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ANTIMAGIC LABELING ON TRIANGULAR FUZZY GRAPHS

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Abstract

The researchers have shown a triangular fuzzy coconut tree graph $CT_{m,n} : (\sigma, \mu)\ \forall\, m, n$ by providing an algorithm. Also the researchers have proved the result that the above mentioned graph satisfies the condition of triangular fuzzy labeling as well as triangular fuzzy anti magic labeling. The researcher has extended the concepts towards the triangular fuzzy anti magic labeled star graph as well as triangular fuzzy anti magic labeled path graph by removing the pendant edge from path P_m of $CT_{m,n}$. Also triangular fuzzy anti magic labeled Y-tree graph has been obtained by removing (n-2)edges from the 'n' pendant edges of $CT_{m,n}$.

Introduction

Alex Rosa [5] introduced labeling technique which is used to assign labels in all networking models which has been represented by graph only for crisp data in real life. But in case of ambiguity in data, fuzzy graph labeling is more useful better than graph labeling to assign labels in the networking model. A. Nagoorgani et al. [1-4] have given the idea and properties of fuzzy graph labeling in 21st century. K. Thirusangu and D. Jeevitha [11] have presented various results on magic labeling and anti magic labeling on fuzzy graphs. S. Vimala et al. [12] have discussed some properties of anti magic labeling on tree like fuzzy graphs. N. Sujatha et al. [6-10] have introduced

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the concept of labeling on fuzzy graph using triangular fuzzy numbers and proved some results using the above concepts on special path related graphs.

In this paper, the researcher has provided a coconut tree fuzzy graph in which labels are triangular fuzzy numbers $CT_{m,n} : (\sigma, \mu) \forall m, n.$ Also the researcher has proved the result that the above mentioned graph satisfies the condition of triangular fuzzy labeling as well as triangular fuzzy anti magic labeling. The researcher has extended the concepts towards the triangular fuzzy anti magic labeled star graph and triangular fuzzy anti magic labeled path graph by removing the pendant edge from path P_m of $CT_{m,n}$. Also triangular fuzzy anti magic labeled Y-tree graph has been obtained by removing (n-2)edges from the 'n' pendant edges of $CT_{m,n}$.

2. Preliminaries

In this section, some basic definitions have been shown which is used to develop the algorithm and theorems.

Definition 2.1 [4]**.** Triangular fuzzy number

It is a fuzzy number represented with three points as follows $\widetilde{A} = [a_1, a_2, a_3]$. This representation is interpreted as membership functions and holds conditions.

(i) a_1 to a_2 is an increasing function

(ii) a_2 to a_3 is a decreasing function

$$\text{and } \mu_{\widetilde{A}}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \dfrac{x - a_1}{a_2 - a_3} & \text{for } a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \le x \le a_3 \\ 0 & \text{for } x > a_3 \end{cases}$$

Definition 2.2 [4]**.** Addition and Division operation of triangular fuzzy number using function principle are given as follows.

Let $\widetilde{A} = [a_1, a_2, a_3]$ and $\widetilde{B} = [b_1, b_2, b_3]$

$$\widetilde{A} + \widetilde{B} = [(a_1 + b_1), (a_2 + b_2), (a_3 + b_3)]$$

$$\frac{\widetilde{A}}{\widetilde{B}} = \begin{bmatrix} Min(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3), (a_2/b_2), \\ Max(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3) \end{bmatrix}.$$

Definition 2.3 [9]**.** The process of assigning triangular fuzzy numbers to each fuzzy labeled vertices and fuzzy labeled edges of a fuzzy labeled graph $G : (\sigma, \mu)$ is known as triangular fuzzy labeling.

Definition 2.4 [9]**.** A fuzzy labeled graph $G : (\sigma, \mu)$ is said to be triangular fuzzy labeled graph if it admits triangular fuzzy labeling with

$\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1] \ni$

$\mu(u, w) < [\sigma(u) \cap \sigma(w)] \ \forall \ u, w \in V.$

3. Main Results

We have proved that the coconut tree graph $CT_{m,n}$ is a triangular fuzzy anti magic labeled graph. Also the researcher has shown the triangular fuzzy anti magic labeled star graph, triangular fuzzy anti magic labeled path graphs and triangular fuzzy anti magic labeled Y-tree graph by removing the edges from $CT_{m,n}$ with the help of following theorems.

Algorithm 3.1.

