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DOMINATION ON BIPOLAR ANTI FUZZY GRAPH

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Abstract

In this paper, we defined size, order and degree of a Bipolar anti fuzzy graph, uninodal anti fuzzy graph is defined with suitable and adequate examples. The properties of bipolar anti fuzzy graph are discussed. The domination number of a bipolar anti fuzzy graph is introduced. Some simple theorems have also been proposed related to this concept.

1. Introduction

The concept of fuzzy graph was proposed by Kaufmann, from the fuzzy relations introduced by Zadeh [11]. In 1975, Rosenfeld [9] introduced another elaborated concept, including fuzzy vertex and fuzzy edges and several fuzzy analogues of graph theoretic concepts such as paths, cycles, connectedness

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and etc. In the year 1998, the concept of domination in fuzzy graphs was investigated by A. Somasundaram, S. Somasundaram [10]. In the year, 2004. In the year 2003, A. Nagoorgani and M. Basheer Ahamed [8] investigated order and Size in fuzzy graph. In 2011, Muhammad Akram [4] introduced Bipolar fuzzy graph. In the year 2017, R. Muthuraj and A. Sasireka [6] introduced Anti fuzzy graph.

2. Preliminaries

In this section, the basic definitions needed to develop the subsequent sections definitions are discussed. Throughout this paper, the dominating set denoted as D, Throughout this paper,

(i) The edge between the vertices u and v as uv.

(ii) G_{BA} be a bipolar anti fuzzy graph, mean that G be a bipolar fuzzy graph with underlying graph $G^* = (V, E)$.

Definition 2.1 [4]**.** A fuzzy subset μ on a set X is a map $\mu : X \to [0, 1]$. A map $v : X \times X \to [0, 1]$ is called a fuzzy relation on X if $v(x, y) \leq \min(\mu(x), \mu(y))$ for all $x, y \in X$. A fuzzy relation v is symmetric if $v(x, y) = v(y, x)$ for $x, y, \in X$.

Definition 2.2[4]**.** Let X be a non-empty set. A bipolar fuzzy set B in X is an object having the form $B = \{(x, \mu_B^P(x), \mu_B^N(x))/x \in X\}$ where $\mu_B^P : X \to [0, 1]$ and $\mu_B^N : X \to [-1, 0]$ are mappings. The positive membership degree $\mu_B^P(x)$ to denote the satisfaction degree of an element x to the property corresponding to a bipolar fuzzy set B, and the negative membership degree $\mu_B^N(x)$ to denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar fuzzy set B. If $\mu_B^P(x) \neq 0$ and $\mu_B^N(x) = 0$. It is the situation that is regarded as having only positive satisfaction for B. If $\mu_B^P(x) = 0$ and $\mu_B^N(x) \neq 0$, it is the situation that x does not satisfy the property of B but somewhat satisfies the counter property of B. It is possible for an element x to be such that $\mu^P(x) \neq 0$ and

$\mu^N(x) \neq 0$. When the membership function of the property overlaps that of its counter property over some portion of X. For the sake of simplicity, we shall use the symbol $B = (\mu_B^P, \mu_B^N)$, for the bipolar fuzzy set $B = \{(x, \mu_B^P(x), \mu_B^N(x))/x \in X\}$.

Definition 2.3 [4]. Let X be a non-empty set. Then we call a mapping $A = (\mu_B^P, \mu_B^N) : X \times X \to [-1, 1] \times [-1, 1]$ a bipolar fuzzy relation on X such that $\mu_B^P(x, y) \in [0, 1]$ and $\mu_B^N(x, y) \in [-1, 0]$.

