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DERIVATIONS OF INTUITIONISTIC FUZZY IMPLICATIVE IDEALS OF BCK-ALGEBRA

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Abstract

The aim of this paper is to apply the concept of Derivation Left-Right (L, R)- Derivation and Right-Left (R, L)- Derivation) to Intuitionistic Fuzzy Sub-Algebra, Intuitionistic Fuzzy Ideal and Intuitionistic Fuzzy Implicative Ideals and introduced the notions of Derivations of Intuitionistic Fuzzy Sub-Algebra, Derivations of Intuitionistic Fuzzy Ideal and Derivations of Intuitionistic Fuzzy Implicative ideals and finding the different results, learning the relation among Derivations of Intuitionistic Fuzzy Sub-Algebra, Derivations of Intuitionistic Fuzzy Ideal and Derivations of Intuitionistic Fuzzy Implicative Ideals and so many properties that are related investigated.

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1. Introduction

In 1966, K. Iseki and Y. Imai [11] introduced a new perception known as BCK-algebras. So many researchers investigated assorted houses of this algebra. Such Algebra generalized the thought of units with the set subtraction as the most effective nonnulary operation. In [9], Iseki and Tanaka introduce the idea of Sub-Algebra, Ideals and Positive Implicative Ideals in BCK-algebras. Meng [15], introduce the notion of Implicative best in BCK-Algebras and investigate the affiliation of it with the concept of Positive Implicative ideals and Commutative Ideals in BCK-Algebras. In Xi [26] functional the concept of Fuzzy Set to BCK-Algebras and had given some houses of it. Later then Jun and Meng examined similarly residences of Fuzzy BCK-Algebra and fuzzy beliefs (see [12, 13]), and because then.

Jun collectively with Hung, Kim, Roh and Song, keep in mind the fuzzification of Sub-Algebras and Ideals in BCK-Algebras (cf. [33, 36, 39, and 38]). In [41], Jun and Kim, the usage of the Atanassov's concept, we located the Intuitionistic Fuzzification of the idea of Sub-Algebra and Ideals in BCK-Algebra, and investigated a few of its houses. In this section, we gave the definitions and examples of Intuitionistic Fuzzy Sub-Algebra and Intuitionistic Fuzzy Ideals of BCK-Algebras and associated effects. Meng [59] establish the conception of Implicative Ideals in BCK-Algebras. In [15, 16], added the idea of fuzzy implicative ideals of BCK-algebras and related residences are investigated. In [25] Satyanarayana and Durga Prasad, added the idea of Intuitionistic Fuzzy Implicative Ideals in BCK-Algebras and associated residences are examined.

The intention of present paper is to use the idea of Derivation (L, R)- derivation and (R, L)- derivation) to Intuitionistic Fuzzy Sub-Algebra, Intuitionistic Fuzzy Ideal and Derivations of Intuitionistic Fuzzy Implicative Ideals and introduced the Derivations of Intuitionistic Fuzzy Sub-Algebra, Derivations of Intuitionistic Fuzzy Ideal and Derivations of Intuitionistic Fuzzy Implicative Ideals and discover the specific outcomes, reading the relation amongst Derivations of Intuitionistic Fuzzy Sub-Algebra, Derivations of Intuitionistic Fuzzy Ideal and Derivations of Intuitionistic Fuzzy Implicative ideals and a number of associated residences are tested.

2. Preliminaries

Definition 2.1. The following are the elementary definitions. A BCK-algebra is an algebra of type (2, 0) if it satisfies the following axioms for all $p, q, r \in K$.

(BCK-1) $((p \otimes q) \otimes (p \otimes r)) \otimes (r \otimes q) = 0$

(BCK-2) $(p \otimes (p \otimes q)) \otimes q = 0$

(BCK-3) $p \otimes q = 0$

(BCK-4) $0 \otimes p = 0$

(BCK-5) $p \otimes q = 0$ and $q \otimes p = 0$ implies $p = q$.

We can define a binary relation $\leq$ on K by assuming $p \leq q$ if and only if $p \otimes q = 0$. In that case $(K, \leq)$ is a partial ordered set with least element 0 and $(K, \otimes, 0)$ is a BCK-algebra if and only if, it satisfies the following axioms: For all $p, q, r \in K$.

