [1] defined some remarks on fuzzy graphs. The operations of union and join on

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two fuzzy graphs were defined by Mordeson. J. N. and Premchand S. Nair, [2] in 2000. Sabu Sebastian, T. V. Ramakrishnan [10] defined Multi fuzzy set in 2010. Multi-fuzzy set theory is useful to characterize the problems. Later on, Multi-fuzzy group and its level subgroups, Anti Fuzzy graph, Multi fuzzy graph and Multi anti fuzzy graph defined by Muthuraj R. et al. [3, 4, 5, 6, 7]. In this paper, we examined some operations on Multi fuzzy graph such as like union, intersection and join which is related to the view of operations on fuzzy graph were defined and also prove some theorems related to them.

2. Preliminaries

Definition 2.1. A *fuzzy graph* $G = (\sigma, \mu)$ defined on the underlying crisp graph $G^* = (V, E)$ where $E \subseteq V \times V$ is a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1], \mu$ is a symmetric fuzzy relation on σ such that $\mu(uv) \leq \min\{\sigma(u), \sigma(v)\}$ for all $u, v \in V$.

Definition 2.2. Let X be a non-empty set. A *Multi Fuzzy set* A in X is defined as a set of ordered sequences: $A = \{(x, \mu_1(x), \mu_2(x), \ldots, \mu_i(x)\ldots) : x \in X\}$ where $\mu_i : X \to [0, 1]$ for all i.

Definition 2.3. A *Multi fuzzy Graph* (*MFG*) of dimension m defined on the underlying crisp graph $G^* = (V, E)$ where $E \subseteq V \times V$, is denoted as $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ and $\sigma_i : V \to [0, 1]$ and $\mu_i : V \times V \to [0, 1], \mu_i$ symmetric fuzzy relation on σ_i such that $\mu_i(uv) \leq \min\{\sigma_i(u), \sigma_i(v)\}$ for all $i = 1, 2, 3, \ldots, m$ where $u, v \in V$ and $uv \in E$.

Definition 2.4. A multi fuzzy graph of dimension m defined on the underlying crisp graph $G^* = (V, E)$ where $E \subseteq V \times V$, is denoted as $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ is said to be a complete multi fuzzy graph if $\mu_i(u, v) = \sigma_i(u) \wedge \sigma_i(v)$ for all $i = 1, 2, 3, \ldots, m$ and for all $u, v \in V$, where G^* is a complete graph.

3. Main Results

Operations on Multi Fuzzy Graph

Throughout this section $G_1 = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ denotes the multi fuzzy graph with dimension m with the underlying crisp graph $G_1^* = (V_1, E_1)$ and $G_2 = ((\alpha_1, \alpha_2, \ldots, \alpha_n), (\beta_1, \beta_2, \ldots, \beta_n))$ denotes the multi fuzzy graph with dimension n with the underlying crisp graph $G_2^* = (V_2, E_2)$.

Definition 3.1. A multi fuzzy graph $H : ((\tau_1, \tau_2, \ldots, \tau_m), (\rho_1, \rho_2, \ldots, \rho_m))$ of dimension m is a multi fuzzy subgraph of $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ of dimension m if $\tau_i(u) \le \sigma_i(u)\ \forall u \in V$ and $\rho_i(u, v) \le \mu_i(u, v) \forall (u, v) \in E$.

Definition 3.2. A multi fuzzy graph $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ of dimension m is said to be a strong multi fuzzy graph if $\mu_i(u, v) = \sigma_i(u) \wedge \sigma_i(v)$ for all $i = 1, 2, 3, \ldots, m$ and for all $(u, v) \in E$.