Definition 2.4 [4]. A Bipolar fuzzy graph, is denoted as a pair $G = (A, B)$, where $A = (\mu_B^P, \mu_B^N)$ and $B = (\mu_B^P, \mu_B^N)$ are bipolar fuzzy sets and $\mu_A^P : V \to [0, 1], \mu_A^N : V \to [-1, 0]$, and $\mu_B^P : V \times V \to [0, 1], \mu_B^N : V \times V \to [-1, 0]$, are bipolar fuzzy mappings such that $\mu_B^P(uv) \leq \min\{\mu_A^P(u), \mu_A^P(v)\}$ and $\mu_B^N(uv) \geq \min\{\mu_A^N(u), \mu_A^N(v)\}$ for all $uv \in E$. A is called the bipolar fuzzy vertex set of V and B the bipolar fuzzy edge set of E respectively. Note that B is a symmetric bipolar fuzzy relation on A. That is, $G = (A, B)$ is a bipolar fuzzy graph of the underlying crisp graph $G^* = (V, E)$, where V is a vertex set and the edge set $E \subseteq V \times V$ such that, $\mu_B^P(uv) \leq \min\{\mu_A^P(u), \mu_A^P(v)\}$ and $\mu_B^N(uv) \geq \min\{\mu_A^N(u), \mu_A^N(v)\}$ for all $uv \in E$.

Example 2.5.

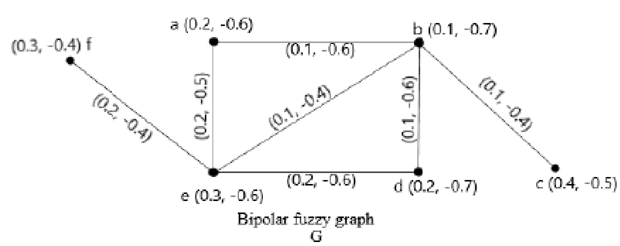


Figure 2.1.

Definition 2.6 [5]. Let $G = (A, B)$ be a bipolar fuzzy graph. Then the cardinality of V or the order of G is defined by

$$p = |V| = \sum_{u \in V} \frac{1 + \mu_A^P(u) + \mu_A^N(u)}{2}.$$

Definition 2.7 [5]**.** Let $G = (A, B)$ be the bipolar fuzzy graph. Then cardinality of E or the size of G is defined as

$$q = |E| = \sum_{uv \in E} \frac{1 + \mu_B^P(uv) + \mu_B^N(uv)}{2}.$$

Definition 2.8 [5]**.** Let $G = (A, B)$ be a bipolar fuzzy graph, then the degree of the vertex is denoted by $\deg(u)$ and it is defined as

$$\deg(u) = \sum_{v \in E} \frac{1 + \mu_B^P(uv) + \mu_B^N(uv)}{2}.$$

Definition 2.9 [5]**.** Let $G = (A, B)$ be a bipolar fuzzy graph. The maximum degree of a bipolar fuzzy graph is denoted by

$$\Delta(G) = \max\{\deg(u)/u \in V\}.$$

Definition 2.10 [5]**.** Let $G = (A, B)$ be a bipolar fuzzy graph. The minimum degree of a bipolar fuzzy graph is denoted by

$$\delta(G) = \min\{\deg(u)/u \in V\}.$$

Definition 2.11 [5]**.** Let $G = (A, B)$ be a bipolar fuzzy graph. The degree of an edge $uv \in E$ is denoted as $\deg(uv)$ and it is defined as,

$$\deg(uv) = \sum_{uv \in E} \frac{1 + \mu_B^P(uv) + \mu_B^N(uv)}{2}.$$

Definition 2.12 [5]**.** Let $G = (A, B)$ be a bipolar fuzzy graph. Then the neighbors (neighborhood) of u or an open neighbor of $u \in V$ of G is denoted by $N(u)$ and is defined as $N(u) = \{v \in V / \mu_B^P(uv) = \min\{\mu_A^P(u), \mu_A^P(v)\}$ and $\mu_B^N(uv) = \max\{\mu_A^N(u), \mu_A^N(v)\}$ and $uv \in E\}$.

The closed neighbors of $u \in V$ of G is denoted by $N[u]$ and is defined as $N[u] = N(u) \cup \{u\}$.