(i) $((p \otimes q) \otimes (p \otimes r)) \leq (r \otimes q)$ (ii) $(p \otimes (p \otimes q)) \leq q$ (iii) $p \leq p$ (iv) $0 \leq p$

(v) $p \leq q$ and $q \leq p$ implies $p = q$, for all $p, q, r \in K$.

The following are the properties in a BCK-algebra:

(P1) $p \otimes 0 = p$, (P2) $p \otimes q \leq p$, (P3) $(p \otimes q) \otimes r = (p \otimes r) \otimes q$, (P4) $(p \otimes r) \otimes (q \otimes r) \leq (p \otimes q)$

(P5) $p \otimes (p \otimes (p \otimes q)) = p \otimes q$, (P6) $p \leq q \Rightarrow p \otimes r \leq q \otimes r$ and $r \otimes q \leq r \otimes p$

(P7) $p \otimes q \leq r \Rightarrow p \otimes r \leq q$ for all $p, q, r \in K$.

A BCK-Algebra K is said to be implicative if $p = p \otimes (q \otimes p)$, for all $p, q, r \in K$. A non-empty subset I of K is said to be sub-algebra of K if $p \otimes q \in I$ whenever $p, q \in K$, an ideal of K if $(I_1) 0 \in I$ and $(I_2)\, p \otimes q$ and $q \in I$ imply that $p \in I$ for all $p, q \in K$, an implicative ideal if (I_1) and

$(I_3)(p \otimes (q \otimes p)) \otimes r \in I$ and $r \in I$ imply $p \in I$ for all $p, q, r \in K$. Once we recollect the contents of fuzzy set and intuitionistic fuzzy set.

A fuzzy set in a set K is a function $M : K \to [0, 1]$ and the complement of M denoted by $\overline{M}$ the fuzzy set on K given by $\overline{M}(p) = 1 - M(p)$ for all $p \in K$. Let M and N be the fuzzy sets on K. For $s, t \in [0, 1]$ the set $U(M, s) = \{p \in K / M_A(p) \geq s\}$ is called upper s-level cut of M and the set $L(N, t) = \{p \in K / N_A(p) \leq t\}$ is called lower t-level cut of N and can be used to characterize of M and N. An intuitionistic fuzzy set (briefly IFS) A in a non-empty set K is an object having the form $A = \{p, M_A, N_A(p)/p \in K\}$ where the functions $M_A : K \to [0, 1]$ and $N_A : K \to [0, 1]$ denoted the degree of membership of each element $p \in K$ to the set A respectively and $0 \leq M_A(p) + N_A(p) \leq 1$ for all $p \in K$. Let K denotes a BCK-algebra.

A map $\Delta : K \to K$ is known as a left-right derivation (briefly (L, R)-derivation of X if:

$$\Delta(p \otimes q) = (\Delta(p) \otimes q) \wedge (p \otimes \Delta(q)), \text{ for all } p, q \in K.$$

A map $\Delta : K \to K$ is known as a right derivation (briefly (R, L)- derivation of K if:

$$\Delta(p \otimes q) = (p \otimes \Delta(q)) \wedge (\Delta(p) \otimes q), \text{ for all } p, q \in K.$$

A map $\Delta : K \to K$ is known as a derivation of K if Δ is both a (L, R)- derivation and (R, L)- derivation of K. Let $(K, \otimes, 0)$ be a BCK-algebra, $\Delta : K \to K$ be a self map. A non-empty subset A of BCK-algebra K and $p, q, r \in K$ is called (i) left derivation ideal of BCK-algebra K if it satisfies: $(D_1)\, 0 \in A$ and $(LD_2) \Delta(p) \otimes q \in A$ and $\Delta(q) \in I$ imply that $\Delta(p) \in I$ for all $p, q \in K$. (ii) right derivation ideal of BCK-algebra K if it satisfies: (D_1) and $(RD_2) p \otimes \Delta(q) \in A$ and $\Delta(q) \in I$ imply that $\Delta(p) \in I$ for all $p, q \in K$ and called derivation ideal of BCK-algebra K, (D_1) and $(D_2)\, \Delta(p \otimes q) \in A$ and $\Delta(q) \in I$ imply that $\Delta(p) \in I$ for all $p, q \in K$. A non-empty subset A of BCK-algebra K and p, q, r, K is called DMI (derivation implicative ideal) of BCK-algebra K if it satisfies: (DMI-1) $0 \in A$

(DMI-2) $\Delta((p \otimes (q \otimes p)) \otimes r) \in A$ and $\Delta(r) \in A$ imply $\Delta(p) \in A$. The same is applied for left and right derivation implicative ideals.