Input: $CT_{m,n} : (\sigma, \mu) \ \forall \ m, n$

Procedure

for $i = 1$ to n

$\{(v_i) \leftarrow$pendant vertices of v_m which is the end vertex of the path P_m in $CT_{m,n}$

$\sigma(v_m) = [a_m, b_m, c_m] = [1.0, 1.1, 2.6]$

$\sigma(v_i) = [a_i, b_i, c_i]$

$a_i = a_m, b_i = b_m$

$c_i = a_i + \frac{i}{10}$

$\mu(v_i, v_m) = [p_{i,}\ q_i, r_i]$

$p_i = Min(a_i/a_m, a_i/c_m, c_i/a_m, c_i/c_m)$

$q_i = (b_i/b_m)$

$r_i = Max(a_i/a_m, a_i/c_m, c_i/a_m, c_i/c_m)$

for $k = m$ to 1

{Let $C_m + 1 = 2.3$

$(v_k) \leftarrow$ vertices of the path P_m in $CT_{m,n}$

$(v_k, v_{k-1}) \leftarrow$ edges of the path P_m in $CT_{m,n}$

$a_k = 1.0, a_{k-1} = 1.0$

$b_k = 1.1, b_{k-1} = 1.1$

$c_k = c_{k+1} + \frac{3}{10}$

$c_{k-1} = c_k + \frac{3}{10}$

for $j = m$ to 1

{$\sigma(v_k) = [a_{k,}\ b_k, c_k]$

$\sigma(v_{k-1}) = [a_{k-1,}\ b_{k-1}, c_{k-1}]$

$\mu(v_k, v_{k-1}) = [p_k, q_k, r_k]$

$p_k = Min(a_k/a_{k-1}, a_k/c_{k-1}, c_k/a_{k-1}, c_k/c_{k-1})$

$q_k = (b_k/b_{k-1})$

$r_k = Max(a_k/a_{k-1}, a_k/c_{k-1}, c_k/a_{k-1}, c_k/c_{k-1})$

}

}

}

end procedure.

Theorem 3.1. *If* $CT_{m,n} : (\sigma, \mu) \; \forall \; m, n$ *be the fuzzy coconut tree graph in which the labels are assigned both vertices and edges are triangular fuzzy numbers. Then* $CT_{m,n} : (\sigma, \mu) \; \forall \; m, n$ *be the triangular fuzzy labeled graph.*

Proof. Given $CT_{m,n} : (\sigma, \mu) \; \forall \; m, n$ be the triangular fuzzy coconut tree graph with a path P_m which has been appended by '*n*' new pendant edges of the end vertex of P_m.

Using algorithm 3.1

for $i = 1$ to n

$$\mu(v_i, v_m) < \sigma(v_i) \text{ and } \mu(v_i, v_m) < \sigma(v_m) \tag{1.1}$$

and

for $k = m$ to 1

$$[\mu(v_i, v_{k-1}) < \sigma(v_k) \text{ and } \mu(v_i, v_{k-1}) < \sigma(v_{k-1}) \tag{1.2}$$

by (1.1) and (1.2) triangular fuzzy coconut tree graph $CT_{m,n} : (\sigma, \mu) \; \forall \; m, n$ satisfies the condition of triangular fuzzy labeling.

Theorem 3.2. *Every triangular fuzzy coconut tree graph* $CT_{m,n} : (\sigma, \mu) \; \forall \; m, n$ *is admitting the condition of anti magic labeling.*

Proof. Given $CT_{m,n} : (\sigma, \mu) \; \forall \; m, n$ be the triangular fuzzy labeled coconut tree graph.

To prove $CT_{m,n}$ admits triangular fuzzy anti magic labeling.

From algorithm 3.1,

Sum of the membership functions of the edges corresponding to any two distinct vertices

$\sigma(v_i)$ and $\sigma(v_{i+1})\ \forall\ i \in n$

$\sigma(v_k)$ and $\sigma(v_{k+1})\ \forall\ k \in m$

are given as

$$\mu(v_i, v_{i+1}) + \mu(v_{i+1}, v_{i+2})$$

$$= [(p_{i,} + p_{i+1}), (q_{i,} + q_{i+1}), (r_{i,} + r_{i+1})] \tag{1.3}$$

Also

$$\mu(v_k, v_{k+1}) + \mu(v_{k+1}, v_{k+2})$$

$$= [p_{k,} q_k, r_k] + [p_{k+1,} q_{k+1}, r_{k+1}]$$

$$= [(p_k + p_{k+1}), (q_k + q_{k+1}), (r_k + r_{k+1})] \tag{1.4}$$

From (1.3) and (1.4), we have the distinct sum of the membership values on the incidence of the edges for $\sigma(v_i)$ and $\sigma(v_{i+1})$ as well as $\sigma(v_k)$ and $\sigma(v_{k-1})$.