Definition 2.13 [5]. Let $G = (A, B)$ be a bipolar fuzzy graph. Then the neighborhood degree of $u \in V$ is denoted as $\deg N(u)$ and is defined as,

$$\deg_N(u) = \sum_{v \in N(u)} \frac{1 + \mu_A^P(v) + \mu_A^N(v)}{2}.$$

Definition 2.14 [5]. Let $G = (A, B)$ be a bipolar fuzzy graph. The maximum neighborhood degree of a bipolar fuzzy graph is denoted by

$$\Delta_N(G) = \max\{\deg_N(u)/u \in V\}.$$

Definition 2.15 [5]. Let $G = (A, B)$ be a bipolar fuzzy graph. The minimum neighborhood degree of a bipolar fuzzy graph is denoted by

$$\delta_N(G) = \min\{\deg_N(u)/u \in V\}.$$

Definition 2.16 [5]. Let $G = (A, B)$ be a bipolar fuzzy graph. An edge of G is said to be an effective edge if $\mu_B^P(uv) = \min\{\mu_A^P(u), \mu_A^P(v)\}$ and $\mu_B^N(uv) = \max\{\mu_A^N(u), \mu_A^N(v)\}$ and $uv \in E$.

Definition 2.17 [5]. Let $G = (A, B)$ be a bipolar fuzzy graph. The minimum effective degree of a bipolar fuzzy graph is denoted by

$$\delta_N(G) = \min\{\deg_E(u)/u \in V\}.$$

Example 2.18.

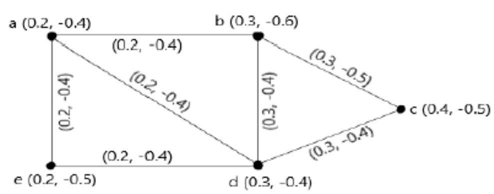


Figure 2.2.

Bipolar fuzzy graph

$$\deg(u) = \sum_{v \in V} \frac{1 + \mu_B^P(uv) + \mu_B^N(uv)}{2}, \ \deg(uv) = \sum_{uv \in V} \frac{1 + \mu_B^P(uv) + \mu_B^N(uv)}{2}.$$

$\deg(a) = 0.4 + 0.4 + 0.4 = 1.2$ $\deg(ab) = 0.4 + 0.4 + 0.4 = 1.2$

$\deg(b) = 0.4 + 0.4 + 0.45 = 1.25$ $\deg(bc) = 0.7$

$\deg(c) = 0.45 + 0.4 = 0.85$ $\deg(cd) = 0.65$

$\deg(d) = 0.4 + 0.45 + 0.45 + 0.4 = 1.7$ $\deg(ad) = 0.6$

$\delta(G) = \min\{\deg(u)/u \in V\} = 1.2$, $\delta_{\deg(uv)}(G) = \min\{\deg(uv)/uv \in E\} = 0.6$

$\Delta(G) = \max\{\deg(u)/u \in V\} = 1.3$, $\Delta_{\deg(uv)}(G) = \max\{\deg(uv)/uv \in V\} = 0.7$

$N(u) = \{v/v \text{ is adjacent to } u\}$, $\deg_N(u) = \sum_{v \in N(u)} \frac{1 + \mu_A^P(v) + \mu_A^N(v)}{2}$.