Definition 2.2. A self mapping of BCK-algebra is called regular if $\Delta(0) = 0$.

Corollary 2.3. *Derivation on BCK-algebra is regular.*

Proposition 2.4. *Let Δ be a regular derivation of BCK-algebra K, then the following are hold for all $p, q \in K$*

(i) $\Delta(p) \leq p$

(ii) $\Delta(p) \otimes q \leq p \otimes \Delta(q)$

(iii) $\Delta(p \otimes q) = \Delta(p) \otimes q \leq \Delta(p) \otimes (q)$

(iv) $\Delta^{-1}(0) = \{p \in K / \Delta(p) = 0\}$ *is a sub algebra of K and* $\Delta^{-1}(0) \subset K_+$

3. DIFSA and DIFI's in BCK-ALGEBRA
(Derivations of Intuitionistic fuzzy sub-algebra and Derivations Intuitionistic fuzzy ideals in BCK-algebra)

In this part, we apply the concept of Derivation $((L, R)$- derivation and (R, L)- derivation) to Intuitionistic fuzzy sub-algebra (DIFSA), Intuitionistic fuzzy Ideals (IFI) and initiated the view of DIFSA, Derivations of Intuitionistic fuzzy Ideal (DIFI) and related properties are investigated.

Derivation 3.1. A derivation $\Delta : K \to K$ is a mapping of BCK-algebra.

Let $A = (K, M_A, N_A)$ be a non-empty IFS of K for all $p, q, r \in K$ is called left derivation intuitionistic fuzzy implicative ideal (briefly LDIFI) of K if it satisfies:

(LDIFI-1) $M_A(0) \geq M_A(p)$ and $N_A(0) \leq N_A(p)$

(LDIFI-2) $M_A(\Delta(p)) \geq \min\{M_A(\Delta(p) \otimes q), M_A(\Delta(q))\}$

(LDIFI-3) $N_A(\Delta(p)) \leq \max\{N_A(\Delta(p) \otimes q), N_A(\Delta(q))\}$.

Right derivation intuitionistic fuzzy ideal (briefly RDIFI) of K if it satisfies:

(RDIFI-1) $M_A(0) \geq M_A(p)$ and $N_A(0) \leq N_A(p)$

(RDIFI-2) $M_A(\Delta(p)) \geq \min\{M_A(p \otimes \Delta(q)), M_A(\Delta(q))\}$

(RDIFI-3) $N_A(\Delta(p)) \leq \max\{N_A(p \otimes \Delta(q)), N_A(\Delta(q))\}$

and derivation intuitionistic fuzzy ideal (briefly DIFI) of K if it satisfies:

(DIFI-1) $M_A(0) \geq M_A(p)$ and $N_A(0) \leq N_A(p)$

(DIFI-2) $M_A(\Delta(p)) \geq \min\{M_A(p \otimes q), M_A(\Delta(q))\}$

(DIFI-3) $N_A(\Delta(p)) \leq \max\{N_A(p \otimes q), N_A(\Delta(q))\}$.

Proposition 3.2. *Every DIFI M_A of K is of reversing order and N_A of K is of preserving order.* (*Or*)

Let $A = (K, M_A, N_A)$ be a DIFI of K. If $\Delta(p) \leq \Delta(q)$ in K, then $M_A(\Delta(p)) \geq M_A(\Delta(q))$ and $N_A(\Delta(p)) \leq N_A(\Delta(q))$ (i.e.) *M_A is of reversing order and N_A is of preserving order.*

Proof. Let $\Delta(p) \leq \Delta(q)$. Since M_A is DIFI on BCK-algebra K.

By DIFI-2, we have $M_A(\Delta(p)) \geq \min\{M_A(\Delta(p \otimes q)), M_A(\Delta(q))\}$.

Since $\Delta(p) \leq \Delta(q)$, then $\Delta(p) \otimes \Delta(q) = 0$, we know that $\Delta(p) \otimes \Delta(q) \geq \Delta(p) \otimes q = \Delta(p \otimes q)$.