Hence $CT_{m,n}\ : (\sigma, \mu)\ \forall\ m, n$ be the triangular anti magic labeled graph.

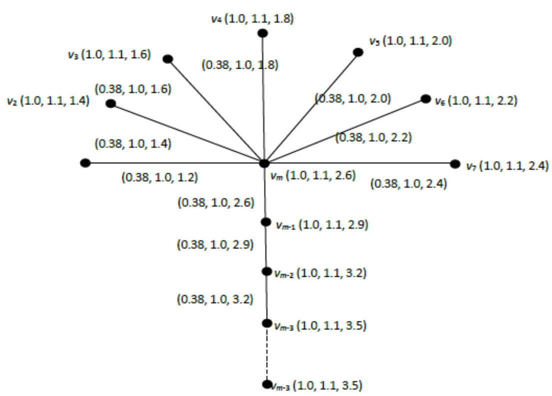

Figure 1. Triangular fuzzy anti magic labeled coconut tree graph $CT_{m,7} : (\sigma, \mu)$.

Theorem 3.3. *If* $CT_{m,n}\ : (\sigma, \mu)\ \forall\ m, n$ *be the triangular fuzzy anti magic labeled coconut tree graph then the removal of an edge* $[\sigma(v_m), \sigma(v_{m-1})]$

from the path P_m which has been appended by 'n' new pendant edges of $CT_{m,n}$ leads to two disconnected graph such as triangular fuzzy anti magic labeled star graph $K_{1,n} : (\sigma, \mu)\ \forall\ n$ and triangular fuzzy anti magic labeled path graph $P_{m\text{-}1} : (\sigma, \mu)\ \forall\ m$.

Proof. Let $CT_{m,n}\ : (\sigma, \mu)\ \forall\ m, n$ be the triangular fuzzy anti magic labeled coconut tree graph.

Remove an edge $[\sigma(v_m), \sigma(v_{m-1})]$ from the path P_m which has been appended by 'n' new pendant edges of $CT_{m,n}$.

Then we have got two disconnected graph such as triangular fuzzy labeled star graph $K_{1,n} : (\sigma, \mu)\ \forall\ n$ and triangular fuzzy labeled path graph $P_{m\text{-}1} : (\sigma, \mu)\ \forall\ m$.

Stage 1. Remove an edge $[\sigma(v_m), \sigma(v_{m-1})]$ from the path P_m which has been appended by 'n' new pendant edges of $CT_{m,n}$.

Then we have got one of the disconnected graphs as the triangular fuzzy labeled star graph $K_{1,n} : (\sigma, \mu)\ \forall\ n$.

By Theorem 3.2 and using (1.3)

for $i = 1$ to n,

$$\mu(v_i, v_{i+1}) + \mu(v_{i+1}, v_{i+2})$$

$$= [p_i, q_i, r_i] + [p_{i+1}, q_{i+1}, r_{i+1}]$$

$$= [(p_{i,} + p_{i+1}), (q_{i,} + q_{i+1}), (r_{i,} + r_{i+1})] \tag{1.5}$$

From (1.5),

$K_{1,n} : (\sigma, \mu)\ \forall\, n$ satisfies the condition of triangular fuzzy anti magic labeling.

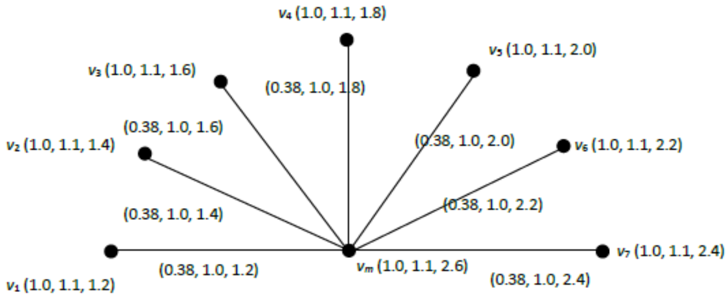

Figure 2. Triangular fuzzy anti magic labeled star graph $K_{1,7} : (\sigma, \mu)$.

Stage 2. By removing an edge $[\sigma(v_m), \sigma(v_{m-1})]$ from the path P_m which has been appended by 'n' new pendant edges of $CT_{m,n}$.