Definition 3.3. The operation *Union* between two MFG G_1 and G_2 is defined as follows, $G_1 \cup G_2 = ((\sigma_1 \cup \alpha_1, \sigma_2 \cup \alpha_2, \ldots, \sigma_k \cup \alpha_k), (\mu_1 \cup \beta_1, \mu_2 \cup \beta_2, \ldots, \mu_k \cup \beta_k))$ with the underlying crispgraph $G_1^* \cup G_2^* = (V_1 \cup V_2, E_1 \cup E_2)$,

$$(\sigma_i \cup \alpha_i)(u) = \begin{cases} \sigma_i(u) \text{ if } u \in V_1 - V_2 \\ \alpha_i(u) \text{ if } u \in V_2 - V_1 \\ \max\{\sigma_i(u), \alpha_i(u)\} \text{ if } u \in V_1 \cap V_2 \end{cases} \quad \text{for all } i = 1, 2, 3, \ldots, k$$

$$(\mu_i \cup \beta_i)(u, v) = \begin{cases} \mu_i(u, v) & \text{if } (u, v) \in E_1 - E_2 \\ \beta_i(u, v) & \text{if } (u, v) \in E_2 - E_1 \\ \max\{\mu_i(u, v), \beta_i(u, v)\} & \text{if } (u, v) \in E_1 \cap E_2 \end{cases}$$

$$\text{for all } i = 1, 2, 3, \ldots, k$$

If $m \ne n$ let $k = \max(m, n)$. Suppose $m < n$ then let us introduce $n - m$ membership values of multi fuzzy graph G_1 into 0 so as to convert the multi fuzzy graphs G_1 and G_2 have the same dimension as k.

Example 3.4.

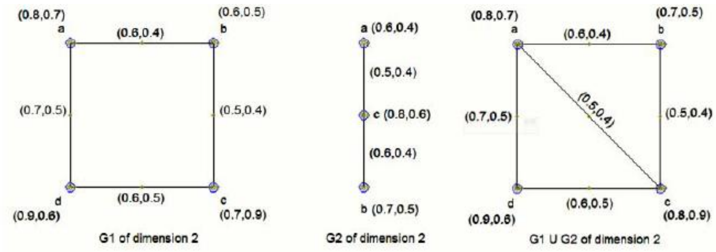

Figure (1).

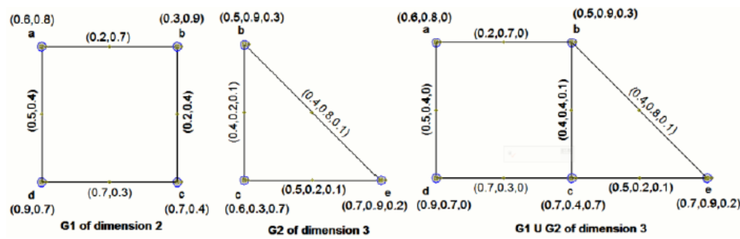

Figure (2).

Definition 3.5. The operation *Intersection* between two MFG G_1 and G_2 is defined as follows, $G_1 \cap G_2 = ((\sigma_1 \cap \alpha_1, \sigma_2 \cap \alpha_2, \ldots, \sigma_k \cap \alpha_k), (\mu_1 \cap \beta_1, \mu_2 \cap \beta_2, \ldots, \mu_k \cap \beta_k))$ with the underlying crisp graph $G_1^* \cap G_2^* = (V_1 \cap V_2, E_1 \cap E_2)$,

$$(\sigma_i \cap \alpha_i)(u) = \min\{\sigma_i(u), \alpha_i(u)\} \text{ if } u \in V_1 \cap V_2$$

$$(\mu_i \cap \beta_i)(u, v) = \min\{\mu_i(u, v), \beta_i(u, v)\} \text{ if } (u, v) \in E_1 \cap E_2 \text{ for all } i = 1, 2, 3, \ldots, k.$$

If $m \neq n$ let $k = \min(m, n)$. Suppose $m < n$ then we take first m dimensions for G_2 so as to convert the MFG G_1 and G_2 have the same dimension k.

Example 3.6.

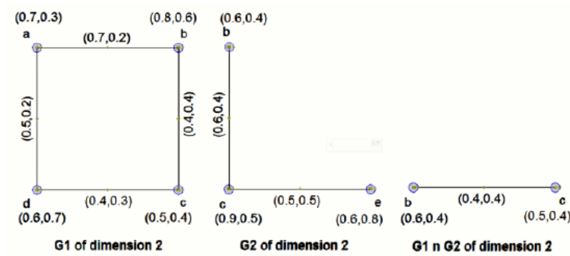

Figure (3).