Therefore,

$$M_A(\Delta(p)) \geq \min\{M_A(\Delta(p \otimes q)), M_A(\Delta(q))\} \geq \min\{M_A(\Delta(p) \otimes \Delta(q)), M_A(\Delta(q))\}$$

$$= \min\{M_A(0), M_A(\Delta(q))\} = M_A(\Delta(q)).$$

By DIFI-3, we have

$$N_A(\Delta(p)) \geq \max\{N_A(\Delta(p \otimes q)), N_A(\Delta(q))\} \geq \max\{N_A(\Delta(p) \otimes \Delta(q)), N_A(\Delta(q))\}$$

$$= \min\{N_A(0), N_A(\Delta(q))\} = N_A(\Delta(q)).$$

Therefore $M_A(\Delta(p)) \geq M_A(\Delta(q))$ and $N_A(\Delta(p)) \leq N_A(\Delta(q))$. Hence proved.

Proposition 3.3. *An IFS* $A = (K, M_A, N_A)$ *satisfying DIFI*-1 *in BCK-algebra K is a DIFI if and only if for all* $p, q, r \in K$, $\Delta(p \otimes q) \leq \Delta(r)$ *implies* $M_A(\Delta(r)) \geq \min\{M_A(\Delta(q)), M_A(\Delta(r))\}$ *and* $N_A(\Delta(r)) \leq \max\{N_A(\Delta(q)), N_A(\Delta(r))\}$.

Proof. Assume $A = (K, M_A, N_A)$ is DIFI of K and $\Delta(p \otimes q) \leq \Delta(r)$. We can say $M_A(\Delta(r)) \leq M_A(\Delta(p \otimes q))$ and $N_A(\Delta(r)) \geq N_A(\Delta(p \otimes q))$ (since M_A is of reversing order and N_A is of preserving order) and by DIFI-2, we have $M_A(\Delta(p)) \geq \min\{M_A(\Delta(p \otimes q), M_A(\Delta(q))\} \geq \min\{M_A(\Delta(q)), M_A(r))\}$ and $N_A(\Delta(p)) \geq \max\{N_A(\Delta(p \otimes q), N_A(\Delta(q))\} \leq \max\{N_A(\Delta(q)), N_A(r))\}$.

Therefore $M_A(\Delta(p)) \geq \min\{M_A(\Delta(q)), M_A(\Delta(r))\}$ and $N_A(\Delta(p)) \leq \max\{N_A(\Delta(q)), N_A(\Delta(r))\}$ if for all $p, q, r \in K$, $\Delta(p \otimes q) \leq \Delta(r)$.

Conversely assume that if for all $p, q, r \in K$, $\Delta(p \otimes q) \leq \Delta(r)$.

$$M_A(\Delta(p)) \geq \min\{M_A(\Delta(q)), M_A(\Delta(r))\} \text{ and}$$

$$N_A(\Delta(p)) \leq \max\{N_A(\Delta(q)), N_A(\Delta(r))\}.$$

Since $\Delta(p \otimes q) \otimes \Delta(r) = 0$ then $\Delta(p \otimes q) \leq \Delta(r)$.

We have $M_A(\Delta(p)) \geq \min\{M_A(\Delta(q), M_A(\Delta(r))\} \geq \min\{M_A(\Delta(q)), M_A(\Delta(p \otimes q))\}$ and $N_A(\Delta(p)) \leq \max\{N_A(\Delta(q), N_A(\Delta(r))\} \leq \max\{N_A(\Delta(q)), N_A(\Delta(p \otimes q))\}$.

Hence $A = (K, M_A, N_A)$ is DIFI of K.

Definition 3.4. Let K be a BCK-algebra. An IFS $A = (K, M_A, N_A)$ is called to be a (left/right) DIFSA (derivation intuitionistic fuzzy sub-algebra) of K if it satisfies:

(DIFSA-1) $M_A(\Delta(p \otimes q)) \geq \min\{M_A(\Delta(p), M_A(\Delta(q))\}$

(DIFSA-2) $N_A(\Delta(p \otimes q)) \geq \min\{N_A(\Delta(p), N_A(\Delta(q))\}$ for all $p, q \in K$.