$N(a) = \{b, d\}$, $\deg_N(a) = 0.7 + 0.7 = 1.4$

$N(b) = \{a\}$, $\deg_N(b) = 0.55 = 0.55$

$N(c) = \{b, d\}$, $\deg_N(c) = 0.7 + 0.7 = 1.4$

$N(d) = \{a, c\}$, $\deg_N(d) = 0.55 + 0.5 = 1.05$

$\Delta_N(G) = \max\{\deg_N(u)/u \in V\} = 1.4$, $\delta_N(G) = \min\{\deg_N(u)/u \in V\} = 0.55$

$\deg_E(u) = \sum_{v \in V} \frac{1 + \mu_B^P(v) + \mu_B^N(v)}{2}$; uv is an effective edge

$\deg_E(a) = 0.6 + 0.6 = 1.2$, $\deg_E(c) = 0.65 = 0.65$

$\deg_E(b) = 0.6 = 0.6$, $\deg_E(d) = 0.6 + 0.65 = 1.25$

$\Delta_E(G) = \max\{\deg_E(u)/u \in V\} = 0.6$, $\delta_E(G) = \min\{\deg_E(u)/u \in V\} = 1.25$

$|V| = 0.55 + 0.7 + 05 + 0.7 = 2.45$, $|E| = 0.6 + 0.7 + 0.65 + 0.6 = 2.55$ and $|G| = 2.45 + 2.5 + 5$

$\sum_{u \in V} \deg(u) = 1.2 + 1.25 + 1.3 + 1.25 + = 5.1 = 2 \times |E|$

Definition 2.19 [5]. A bipolar fuzzy graph $G = (A, B)$ is said to be a complete bipolar fuzzy graph if

(i) Every vertex of V in G is adjacent to every other vertex of V in G.

(ii) $\mu_B^P(uv) = \min\{\mu_A^P(u), \mu_A^P(v)\}$ and $\mu_B^N(uv) = \max\{\mu_A^N(u), \mu_A^N(v)\}$ and $uv \in E$.

Example 2.20.

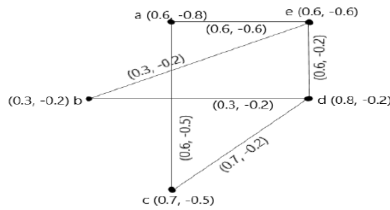


Figure 2.3.

Definition 2.21 [5]. Let $G = (A, B)$ be a BFG. Then is said to be a bipartite BFG if the vertex set M of a BFG can be partitioned into two subsets M_1 and M_2 such that $\alpha_B^P(rt) = 0$ and $\alpha_B^N(rt) = 0$, for all $r, t \in M_1$ or $r, t \in M_2$.

A bipartite BFG $G = (A, B)$ is said to be a complete BFG if, $\alpha_B^P(rt) = \min\{\alpha_A^P(r), \alpha_A^P(t)\}$ and $\alpha_B^N(rt) = \max\{\alpha_A^N(r), \alpha_A^N(t)\}$ for all $r \in M_1$ and $t \in M_2$.

Example 2.22.

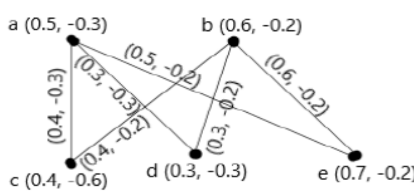


Figure 2.4.

Definition 2.23[5]. Let $G = (A, B)$ be a bipolar fuzzy graph. Let $u, v \in V$. The vertex u is said to be dominates v in G if

$\mu_B^P(uv) = \min\{\mu_A^P(u), \mu_A^P(v)\}$ and $\mu_B^N(uv) = \max\{\mu_A^N(u), \mu_A^N(v)\}$ and $uv \in E$.

A subset D of V is said to be a dominating set in G if for every $v \in V - D$ there exist $u \in D$ such that u dominates v.

A dominating set D of V is said to be a minimal dominating set if no proper subset of D is a dominating set of G.

The minimum fuzzy cardinality of a minimal dominating set in G is called the domination number of G and is denoted by $\gamma(G)$ or simply γ.

Definition 2.24 [6]**.** A fuzzy graph $G = (\sigma, \mu)$ is said to be an anti fuzzy graph with a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$, where for all $u, v \in V$, we have $\mu(u, v) \geq \sigma(u) \vee \sigma(v)$ and it is denoted by $G_A(\sigma, \mu)$.