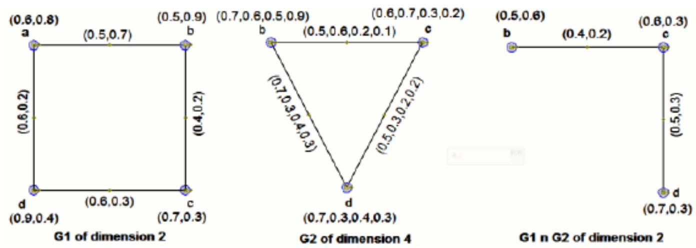

Figure (4).

Definition 3.7. The operation *Join* between two MFG G_1 and G_2 is defined as follows, $G_1 + G_2 = ((\sigma_1 + \alpha_1, \sigma_2 + \alpha_2, \ldots, \sigma_k + \alpha_k), (\mu_1 + \beta_1, \mu_2 + \beta_2, \ldots, \mu_k + \beta_k))$ with the underlying crisp graph $G_1^* + G_2^* = (V_1 \cup V_2, E_1 \cup E_2 \cup E')$, where E' is the set of all edges joining vertices of V_1 with vertices of V_2 and we assume that $V_1 \cap V_2 = \phi$

$$(\sigma_i + \alpha_i)(u) = \begin{cases} \sigma_i(u) \text{ if } u \in V_1 \\ \alpha_i(u) \text{ if } u \in V_2 \end{cases} \text{ and}$$

$$(\mu_i + \beta_i)(u, v) = \begin{cases} \mu_i(u, v) \text{ if } (u, v) \in E_1 \\ \beta_i(u, v) \text{ if } (u, v) \in E_2 \\ \min(\sigma_i(u), \alpha_i(v)) \text{ if } (u, v) \in E' \end{cases} \quad \text{for all } i = 1, 2, 3, \ldots, k$$

If $m \neq n$ let $k = \max(m, n)$. Suppose $m < n$ then let us introduce $n - m$ membership values of multi fuzzy graph G_1 into 0 so as to convert the multi fuzzy graphs G_1 and G_2 have the same dimension as k.

Example 3.8.

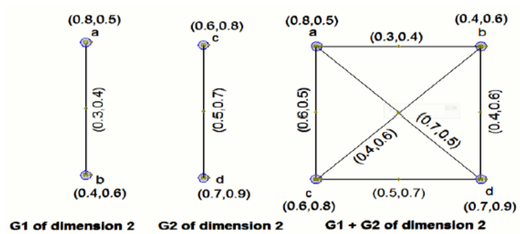

Figure (5).

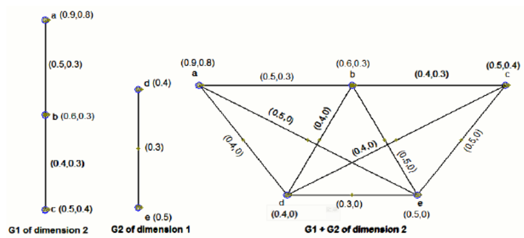

Figure (6).

Theorem 3.9. *The union of two multi fuzzy graphs is also a multi fuzzy graph.*

Proof. Let $G_1 = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ and $G_2 = ((\alpha_1, \alpha_2, \ldots, \alpha_n), (\beta_1, \beta_2, \ldots, \beta_n))$ be the multi fuzzy graph with dimension m and n respectively

To prove: $G = G_1 \cup G_2$ is also multi fuzzy graph with dimension k where $k = \max(m, n)$

Case I. Let $(u, v) \in E_1$

Subcase (i) $u, v \in V_1$

$(\sigma_i \cup \alpha_i)(u) = \sigma_i(u)$ and $(\sigma_i \cup \alpha_i)(v) = \sigma_i(v)$

$(\mu_i \cup \beta_i)(u, v) = \mu_i(u, v) \leq \min\{\sigma_i(u), \sigma_i(v)\}$

$$= \min\{(\sigma_i \cup \alpha_i)(u), (\sigma_i \cup \alpha_i)(v)\}$$

$\therefore (\mu_i \cup \beta_i)(u, v) \leq \min\{(\sigma_i \cup \alpha_i)(u), (\sigma_i \cup \alpha_i)(v)\}$