Theorem 3.5. *Any DIFI* $A = (K, M_A, N_A)$ *must be a DIFS of K.*

Proof. Since $\Delta(p \otimes q) \leq \Delta(r)$ since, M_A is of reversing order, so that

$$M_A(\Delta(p \otimes q)) \geq M_A(\Delta(p)).$$

So, by DIFI properties, we have

$$M_A(\Delta(p \otimes q)) \geq M_A(\Delta(p)) \geq \min\{M_A(\Delta(p \otimes q)), M_A(\Delta(q))\}$$

$$\geq \min\{M_A(\Delta(p)), M_A(\Delta(q))\}.$$

Since $\Delta(p \otimes q) \leq \Delta(r) N_A$ is order preserving, so that $N_A(\Delta(p \otimes q)) \leq N_A(\Delta(p))$.

So, by DIFI properties, we have

$$N_A(\Delta(p \otimes q)) \leq N_A(\Delta(p)) \leq \max\{N_A(\Delta(p \otimes q)), N_A(\Delta(q))\}$$

$$\leq \max\{N_A(\Delta(p)), N_A(\Delta(q))\}.$$

This shows that A is DIFS of K.

We are giving a clause now for an IFSA to be an IFI.

Theorem 3.6. *A DIFSA* $A = (K, M_A, N_A)$ *of K is a DIFI of K if for all* $p, q, r \in K$, *the inequality* $\Delta(p \otimes q) \leq \Delta(r)$ *in* K *implies that* $M_A(\Delta(p)) \geq \min\{M_A(\Delta(q)), M_A(\Delta(r))\}$ *and* $N_A(\Delta(p)) \leq \max\{N_A(\Delta(q)), N_A(\Delta(r))\}$.

Proof. Suppose that A is DIFSA of K, and satisfying $\Delta(p \otimes q) \leq \Delta(r)$ in K implies

$$M_A(\Delta(p)) \geq \min\{M_A(\Delta(q)), M_A(\Delta(r))\} \text{ and}$$

$$N_A(\Delta(p)) \leq \max\{N_A(\Delta(q)), N_A(\Delta(r))\}.$$

Therefore, $\Delta(p \otimes (p \otimes q)) \leq \Delta(q)$, it follows that $M_A(\Delta(p)) \geq \min\{M_A(\Delta(p \otimes q)), M_A(\Delta(q))\}$ and $N_A(\Delta(p)) \leq \max\{N_A(\Delta(p \otimes q)), N_A(\Delta(q))\}$. Hence A is DIFI.

4. DIFII of BCK-Algebras
(Derivation of Intuitionistic fuzzy implicative ideal of BCK-algebras)

In this part we pertain the idea of Derivation to IFII and introduced the view of Derivations of IFII's and find the different consequences, reading the relation among DIFSA's, DIFI's and DIFII's and a number of related properties are examined.

Definition 4.1. A derivation $\Delta : K \to K$ is a mapping of BCK-algebra. Let $A = (K, M_A, N_A)$ be a non-empty IFS of K for all $p, q, r \in K$ is called left derivation intuitionistic fuzzy implicative ideal (briefly LDIFII) of K if it satisfies:

(LDIFII-1) $M_A(0) \geq M_A(p)$ and $N_A(0) \geq N_A(p)$

(LDIFII-2) $M_A(\Delta(a)) \geq \min\{M_A((\Delta(p \otimes (q \otimes p))) \otimes r), M_A(\Delta(r))\}$

(LDIFII-3) $N_A(\Delta(a)) \leq \max\{N_A((\Delta(p \otimes (q \otimes p))) \otimes r), N_A(\Delta(r))\}$.

Right derivation intuitionistic fuzzy implicative ideal (briefly RDIFII) of K if it satisfies:

(RDIFII-1) $M_A(0) \geq M_A(p)$ and $N_A(0) \leq N_A(p)$

(RDIFII-2) $M_A(\Delta(a)) \geq \min\{M_A(((p \otimes (q \otimes p))) \otimes \Delta(r)), M_A(\Delta(r))\}$

(RDIFII-3) $N_A(\Delta(a)) \leq \max\{N_A(((p \otimes (q \otimes p))) \otimes \Delta(r)), N_A(\Delta(r))\}$.

and derivation intuitionistic fuzzy implicative ideal (briefly DIFII) of K if it satisfies:

(DIFII-1) $M_A(0) \geq M_A(p)$ and $N_A(0) \geq N_A(p)$

(DIFII-2) $M_A(\Delta(a)) \geq \min\{M_A(\Delta((p \otimes (q \otimes p)) \otimes r)), M_A(\Delta(r))\}$

(DIFII-3) $N_A(\Delta(a)) \geq \min\{N_A(\Delta((p \otimes (q \otimes p)) \otimes r)), N_A(\Delta(r))\}$.