Example 2.25.

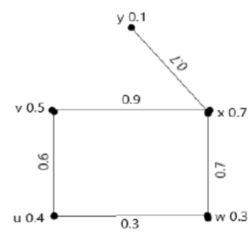


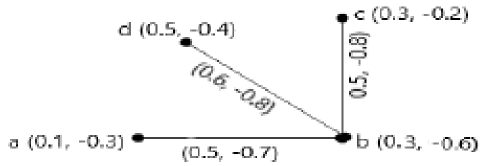
Figure 2.5.

Definition 2.26 [6]**.** Every vertex in an anti fuzzy graph G_A has a unique fuzzy values, then G_A is said to be v-nodal anti fuzzy graph. (i.e.) $\sigma(u) = c$ for all $u \in V(G_A)$.

3. Domination on Bipolar Anti Fuzzy Graph

Definition 3.1. A Bipolar anti fuzzy graph, is denoted as a pair $G_{BA} = (A, B)$, where $A = (\mu_B^P, \mu_B^N)$ and $B = (\mu_B^P, \mu_B^N)$ are bipolar fuzzy sets and $\mu_A^P : V \to [0, 1], \mu_A^N : V \to [-1, 0]$, and $\mu_B^P : V \times V \to [0, 1]$, $\mu_B^N : V \times V \to [-1, 0]$, are bipolar fuzzy mappings such that $\mu_B^P(uv) \geq \max\{\mu_A^P(u), \mu_A^P(v)\}$ and $\mu_B^N(uv) \leq \min\{\mu_A^N(u), \mu_A^N(v)\}$ for all $uv \in E$. A is called the bipolar anti fuzzy vertex set of V and B the bipolar fuzzy edge set of E respectively.

Example 3.2.



Bipolar anti fuzzy graph G_{BA}

Figure 3.1.

Definition 3.3. Let G_{BA} be a bipolar anti fuzzy graph. Then the cardinality of V or the order of G_{BA} is defined by

$$p = |V| = \sum_{u \in V} \frac{1 + \mu_A^P(u) + \mu_A^N(u)}{2}.$$

Definition 3.4. Let G_{BA} be a bipolar anti fuzzy graph. Then cardinality of E or the size of G_{BA} is defined as

$$q = |E| = \sum_{uv \in E} \frac{1 + \mu_B^P(uv) + \mu_B^N(uv)}{2}.$$

Definition 3.5. Let G_{BA} be a bipolar anti fuzzy graph, then the degree of the vertex is denoted by $\deg(u)$ and it is defined as

$$\deg(u) = \sum_{v \in V} \frac{1 + \mu_B^P(uv) + \mu_B^N(uv)}{2}.$$

Definition 3.6. Let G_{BA} be a bipolar anti fuzzy graph. The maximum degree of a bipolar fuzzy graph is denoted by $\Delta(G_{BA}) = \max\{\deg(u)/u \in V\}$.

Definition 3.7. Let G_{BA} be a bipolar anti fuzzy graph. The minimum degree of a bipolar fuzzy graph is denoted by $\delta(G_{BA}) = \min\{\deg(u)/u \in V\}$.

Definition 3.8. Let G_{BA} be a bipolar anti fuzzy graph. The degree of an edge $uv \in E$ is denoted as $\deg(uv)$ and it is defined as,

$$\deg(uv) = \sum_{uv \in E} \frac{1 + \mu_B^P(uv) + \mu_B^N(uv)}{2}.$$

Definition 3.9. Let G_{BA} be a bipolar anti fuzzy graph. Then the neighbors (neighborhood) of u or an open neighbor of $u \in V$ of G_{BA} is denoted by $N(u)$ and is defined as

$N(u) = \{v \in V / \mu_B^P(uv) = \min\{\mu_A^P(u), \mu_A^P(v)\}$ and $\mu_B^N(uv) = \max\{\mu_A^N(u), \mu_A^N(v)\}$ and $uv \in E\}$.