Subcase (ii) $u \in V_1$ and $v \in V_1 \cap V_2$

$(\sigma_i \cup \alpha_i)(u) = \sigma_i(u)$

$(\sigma_i \cup \alpha_i)(v) = \max\{\sigma_i(v), \alpha_i(v)\}$

$$\begin{aligned}(\mu_i \cup \beta_i)(u, v) &= \mu_i(u, v)\\ &\leq \min\{\sigma_i(u), \sigma_i(v)\} \text{ since } (u, v) \in E_1\\ &= \min\{(\sigma_i \cup \alpha_i)(u), \max(\sigma_i(u), \alpha_i(v))\}\\ &= \min\{(\sigma_i \cup \alpha_i)(u), (\sigma_i \cup \alpha_i)(v)\}\end{aligned}$$

$\therefore (\mu_i \cup \beta_i)(u, v) \leq \min\{(\sigma_i \cup \alpha_i)(u), (\sigma_i \cup \alpha_i)(v)\}$

Subcase (iii) $u, v \in V_1 \cap V_2$

$(\sigma_i \cup \alpha_i)(u) = \max\{\sigma_i(u), \alpha_i(u)\}$

$(\sigma_i \cup \alpha_i)(v) = \max\{\sigma_i(v), \alpha_i(v)\}$

$$\begin{aligned}(\mu_i \cup \beta_i)(u, v) &= \mu_i(u, v) \text{ since } (u, v) \in E_1\\ &\leq \min\{\sigma_i(u), \sigma_i(v)\}\\ &= \min\{\max\{\sigma_i(u), \alpha_i(u)\}, \max(\sigma_i(v), \alpha_i(v))\}\\ &= \min\{(\sigma_i \cup \alpha_i)(u), (\sigma_i \cup \alpha_i)(v)\}\end{aligned}$$

$\therefore (\mu_i \cup \beta_i)(u, v) \leq \min\{(\sigma_i \cup \alpha_i)(u), (\sigma_i \cup \alpha_i)(v)\}$

Case II. $(u, v) \in E_2$

Subcase (i) $u, v \in V_2$

$(\sigma_i \cup \alpha_i)(u) = \alpha_i(u)$

$(\sigma_i \cup \alpha_i)(v) = \alpha_i(v)$

$$\begin{aligned}(\mu_i \cup \beta_i)(u, v) &= \beta_i(u, v) \text{ since } (u, v) \in E_2\\ &\leq \min\{\alpha_i(u), \alpha_i(v)\}\\ &= \min\{(\sigma_i \cup \alpha_i)(u), (\sigma_i \cup \alpha_i)(v)\}\end{aligned}$$

$\therefore (\mu_i \cup \beta_i)(u, v) \leq \min\{(\sigma_i \cup \alpha_i)(u), (\sigma_i \cup \alpha_i)(v)\}$

Subcase (ii) $u \in V_2$ and $v \in V_1 \cap V_2$

$(\sigma_i \cup \alpha_i)(u) = \alpha_i(u)$

$(\sigma_i \cup \alpha_i)(v) = \max\{\sigma_i(v), \alpha_i(v)\}$

$$\begin{aligned}(\mu_i \cup \beta_i)(u, v) &= \beta_i(u, v) \text{ since } (u, v) \in E_2 \\ &\leq \min\{\alpha_i(u), \alpha_i(v)\} \\ &= \min\{(\sigma_i \cup \alpha_i)(u), \max(\sigma_i(u), \alpha_i(v))\} \\ &= \min\{(\sigma_i \cup \alpha_i)(u), (\sigma_i \cup \alpha_i)(v)\}\end{aligned}$$

$\therefore (\mu_i \cup \beta_i)(u, v) \leq \min\{(\sigma_i \cup \alpha_i)(u), (\sigma_i \cup \alpha_i)(v)\}$

Subcase (iii) $u, v \in V_1 \cap V_2$

$(\sigma_i \cup \alpha_i)(u) = \max\{\sigma_i(u), \alpha_i(u)\}$

$(\sigma_i \cup \alpha_i)(v) = \max\{\sigma_i(v), \alpha_i(v)\}$

$$\begin{aligned}(\mu_i \cup \beta_i)(u, v) &= \beta_i(u, v) \text{ since } (u, v) \in E_2 \\ &\leq \min\{\alpha_i(u), \alpha_i(v)\} \\ &= \min\{\max\{\sigma_i(u), \alpha_i(u)\}, \max\{\sigma_i(v), \alpha_i(v)\}\} \\ &= \min\{(\sigma_i \cup \alpha_i)(u), (\sigma_i \cup \alpha_i)(v)\}\end{aligned}$$