Example 4.2. Consider a BCK-algebra $K = \{0, 1, 2, 3, 4\}$ with the following Cayley table

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0

3	3	3	3	0	0
4	4	3	4	1	0

Define a mapping $\Delta : K \to K$ by $\Delta(p) = \begin{cases} 0 \; if \; p = 0, 1, 2, 3 \\ \\ 4 \; if \; p = 4. \end{cases}$

Then it is clearly that Δ is derivation of K and we define a IFS $A = (K, M_A, N_A)$ in K defined by $M_A(0) = M_A(2) = s_0$, $M_A(1) = M_A(3) = M_A(4) = s_1$ and $N_A(0) = N_A(2) = t_0$, $N_A(1) = N_A(3) = N_A(4) = t_1$, where $t_i, s_j \in [0, 1]$ and $t_i + s_j \leq 1$, where $i, j \in \{0, 1\}$ and if you define derivation on the IFS by $M_A : K \to K$ and $N_A : K \to K$ such that $M_A(\Delta(0)) = M_A(\Delta(2)) = 1$, $M_A(\Delta(1)) = M_A(\Delta(3)) = M_A(\Delta(4)) = 0.4$ and $N_A(\Delta(0)) = N_A(\Delta(2)) = 0$, $N_A(\Delta(1)) = N_A(\Delta(3)) = N_A(\Delta(4)) = 0.6$.

Then it is easily to check that $A = (K, M_A, N_A)$ is DIFII of K.

Theorem 4.3. *Every DIFII of K is an DIFI of K.*

Proof. As given $A = (K, M_A, N_A)$ be DIFII of K, then we have

(DIFII -2) $M_A(\Delta(a)) \geq \min\{M_A(\Delta((p \otimes (q \otimes p)) \otimes r)), M_A(\Delta(r))\}$

(DIFII- 3) $N_A(\Delta(a)) \leq \max\{N_A(\Delta((p \otimes (q \otimes p)) \otimes r)), N_A(\Delta(r))\}$.

By taking $q = 0$ in (DIFII -2) and (DIFII- 3),

We obtain

$$M_A(\Delta(p)) \geq \min\{M_A(\Delta((p \otimes (0 \otimes p)) \otimes r)), M_A(\Delta(r))\}$$

$$= \min\{M_A(\Delta((p \otimes 0) \otimes r)), M_A(\Delta(r))\}$$

$$= \min\{M_A(\Delta((p \otimes r) \otimes r)), M_A(\Delta(r))\}$$

and

$$N_A(\Delta(p)) \leq \max\{N_A(\Delta((p \otimes (0 \otimes p)) \otimes r)), N_A(\Delta(r))\}$$

$$= \max\{N_A(\Delta((p \otimes 0) \otimes r)), N_A(\Delta(r))\}$$

$$= \max\{N_A(\Delta((p \otimes r) \otimes r)), N_A(\Delta(r))\}.$$

This shows that $A = (K, M_A, N_A)$ is DIFI of K.

Theorem 4.4. *If K is an implicative BCK-algebra, then every DIFI of K is a DIFII of K.*

Proof. Since K is an implicative BCK-algebra, it follows that $p = p \otimes (q \otimes p)$, for all $p, q, r \in K$.

Let $A = (K, M_A, N_A)$ DIFI of K.

Then $M_A(\Delta(p)) \geq \min\{M_A(\Delta(p \otimes r)), M_A(\Delta(r))\} \geq \min\{M_A(\Delta((p \otimes (q \otimes p)) \otimes r)), M_A(\Delta(r))\}$ and $N_A(\Delta(p)) \leq \max\{N_A(\Delta(p \otimes r)), N_A(\Delta(r))\} \leq \max\{N_A(\Delta((p \otimes (q \otimes p)) \otimes r)), N_A(\Delta(r))\}$ for all $p, q, r \in K$.

Thus $A = (K, M_A, N_A)$ is a DIFII of K.