The closed neighbors of $u \in V$ of G_{BA} is denoted by $N[u]$ and is defined as $N[u] = N(u) \cup \{u\}$.

Definition 3.10. Let G_{BA} be a bipolar anti fuzzy graph. Then the neighborhood degree of $u \in V$ is denoted as $\deg_N(u)$ and is defined as,

$$\deg_N(u) = \sum_{v \in N(u)} \frac{1 + \mu_A^P(v) + \mu_A^N(v)}{2}.$$

Definition 3.11. Let G_{BA} be a bipolar anti fuzzy graph. The maximum neighborhood degree of a bipolar fuzzy graph is denoted by

$$\Delta_N(G_{BA}) = \max\{\deg_N(u) / u \in V\}.$$

Definition 3.12. Let G_{BA} be a bipolar anti fuzzy graph. The minimum neighborhood degree of a bipolar fuzzy graph is denoted by

$$\delta_N(G_{BA}) = \min\{\deg_N(u) / u \in V\}.$$

Definition 3.13. Let G_{BA} be a bipolar anti fuzzy graph. An edge of G_{BA} is said to be an effective edge if $\mu_B^P(uv) = \min\{\mu_A^P(u), \mu_A^P(v)\}$ and $\mu_B^N(uv) = \max\{\mu_A^N(u), \mu_A^N(v)\}$ for all $uv \in E$.

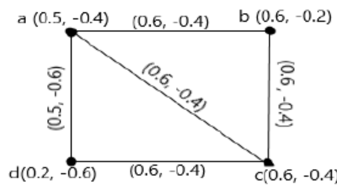
Definition 3.14. Let G_{BA} be a bipolar anti fuzzy graph. The minimum effective degree of a bipolar fuzzy graph is denoted by

$$\delta_N(G_{BA}) = \min\{\deg_E(u) / u \in V\}.$$

Definition 3.15. Every vertex in a bipolar anti fuzzy graph G_{BA} has the equal cardinality, then G_{BA} is said to be uninodal bipolar anti fuzzy graph.

Definition 3.16. A bipolar anti fuzzy graph G_{BA} is said to be a strong bipolar fuzzy graph if $\mu_B^P(uv) = \max\{\mu_A^P(u), \mu_A^P(v)\}$ and $\mu_B^N(uv) = \min\{\mu_A^N(u), \mu_A^N(v)\}$ and $uv \in E$.

Example 3.17.



Strong bipolar anti fuzzy graph

Figure 3.2.

Here all the edges are strong edges.

Definition 3.18. The strong neighborhood of an edge e_i in an bipolar anti fuzzy graph G_{BA} is $N(e_i) = \{e_j \in E(G_{BA})/e_j$ is an effective edge which have maximum fuzzy value in neighborhood of e_i in G_{BA}.

Definition 3.19. A set $D \subseteq V(G_{BA})$ is said to be a dominating set of a bipolar anti fuzzy graph if for every vertex $V \epsilon V(G_{BA}) \setminus D$, there exist u in D such that V is a strong neighborhood of u with $\mu(u, v) = \sigma(u) \vee \sigma(v)$. Otherwise, it dominates itself.

A subset D of V is said to be a dominating set in G_{BA} if for every $v \in V - D$ there exist $u \in D$ such that u dominates v.

A dominating set D of V is said to be a minimal dominating set if no proper subset of D is a dominating set of G_{BA}.

The maximum fuzzy cardinality of a minimal dominating set in G is called the domination number of G_{BA} and is denoted by $\gamma_{BA}(G)$.

Example 3.20.

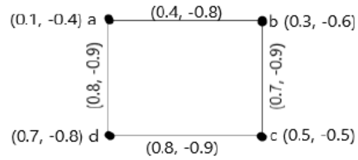
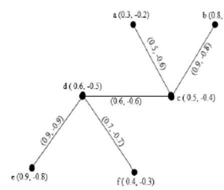


Figure 3.3.