$\therefore (\mu_i \cup \beta_i)(u, v) \leq \min\{(\sigma_i \cup \alpha_i)(u), (\sigma_i \cup \alpha_i)(v)\}$

Case III. $(u, v) \in E_1 \cap E_2$

$(\sigma_i \cup \alpha_i)(u) = \max\{\sigma_i(u), \alpha_i(u)\}$

$(\sigma_i \cup \alpha_i)(v) = \max\{\sigma_i(v), \alpha_i(v)\}$

$$\begin{aligned}(\mu_i \cup \beta_i)(u, v) &= \max\{\mu_i(u, v), \beta_i(u, v)\} \\ &\leq \max\{\min\{\sigma_i(u), \sigma_i(v)\}, \min\{\alpha_i(u), \alpha_i(v)\}\} \\ &= (\sigma_i(u) \wedge \sigma_i(v)) \vee (\alpha_i(u) \wedge \alpha_i(v))\end{aligned}$$

$$= (\sigma_i(u) \vee \alpha_i(u)) \wedge (\sigma_i(v) \vee \alpha_i(v))$$

$$= (\sigma_i \cup \alpha_i)(u) \wedge (\sigma_i \cup \alpha_i)(v)$$

$$\therefore (\mu_i \cup \beta_i)(u, v) \leq \min\{(\sigma_i \cup \alpha_i)(u), (\sigma_i \cup \alpha_i)(v)\}$$

Theorem 3.10. *The intersection of two strong multi fuzzy graphs is also a strong multi fuzzy graph.*

Proof. Let $G_1 = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ and $G_2 = ((\alpha_1, \alpha_2, \ldots, \alpha_n), (\beta_1, \beta_2, \ldots, \beta_n))$ be the strong multi fuzzy graph with dimension m and n respectively

To prove: $G = G_1 \cap G_2$ is also a strong multi fuzzy graph with dimension k where $k = \min(m, n)$

$$(\sigma_i \cap \alpha_i)(u) = \min\{\sigma_i(u), \alpha_i(u)\}$$

$$(\mu_i \cap \beta_i)(u, v) = \min\{\mu_i(u, v), \beta_i(u, v)\}$$

$$= \min\{\min\{\sigma_i(u), \sigma_i(v)\}, \min\{\alpha_i(u), \alpha_i(v)\}\}$$

$$= (\sigma_i(u) \wedge \sigma_i(v)) \wedge (\alpha_i(u) \wedge \alpha_i(v))$$

$$= (\sigma_i(u) \wedge \alpha_i(u)) \wedge (\sigma_i(v) \wedge \alpha_i(v))$$

$$\therefore (\mu_i \cap \beta_i)(u, v) = \min\{(\sigma_i \cap \alpha_i)(u), (\sigma_i \cap \alpha_i)(v)\}$$

Theorem 3.11. *The join of two multi fuzzy graphs is also a multi fuzzy graph.*

Proof. Let $G_1 = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ and $G_2 = ((\alpha_1, \alpha_2, \ldots, \alpha_n), (\beta_1, \beta_2, \ldots, \beta_n))$ be the multi fuzzy graph with dimension m and n respectively

To prove: $G = G_1 + G_2$ is also multi fuzzy graph with dimension k where $k = \max(m, n)$

Case I. $(u, v) \in E_1 \cup E_2$

$$(\sigma_i + \alpha_i)(u) = \begin{cases} \sigma_i(u) \text{ if } u \in V_1 \\ \alpha_i(u) \text{ if } u \in V_2 \end{cases}$$

$(\mu_i + \beta_i)(u, v) \leq \min\{(\sigma_i + \alpha_i)(u), (\sigma_i + \alpha_i)(v)\}$ (by theorem 3.9)