Theorem 4.5. *The intersection of any set of DIFII on BCK-algebra K is also DIFII.*

Proof. Let $A_i = (K, M_{A_i}, N_{A_i})$ be a family of DIFII on BCK-algebra K and for any $p, q \in r \in K$ we will show that the intersection of A_i is a DIFII.

(DIFII-1): $(\bigcap M_{A_i})(0) = \inf(M_{A_i})(0) \geq \inf(M_{A_i})(p) = (\bigcap M_{A_i})(p)$

(DIFII-2): $(\bigcap M_{A_i})(\Delta(p)) = \inf(M_{A_i})(\Delta(p)) \geq \inf(\min\{M_{A_i}(\Delta((p \otimes (q \otimes p) \otimes r)), M_{A_i}(\Delta(r))\} = (\min\{\inf(M_{A_i}(\Delta((p \otimes (q \otimes p) \otimes r)), \inf(M_{A_i}(\Delta(r)))\} = (\min\{\bigcap(M_{A_i}(\Delta((p \otimes (q \otimes p) \otimes r)), \bigcap(M_{A_i}(\Delta(r))\}$.

Therefore, $(\bigcap M_{A_i})(\Delta(p)) \geq (\min\{\bigcap M_{A_i}(\Delta((p \otimes (q \otimes p) \otimes r))), \bigcap(\mu_{A_i}(\Delta(r)))\}$ and that is

DIFII-2.

(DIFII-1): $(\bigcap N_{A_i})(0) = \sup(N_{A_i})(0) \leq \sup(N_{A_i})(p) = \sup(N_{A_i})(p)$

(DIFII-2): $(\bigcap N_{A_i})(\Delta(p)) = \sup(N_{A_i})(\Delta(p)) \leq \sup(\max\{N_{A_i}(\Delta((p \otimes (q \otimes p) \otimes r)), N_{A_i}(\Delta(r))\} = \max(\sup\{N_{A_i}(\Delta((p \otimes (q \otimes p) \otimes r))), \sup(N_{A_i}(\Delta(r))\} = \max\{\bigcap(N_{A_i}(\Delta((p \otimes (q \otimes p) \otimes r)), \bigcap(N_{A_i}(\Delta(r)))\}$.

Therefore, $(\bigcap N_{A_i})(\Delta(p)) \leq (\max\{\bigcap(N_{A_i}(\Delta((p \otimes (q \otimes p) \otimes r))), \bigcap(N_{A_i}(\Delta(r)))\}$ and that is DIFII-3. The same result is applied for left and right DIFII of BCK-algebra *K*.

Theorem 4.6. *The union of many DIFII on BCK-algebra K is also DIFII.*

Proof. Let $A_1 = (K, M_{A_i}, N_{A_i})$ and $A_2 = (K, M_{A_2}, N_{A_2})$ be two DIFMI on BCK-algebra *K*. We will show that $M_{A_1} \cup M_{A_2}$ is also DIFII of *K*.

(DIFMI-1): $((M_{A_1} \cup M_{A_2})(0) = M_{A_1}(0) \cup M_{A_2}(0) \geq M_{A_1}(p) \cup M_{A_2}(p) = (M_{A_1} \cup M_{A_2})(p)$

(DIFMI-2): $(M_{A_1} \cup M_{A_2})(\Delta(p)) = M_{A_1}(\Delta(p)) \cup M_{A_2}(\Delta(p))$

$\geq \min\{M_{A_1}(\Delta((p \otimes (q \otimes p) \otimes r))), M_{A_1}(\Delta(r))\} \cup \min\{M_{A_2}(\Delta((p \otimes (q \otimes p) \otimes r))), M_{A_2}(\Delta(r))\}$

$\geq \min\{M_{A_1}(\Delta((p \otimes (q \otimes p) \otimes r))) \cup M_{A_2}(\Delta((p \otimes (q \otimes p) \otimes r))), M_{A_1}(\Delta(r)) \cup M_{A_2}(\Delta(r))\}$

$= \min\{(M_{A_1} \cup M_{A_2})(\Delta((p \otimes (q \otimes p) \otimes r), (M_{A_1} \cup M_{A_2})(\Delta(r))\}$

$(M_{A_1} \cup M_{A_2})(\Delta(p)) \geq \min\{(M_{A_1} \cup M_{A_2})(\Delta((p \otimes (q \otimes p) \otimes r), (M_{A_1} \cup M_{A_2})(\Delta(r))\}$

and that is DIFII-2.

