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# SHEAR WAVE PROPAGATION WITH RECTANGULAR IRREGULARITY IN DIFFERENT MEDIA: A REVIEW

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## Abstract

This paper throws light on the effect of irregularities on propagation of shear waves in different media. This article mainly focuses on obtaining the frequency relation for shear wave propagation by using perturbation method for some standard models in different layers with and without rectangular irregularity present at the interface along with suitable boundary conditions. Since, the wave propagation is an essential and important phenomenon in various fields like geophysics, seismology, mechanics, fluid dynamics etc. The models reviewed in this paper are very helpful to new researchers to do their research as it provides a platform for them to understand the basic concepts of wave propagation with irregularity.

### Nomenclature

| | |
|---|---|
| $e_{ij}$ : | Strain tensor |
| $\tau_{ij}$ : | Stress tensor |
| $F_i$ : | Components of the vector of the electromagnetic force directed to the body |
| $\rho$: | Mass Density |
| $c_{ijkl}$ : | Elasticity constants |
| $\lambda$, $\mu$ : | Lame's constants |

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|---|---|
| $\delta_{ij}$ : | Kronecker delta |
| $U_i$, $u_i$ : | Displacement vector of fluid and solid respectively |
| $\rho_{11}, \rho_{22}, \rho_{12}$ : | Dynamic coefficients in inertia effect of moving fluid |
| $\nabla$ : | Differential operator del |
| $\mu_e$ : | Magnetic permeability |
| $s$: | Length of irregularity |
| $H'$ : | Depth of irregularity |
| $\boldsymbol{B}$: | Magnetic field vector |
| $\boldsymbol{E}$: | Electric field vector |
| $\boldsymbol{J}$: | Current density |

## 1. Introduction

The study of earthquakes is called seismology. Earthquake is a natural disaster which occurs by shaking of the Earth's surface for mainly short period of time and causes damage to natural as well as man-made structure such as buildings, bridges etc. So the study of earthquakes is very important for seismologists because at the time of earthquakes, a large amount of energy is unrestricted inside the Earth that released energy transmitted in the form of waves, called the seismic waves. The study of these waves is helpful for predicting about the interior of the Earth, as it is assumed to be layered structure and thus, we are able to understand the properties of the materials by which the Earth is made of and through which the waves can propagate.

Seismic waves were discovered by Robert Mallet in 1857 and further work was done on seismic wave propagation in different mediums by Sato [1] and Kennett [2]. Richard Oldham, a British geologist discovered there are two kinds of seismic waves i.e. body and surface waves. Body waves travel in internal part of the Earth. These waves can be further classified into two kinds: (i) Primary waves ($P$-waves) and (ii) Secondary waves ($S$-waves). Shear waves are elastic body waves in which the oscillations of the particles of the medium in the direction perpendicular to the disturbance and therefore they are called the transverse waves. These waves can be polarised into vertical and horizontal directions, which are known as shear horizontal waves (or *SH*-waves) and shear vertical waves (or *SV*-waves) respectively. The surface waves were first mathematically described by a young British mathematician, John William Strutt. Surface waves are also of two kinds Love waves, after the name of the discoverer Love A.E.H. in 1911 and Rayleigh waves, after the name of Lord Rayleigh in 1887 respectively. Stoneley [3] developed wave propagation in separation of elastic solids. Some of noble books explaining the concept of the wave propagation are Bullen [4], Ewing et al. [5], Hunter [6] and Jeffreys [7].

A porous material is a material containing porous voids. The skeleton segment of materials are frequently called frame or network of pores. The porous are typically filled with fluid, liquid or gases. The skeleton materials are usually solids. A porous medium is categorized on the basis of its mainly two properties such as porosity and permeability. Porosity is the amount of pore space (holes) in a material compared with its volume. The porosity is affected by rocks solid sentiment stored in the soil, actual size, degree of packing, particle shape and degree of sorting. Permeability is the capacity of water to flow through Earth's surface. In fluid mechanics, the phenomenon of fluid saturated medium is combination of two phases comprising incompressible fluid phase and elastic solids.

The main aim of this paper is to review the effects on various parameters like magnetic field, irregular boundary, non-homogeneity, homogeneity when the shear waves propagate in different medium with rectangular type of irregularity over a half-space. This paper is organized in seven sections: Section 2 comprises of basic equations. Section 3 and 4 is dedicated to shear wave propagation in homogeneous monoclinic medium and non-homogeneous monoclinic medium at the interface respectively. Shear wave propagation in fluid saturated porous medium and multilayered fluid saturated porous medium including irregularity was discussed in Section 5 and 6 respectively. Section 7 is dedicated to observations and future scope along with the nomenclature and list of references.