Case II. $(u, v) \in E'$

$(\mu_i + \beta_i)(u, v) = \min\{(\sigma_i(u), \alpha_i(v))\}$

$= \min\{(\sigma_i + \alpha_i)(u), (\sigma_i + \alpha_i)(v)\}$

$\therefore (\mu_i + \beta_i)(u, v) = \min\{(\sigma_i + \alpha_i)(u), (\sigma_i + \alpha_i)(v)\}$

Theorem 3.12. *The join of two strong multi fuzzy graphs is also a strong multi fuzzy graph.*

Proof. Case I. $(u, v) \in E_1 \cup E_2$

Subcase (i) $(u, v) \in E_1 - E_2$

$(\mu_i + \beta_i)(u, v) = (\mu_i \cup \beta_i)(u, v)$

$= \mu_i(u, v)$

$= \min\{\sigma_i(u), \alpha_i(v)\} = \min\{(\sigma_i \cup \alpha_i)(u), (\sigma_i \cup \alpha_i)(v)\}$

$= \min\{(\sigma_i + \alpha_i)(u), (\sigma_i + \alpha_i)(v)\}$

$\therefore (\mu_i + \beta_i)(u, v) = \min\{(\sigma_i + \alpha_i)(u), (\sigma_i + \alpha_i)(v)\}$

Subcase (ii) $(u, v) \in E_2 - E_1$

$(\mu_i + \beta_i)(u, v) = (\mu_i \cup \beta_i)(u, v)$

$= \beta_i(u, v)$

$= \min\{\alpha_i(u), \alpha_i(v)\}$

$= \min\{(\sigma_i \cup \alpha_i)(u), (\sigma_i \cup \alpha_i)(v)\}$

$\therefore (\mu_i + \beta_i)(u, v) = \min\{(\sigma_i + \alpha_i)(u), (\sigma_i + \alpha_i)(v)\}$

Case II. $(u, v) \in E'$

$(\mu_i + \beta_i)(u, v) = \min\{\sigma_i(u), \alpha_i(v)\}$

$= \min\{(\sigma_i \cup \alpha_i)(u), (\sigma_i \cup \alpha_i)(v)\}$

$$= \min\{(\sigma_i + \alpha_i)(u), (\sigma_i + \alpha_i)(v)\}$$

$$\therefore (\mu_i + \beta_i)(u, v) = \min\{(\sigma_i + \alpha_i)(u), (\sigma_i + \alpha_i)(v)\}$$

Theorem 3.13. *If G is a union of two multi fuzzy graphs G_1 and G_2 then every multi fuzzy subgraph $H = ((\tau_1, \tau_2, \ldots, \tau_m), (\rho_1, \rho_2, \ldots, \rho_m))$ of G is the union of multi fuzzy subgraphs of G_1 and G_2.*

Proof. Let $G_1 = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ and $G_2 = ((\alpha_1, \alpha_2, \ldots, \alpha_n), (\beta_1, \beta_2, \ldots, \beta_n))$ be the multi fuzzy graph with dimension m and n respectively and G is a union of two multi fuzzy graphs G_1 and G_2 of dimension k where $k = \max(m, n)$. Define the multi fuzzy subsets $(\sigma'_1, \sigma'_2, \ldots, \sigma'_m), (\mu'_1, \mu'_2, \ldots, \mu'_m), (\alpha'_1, \alpha'_2, \ldots, \alpha'_n), (\beta'_1, \beta'_2, \ldots, \beta'_n)$ on V_1, E_1, V_2 and E_2 respectively as follows:

$$\sigma'_i(u) = \tau_i(u),\ u \in V_1 \text{ and } \alpha'_i(u) = \tau_i(u),\ u \in V_2$$

$$\mu'_i(u, v) = \rho_i(u, v),\ (u, v) \in E_1 \text{ and } \beta'_i(u, v) = \rho_i(u, v),\ (u, v) \in E_2$$

$$\mu'_i(u_i, v_i) = \rho_i(u_i, v_i)$$

$$\leq \min\{\tau_i(u_i), \tau_i(v_i)\}$$

$$= \min\{\sigma'_i(u_i), \sigma'_i(v_i)\}$$

$$\mu'_i(u_i, v_i) \leq \min\{\sigma'_i(u_i), \sigma'_i(v_i)\} \text{ where } (u_i, v_i) \in E_1$$

$$\beta'_i(u_i, v_i) = \rho_i(u_i, v_i)$$

$$\leq \min\{\tau_i(u_i), \tau_i(v_i)\}$$

$$= \min\{\alpha'_i(u_i), \alpha'_i(v_i)\}$$

$$\beta'_i(u_i, v_i) \leq \min\{\alpha'_i(u_i), \alpha'_i(v_i)\} \text{ where } (u_i, v_i) \in E_2$$