Here, the dominating set $D = \{d, c\}$

The domination number $\gamma_{BA}(G) = 0.95$.

Example 3.21.



Uninodal Bipolar Anti fuzzy graph

Figure 3.4.

Theorem 3.22. *Let* $G_{BA} = (A, B)$ *be a bipolar anti fuzzy graph. A dominating set D of* G_{BA} *is a minimal dominating set if and only if for each* $u \in D$, *one of the following two conditions holds.*

(i) $N(u) \cap D = \phi$.

(ii) *There is a vertex* $v \in V - D$ *such that* $N(v) \cap D = \{u\}$.

Proof. Let $G_{BA} = (A, B)$ be a bipolar anti fuzzy graph. Let D be a minimal dominating set of G_{BA} and $u \in D$. Let $D_u = D - \{u\}$. Then D_u is not a dominating set as D is a minimal dominating set. Hence there exists $v \in V - D_u$ such that v is not dominated by any element of D_u.

Case i. If $v = u$, then $v = u$ is not dominated by any element of D_u and hence it is not dominated by any element of D and hence, $N(u) \cap D = \phi$.

Case ii. If $v \neq u$ then u dominates v as D is a minimal dominating set of G_{BA} and hence, $N(v) \cap D = \{u\}$.

Conversely, let D be a dominating set of G_{BA} and for each $u \in D$, one of the following two conditions holds.

(i) $N(u) \cap D = \phi$.

(ii) There is a vertex $v \in V - D$ such that $N(v) \cap D = \{u\}$. Suppose if D is not a minimal dominating set of G then $D_1 \subset D$ is a dominating set of G_{BA}.

Consider an element $u \in D$ and $u \notin D_1$. Then $u \in V - D_1$ and there exists $w \in D_1$ such that w dominates u and so $w \in N(u)$. Also $w \in D_1 \subset D$ and hence $N(v) \cap D \neq \phi$.

Given D is not a minimal dominating set, then there is a vertex $v \in V - D$ such that either v is dominated by more than one vertex of D or there exist an element $u \in D$ such that u does not dominate any v for all $v \in V - D$.

Case i. Let $u, w \in D$ dominates v and $u, w \in N(v)$. Then $N(v) \cap D = \{u, w\} \neq \{u\}$.

Case ii. Then for this $u \in D$, $N(v) \cap D \neq \{u\}$ for all $v \in V - D$.

Hence, conditions (i) and (ii) do not hold because of the assumption that D is not a minimal dominating set of G. Hence D is a minimal dominating set of G_{BA}.

Theorem 3.23. *If G_{BA} is an bipolar anti fuzzy graph without isolated vertices and D is a minimal dominating set then $V \setminus D$ is a dominating set in G_{BA}.*

Proof. Let G_{BA} be a bipolar anti fuzzy graph without isolated vertices. Let D be the minimal dominating set of G_{BA}. Let $u \in D$. Since G has no isolated vertices then $v \in N(u)$.

Case i. If $v \in V \setminus D$, then, every element of D is dominated by some element of $V - D$.

Hence, $V \setminus D$ is a dominating set of G_{BA}.

Case ii. If $v \in D$ and D is a minimal dominating set, then, there exists an element $x \in V \setminus D$ such that $x \in N(u)$.

That is, for every element $u \in D$, there exists an element $x \in V \setminus D$ such that x dominates u. Hence $V \setminus D$ is a dominating set of G_{BA}.