## 2. Basic Equations

Some basic equations which are used in the research papers in the field of wave propagation are given below:

2.1. The general equation of motion with body forces is

$$\rho \ddot{u}_i = \sigma_{ij,j} + \rho F_i.$$

2.2. The equation of motion without body forces is

$$\rho \ddot{u}_i = \sigma_{ij,j}.$$

2.3. For fluid saturated medium, the equation of motion without body forces is

$$\sigma_{ij,j} = \rho_{11}\ddot{u}_i + \rho_{12}\ddot{U}_i - b_{ij}(\dot{U}_j - \dot{u}_j),$$

$$\sigma_{,j} = \rho_{12}\ddot{u}_i + \rho_{22}\ddot{U}_i - b_{ij}(\dot{U}_j - \dot{u}_j).$$

2.4. The strain-displacement relation

$$e_{ij} = \frac{1}{2}(u_{ij} + u_{ji}).$$

2.5. Generalized Hooke's law

$$\tau_{ij} = c_{ijkl}e_{kl}.$$

2.6. Hooke's law for isotropic elastic medium

$$\tau_{ij} = \lambda\delta_{ij}\upsilon + 2\mu e_{ij}.$$

2.7. Fourier transform of a function $U_r(y, z)(r = 1, 2, 3)$ is defined as

$$\overline{U}_r(y, \eta) = \int_{\infty}^{-\infty} U_r(y, z)e^{-i\eta z}dz.$$

2.8. Inverse Fourier transform of a function $U_r(y, z)(r = 1, 2, 3)$ is defined as

$$U_r(y, z) = \frac{1}{2\pi}\int_{\infty}^{-\infty} \overline{U}_r U_r(y, \eta)e^{-i\eta z}d\eta.$$

2.9. Maxwell's equations for an electromagnetic field are

$$\nabla \cdot \mathbf{B} = 0,\ \nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t},\ \nabla \times \mathbf{H} = \mathbf{J},\ \mathbf{B} = \mu_e \mathbf{H},\ \mathbf{J} = \sigma\left(\mathbf{E} + \frac{\partial u_i}{\partial t} \times \mathbf{B}\right).$$

2.10. The equation of rectangular irregular interface between two mediums at $y = \varepsilon h(z)$ where

$$h(z) = \begin{cases} 0 & ; \quad z \leq -\frac{s}{2},\ z \geq \frac{s}{2} \\ f(z) & ; \quad -\frac{s}{2} \leq z \geq \frac{s}{2}, \end{cases}$$

where $\varepsilon = \frac{H'}{s}$ and $\varepsilon << 1$.

## 3. Shear Wave Propagation in Homogeneous Monoclinic Medium

As the structure of Earth is complex and its characteristics are not same in different layers, so investigation of wave propagation in monoclinic medium became the core of interest for seismologist. Bhattacharya [8] discussed the propagation of Love waves and plotted the dispersion equation graphically to compare the obtained results with existing one for an in consistent transversely isotropic crustal layer at the interface. The irregularity effects on shear wave propagation were investigated by Chattopadhyay et al. [9]. Singh and Tomar [10] investigated wave propagation between two different monoclinic half-spaces at the interface of quasi-*P* waves. Chattopadhyay [11] studied the effects of phase velocity, wave number, shape and size of the irregularity on the propagation of SH waves in a crustal layer and the frequency equation was obtained with the help of equations 2.7, 2.8, 2.9 and 2.10 in the absence of irregularity lying in isotropic semi-infinite medium with monoclinic layer. The shear wave propagation in a model having anirregular monoclinic layerlying between two isotropic half-spaces had been studied by Chattopadhyay et al. [12]. The effect of presence of magnetoelastic irregular monoclinic layer had been seen on different parameters with the help of graphs. The result showed that the phase velocity decreases as the size of irregularity and monoclinic elastic coupling parameters increases. Chattopadhyay et al. [13] obtained the dispersion equation for shear wave propagation in a monoclinic layer resting between two isotropic half-spaces by using equations 2.8, 2.9 and 2.10 and the dispersion equation was plotted graphically and studied the effect of wave number, phase velocity, size and shape of irregularity the propagation of shear waves in monoclinic layer.