Thus $((\sigma'_1, \sigma'_2, \ldots, \sigma'_m), (\mu'_1, \mu'_2, \ldots, \mu'_m))$ and $((\alpha'_1, \alpha'_2, \ldots, \alpha'_n), (\beta'_1, \beta'_2, \ldots, \beta'_n))$ are the multi fuzzy subgraph of G_1 and G_2. Clearly $\tau_i = \sigma'_i \cup \alpha'_i$ and $\rho_i = \mu'_i \cup \beta'_i$ Therefore union of two multi fuzzy subgraphs of G_1 and G_2 is a multi fuzzy subgraph of G.

Theorem 3.14. *If G is the join of two multi fuzzy graphs G_1 and G_2 then every strong multi fuzzy subgraph $H = ((\tau_1, \tau_2, \ldots, \tau_m), (\rho_1, \rho_2, \ldots, \rho_m))$ of G is the join of a strong multi fuzzy subgraphs of G_1 and G_2.*

Proof. Let $G_1 = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ and $G_2 = ((\alpha_1, \alpha_2, \ldots, \alpha_n), (\beta_1, \beta_2, \ldots, \beta_n))$ be the multi fuzzy graph with dimension m and n respectively and G is a union of two multi fuzzy graphs G_1 and G_2 of dimension k where $k = \max(m, n)$. Define the multi fuzzy subsets $(\sigma'_1, \sigma'_2, \ldots, \sigma'_m), (\mu'_1, \mu'_2, \ldots, \mu'_m), (\alpha'_1, \alpha'_2, \ldots, \alpha'_n), (\beta'_1, \beta'_2, \ldots, \beta'_n)$ on V_1, E_1, V_2 and E_2 respectively as follows:

$\sigma'_i(u) = \tau_i(u), u \in V_1$ and $\alpha'_i(u) = \tau_i(u), u \in V_2$

$\mu'_i(u, v) = \rho_i(u, v), (u, v) \in E_1$ and $\beta'_i(u, v) = \rho_i(u, v), (u, v) \in E_2$

Clearly $((\sigma'_1, \sigma'_2, \ldots, \sigma'_m), (\mu'_1, \mu'_2, \ldots, \mu'_m))$ and $((\alpha'_1, \alpha'_2, \ldots, \alpha'_n), (\beta'_1, \beta'_2, \ldots, \beta'_n))$ are the multi fuzzy subgraph of G_1 and G_2 and $\tau_i = \sigma'_i \cup \alpha'_i$ and $\rho_i = \mu'_i \cup \beta'_i$ (by the theorem 3.13.) Hence $\tau_i = \sigma'_i + \alpha'_i$ and $\rho_i = \mu'_i + \beta'_i$.

If $(u, v) \in E_1 \cup E_2$ then $\rho_i(u, v) = (\mu'_i \cup \beta'_i)(u, v) = (\mu'_i + \beta'_i)(u, v)$

If $(u, v) \in E'$ where $u \in V_1$ and $v \in V_2$ then

$(\mu'_i + \beta'_i)(u, v) = \min(\sigma'_i(u), \alpha'_i(v)) = \min(\tau_i(u), \tau_i(v)) = \rho_i(u, v)$

Therefore join of two strong multi fuzzy subgraphs of G_1 and G_2 is also a strong multi fuzzy subgraph of G.

4. Conclusion

This investigates on different type of operations showing various formations of multi fuzzy graph and their theorems. Some operations such as like union, intersection and join are defined on Multi fuzzy graph. Several theorems are explained on them. In future, work on other different operations on multi fuzzy graph and the usages of operations on multi fuzzy graph will be obtained.

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