Theorem 3.24. *Let G_{BA} be a bipolar anti fuzzy graph with the order p, size q and the minimum degree δ, then $q - \mathrm{p} \leq \gamma_{\mathrm{BA}} \leq p - \delta$.*

Proof. Let G_{BA} be a bipolar anti fuzzy graph. Let $D \subseteq M$ be a dominating set and $D \subseteq M$ be a dominating set of G_{BA}. Let q be the sum of the fuzzy cardinality of all the edges and let p be the sum of all the vertices. Therefore, the difference between the order and size of the bipolar anti fuzzy graph is minimum and the domination number of the bipolar anti fuzzy graph is the minimum cardinality over all the vertices. Therefore, $|V \setminus D| = p - \gamma_{BA}$. Then there exists at most $\deg(G_{BA})/2$ edges incident from $V \setminus D$ to D. Hence, $p - \gamma_{BA} \leq q$.

This implies that $q - \mathrm{p} \leq \gamma_{\mathrm{BA}}$ (1). Let u be the vertex with minimum degree δ. U must be adjacent to strongly dominate vertices in G_{BA}.

Hence, $V \setminus N(u)$ is a dominating set. Therefore, $\gamma_{BA} \leq p - \delta$ (2). From (1) and (2) we have, $q - \mathrm{p} \leq \gamma_{\mathrm{BA}} \leq p - \delta$.

Hence the Proof.

Theorem 3.25. *For any bipolar anti fuzzy graph without isolated vertices, $\gamma \leq \frac{p}{2}$, where p is the order of BAFG.*

Proof. Any bipolar anti fuzzy graph without isolated vertices has two disjoint dominating sets and hence, $\gamma \leq \frac{p}{2}$, where p is the order of BAFG.

Theorem 3.26. *Let G_{BA} be a complete bipolar anti fuzzy graph and let D be the minimal dominating set in G_{BA}, then $(V \setminus D)$ is also complete.*

Proof. Let G_{BA} be a complete bipolar anti fuzzy graph with vertex set $v_i \in V$, $i = 1, 2, 3, \ldots, n$. Let D be the minimal dominating set in G_{BA}.

Hence, the resultant vertices in $(V \setminus D)$ are dominates every other vertex. Therefore, it is complete.

Hence the proof.

Theorem 3.27. *Let G_{BA} any completely bipartite bipolar anti fuzzy graph with vertex sets $M = \{u_1, u_2, \ldots, u_n\}$ and $N = \{v_1, v_2, \ldots, v_n\}$, then $\gamma(G_{BA}) = \min\{\sigma(u_i)\} + \min\{\sigma(v_i)\}$ for all $u_i \in M$ and $v_i \in N$.*

Proof. Let G_{BA} be any completely bipartite bipolar anti fuzzy graph. Then, $V = M \cup N$ and $M \cap N = \phi$. Let $D = u_i, v_j$, where $u_i \in M$ and $v_j \in N$ be a dominating set of G_{BA}. By the definition of completely bipartite bipolar anti fuzzy graph every vertex in M dominates every vertex in N and vice versa. The minimal dominating set dominates exactly one vertex in M and N. Hence $\gamma(G_{BA}) = \min\{\sigma(u_i)\} + \min\{\sigma(v_i)\}$ for all $u_i \in M$ and $v_i \in N$.

Theorem 3.28. *Let G_{BA} is and uninodal bipolar fuzzy graph with no isolated vertices then, $\gamma(G_{BA}) = p - \Delta(G_{BA})$ where $\Delta(G_{BA})$ is the maximum degree of G_{BA}.*

Proof. Let G_{BA} is and uninodal bipolar fuzzy graph with no isolated vertices and let D be the dominating set in G_{BA} then every vertex in D dominates at most $\Delta(G_{BA}) - 1$ vertices in $V \setminus D$ and dominates at least one vertex in D. Hence,

$$p = \Delta(G_{BA}) - 1 + |D| + 1p = \Delta(G_{BA}) + \gamma(G_{BA})$$

$\gamma(G_{BA}) = p - \Delta(G_{BA})$. Hence the Proof.

4.0 Conclusion

We introduced the concept of domination of bipolar anti fuzzy graphs and its based definitions. The perfect bipolar anti fuzzy graph and operations and other dominations on bipolar anti fuzzy graph will be described in the accessible papers.

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