We have got another disconnected graph such as the triangular fuzzy labeled $P_{m\text{-}1} : (\sigma, \mu)\ \forall\ m$.

By Theorem 3.1.2 and using (1.4)

$$\mu(v_k, v_{k+1}) + \mu(v_{k+1}, v_{k+2})$$

$$= [p_k,\ q_k,\ r_k] + [p_{k+1},\ q_{k+1},\ r_{k+1}]$$

$$= [(p_k,\ +\ p_{k+1}),\ (q_k,\ +\ q_{k+1}),\ (r_k\ +\ r_{k+1})] \tag{1.6}$$

Hence by (1.6), $P_{m\text{-}1} : (\sigma, \mu)\ \forall\, m$ be the triangular fuzzy anti magic labeled path graph $P_{m\text{-}1} : (\sigma, \mu)\ \forall\, m$.

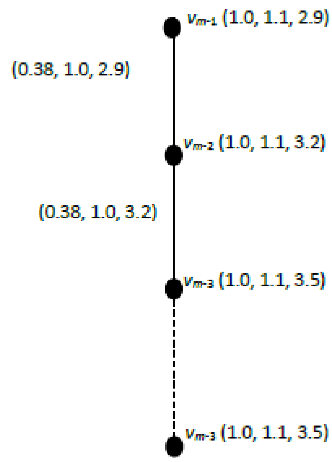

Figure 3. Triangular fuzzy anti magic labeled path graph $P_{m\text{-}1}$: (σ, μ).

Theorem 3.4. *If* $CT_{m,n}$: (σ, μ) ∀ m, n *be the triangular fuzzy anti magic labeled coconut tree graph then the removal of n-2 edges from 'n' pendant edges from* $CT_{m,n}$ *resulting a triangular fuzzy anti magic labeled Y-tree graph* Y_m : (σ, μ) ∀ m.

Proof. Given $CT_{m,n}$: (σ, μ) ∀ m, n be the triangular fuzzy anti magic labeled coconut tree graph.

Let us remove (n-2) edges from $CT_{m,n}$ such as $[\sigma(v_n), \sigma(v_{n-1}), \ldots, \sigma(v_3)]$ which has been reduced into a triangular fuzzy Y-tree graph Y_m : (σ, μ) ∀ m. By Theorem 3.2 and using (1.3) and (1.4).

We have got triangular fuzzy Y-tree graph which satisfies the condition of anti magic labeling.

Hence the resultant graph is a triangular fuzzy anti magic labeled Y-tree graph.

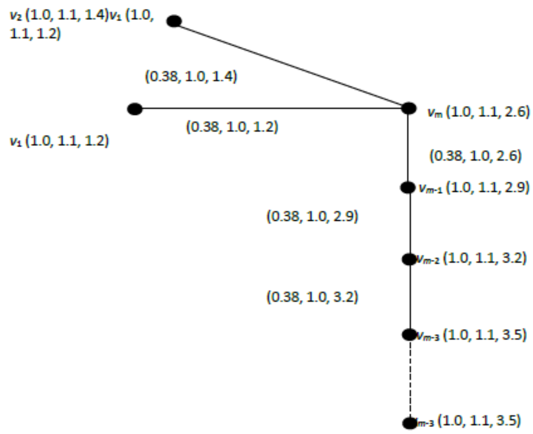

Figure 4. Triangular fuzzy anti magic labeled Y-tree graph $Y_m : (\sigma, \mu)\ \forall\ m.$

4. Conclusion

Fuzzy graph labeling has been taken part a major role in the telecommunication system (both wired and wireless) in case of vagueness while receiving and transmitting signals in everyday life. Here we have shown triangular fuzzy coconut tree graph $CT_{m,n}\ : (\sigma, \mu)\ \forall\ m, n.$ Also the concept has been extended that the above mentioned graph admits triangular fuzzy anti magic labeling. Triangular fuzzy anti magic labeled star graph and triangular fuzzy anti magic labeled path graph have been obtained by removing the pendant edge from the path P_m of $CT_{m,\, n} : (\sigma, \mu)\ \forall\ m, n.$ Also by removing (n-2) edges from the 'n' pendant edges of $CT_{m,\, n} : (\sigma, \mu)\ \forall\ m, n$ we have attained a triangular fuzzy anti magic labeled Y-tree graph. The above results can be expanded with trapezoidal fuzzy number.

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