## 4. Shear Wave Propagation in Non-Homogeneous Monoclinic Medium

Since, the surface of Earth is covered by rocks and water bodies. Crystals are the solids whose molecules (or atoms) are arranged in a repeating pattern and crystalline media plays an important role for geophysicists mainly in signal processing and ultrasonic. As the Earth's surface is not homogeneous so the study of irregular boundaries on propagation of elastic waves gained much importance due to varying environmental conditions and it helps to understand seismological behaviour at continental margins and mountain roots. Chattopadhyay [14] discussed the irregularities effect on Love wave propagation in an inhomogeneous crustal layer and obtained the corresponding frequency equation. Chattopadhyay et al. [15] studied the propagation of SH waves in an inhomogeneous irregular monoclinic layer over semi-infinite monoclinic medium. The frequency relation was obtained by using equations 2.5, 2.8, 2.9 and 2.10 and was observed that in the isotropic case, the frequency equation reduces to standard shear waves in the absence of non-homogeneity and irregularity. The effect of rectangular irregularity and in homogeneity parameter with the help of graphs on the propagation of plane shear waves in the considered model had also been observed. It was also observed from graphs that the size of irregularity increases as the phase velocity decreases and the phase velocity increases with wave number. Sethi et al. [16] discussed the effect of elastic parameters on the non-homogeneous monoclinic layer which is regular in nature measuring the rigidity sandwiched between isotropic semi-infinite medium lying in the half-space and hence obtained the frequency relation by using 2.5 and 2.6.

## 5. Shear Wave Propagation in Fluid Saturated Porous Medium (FSP)

A porous medium is difficult to be defined geometrically due to the fact by means of a number of geometrical qualities inclusive of porosity, particular internal area and therefore is complicated in nature. FSP rocks lie on or under of the Earth's surface in the form of oil sentiments, and stone and groundwater. So, the study of propagation of shear waves in FSP medium has received high impact. Biot [17] discussed the elastic wave propagation in single-phase for liquid fill edporous solids. The frequency relation was derived with the help of equation 2.2 and 2.3 for different size of irregularities by using perturbation technique and their effects on phase velocities curves were studied by Biot [18]. Pal and Mandal [19] studied the propagation of surface waves in FSP layer between orthotropic half-space and homogeneous liquid layer and obtained the resulting frequency equation. Kumar et al. [20] also obtained the frequency equation by using perturbation technique given by Eringen and Samuels [21] for shear wave propagation in transversely isotropic FSP half space with three different mediums taken in the model and suitable boundary conditions were considered for the problem solved by using 2.8 and 2.9. With the help of graphs, the effect of phase velocity on wave number and the size of irregularity were discussed. It was observed that the wave number increases when the phase velocity and their regularity size in transversely isotropic FSP decreases. The effect of irregularity, in homogeneity and rigidity in FSP layer over non-homogeneous elastic half space had been studied by Kumar et al. [22, 23]. Recent work was done by Sahuet al. [24] on effects on different parameters like heterogeneity and wave number on phase velocity when the shear wave propagate in FSP medium sandwiched between two heterogeneous half-spaces by surface plot. The frequency equation was also obtained with the help of perturbation technique and on the equation other particular cases were studied. Sahu et al. [25] studied the propagation of SH waves with the help of surface plot on the FSP layer sandwiched between heterogeneous half-spaces by the technique of separable variables. For the particular cases, the Whittaker's function is used to derive the frequency equation by using 2.8 and 2.9. The drawbacks on porosity and heterogeneity were seen for the considered model.

## 6. Propagation of Shear Waves in Multilayered FSP Stratum

Earth is heterogeneous body consisting of different layers, a rigid layer due to its hardness has great importance in earthquake engineering to understand the behaviour of waves on the surface surroundings. FSP medium is mainly used in engineering branches including soil mechanics, material science, petroleum engineering, chemical engineering and measuring of water table etc. One of the model related to shear wave propagation in layered stratum consisting a transversely isotropic FSP layer lying between an in homogeneous elastic half space and an elastic isotropic homogeneous layer with free surface had been developed by Konczak [26] and derived the corresponding dispersion relation with the help of equation 2.2, 2.3, 2.4 and 2.6. Further, this work was extended by Kumar et al. [27] by constructing the frequency equation for surface waves in a FSP half-space with the help of dual layer, out of which one is inhomogeneous and other one is homogeneous liquid with irregularity present at the interface. Madan et al. [28] investigated the effect of irregularity on Love wave propagation in a FSP medium. The frequency equation for shear in multi-layered medium with rectangularity irregularity under the effect of rigid boundary over the half space had been derived by Kumar et al. [29] by considering three mediums out of which the topmost is isotropic homogeneous elastic layer given by equation 2.2, intermediate is transversely isotropic FSP layer given by equations 2.4 and 2.6 and lower medium is homogeneous elastic semi-infinite medium given by equation 2.10. Poonia et al. [30] plotted the phase velocity graphically with the help of MATLAB graphical routines verses wave number and different size of rectangular irregularity present at the interface in layered stratum with FSP layer.

## 7. Observations and Future Scope

This paper is review in nature in which we have studied many research articles based on the propagation of shear waves in different media over the half-space with rectangular irregularity present at the interface. It has been observed that rectangular irregularity has a significant effect on wave propagation for different elastic models. These types of models can be further extended by introducing the effect of heat and initial stress. The results obtained by many researchers time to time as discussed in this review paper can be useful to the study of deformation due to seismic sources present in the elastic medium on the interface with irregularity.

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