(DIFII-1): $((N_{A_1} \cup N_{A_2})(0) = N_{A_1}(0) \cup N_{A_2}(0) \geq N_{A_1}(p) \cup N_{A_2}(p) = (N_{A_1} \cup N_{A_2})(p)$

(DIFII-2): $(N_{A_1} \cup N_{A_2})(\Delta(p)) = N_{A_1}(\Delta(p)) \cup N_{A_2}(\Delta(p))$

$\leq \max\{N_{A_1}(\Delta((p \otimes (q \otimes p) \otimes r))), N_{A_1}(\Delta(r))\} \cup \max\{N_{A_2}(\Delta((p \otimes (q \otimes p) \otimes r))), N_{A_2}(\Delta(r))\}$

$\leq \max\{N_{A_1}(\Delta((p \otimes (q \otimes p) \otimes r))) \cup N_{A_2}(\Delta((p \otimes (q \otimes p) \otimes r))), N_{A_1}(\Delta(r)) \cup N_{A_2}(\Delta(r))\}$

$= \max\{(N_{A_1} \cup N_{A_2})(\Delta((p \otimes (q \otimes p) \otimes r), (N_{A_1} \cup N_{A_2})(\Delta(r))\}$

$(N_{A_1} \cup N_{A_2})(\Delta(p)) \leq \max\{(N_{A_1} \cup N_{A_2})(\Delta((p \otimes (q \otimes p) \otimes r),$
$(N_{A_1} \cup N_{A_2})(\Delta(r))\}$

and that is DIFII-3

Therefore, the union of many DIFII on BCK-algebra K is also DIFII. The same result is applied for left and right DIFII of BCK-algebra K.

Corollary. 4.7. *Every IFII of K is an IFSA of K.*

Corollary. 4.8. *Let* $A = (K, M_A, N_A)$ *be an IFII of K, if* $p \leq q$ *in K, then* $M_A(p) \geq M_A(q)$, $N_A(p) \geq N_A(q)$, *that is,* M_A *is of reversing order and* N_A *is of preserving order.*

Theorem 4.9. *An IFS* $A = (K, M_A, N_A)$ *of BCK-algebra K is DIFII of K if and only if K if and only if the non-empty upper s-level cut* $U(M_A; s)$ *and the non-empty lower t-level cut* $L(N_A; t)$ *are DII of K, for any* $s, t \in [0, 1]$.

Proof. Assume that $A = (K, M_A, N_A)$ be DIFII of BCK-algebra K then $M_A(0) \geq M_A(p)$ for any $p \in K$ therefore $M_A(0) \geq M_A(p) \geq s$ then $p \in U(M_A; s)$ and so $0 \in U(M_A; s)$ and this gives $U(M_A; s) \notin \Phi$. Let $\Delta((p \otimes (q \otimes p)) \otimes r.\ \Delta(r) \in U(M_A; s))$. So $M_A(\Delta((p \otimes (q \otimes p)) \otimes r) \geq s$, $M_A(\Delta(r)) \geq s$.

Since A is DIFII we have: $M_A(\Delta(p)) \geq \min\{M_A(\Delta((p \otimes (q \otimes p)) \otimes r),$ $M_A(\Delta(r)) \geq s$. Implies $M_A\Delta(p) \geq s$, so that $\Delta(p) \in U(M_A; s)$ and $N_A(0) \leq N_A(p)$ for any $p \in K$ therefore $N_A(0) \leq N_A(p) \leq t$ then $p \in L(N_A; t)$ and so $0 \in L(N_A; t)$ and this gives $L(N_A; t) \notin \Phi$. Let $N_A(\Delta((p \otimes (q \otimes p)) \otimes r), \Delta(r) \in L(N_A; t)$. So $N_A(\Delta((p \otimes (q \otimes p)) \otimes r) \geq t$, $N_A(\Delta(r)) \leq t$. Since A is DIFII we have $N_A(\Delta(p)) \leq \max\{N_A(\Delta((p \otimes (q \otimes p)) \otimes r), N_A(\Delta(r)) \leq t$. Hence $N_A(\Delta(p)) \leq t$ so $\Delta(p) \in L(N_A; t)$. Therefore, upper s-level cut $U(M_A; s)$ and the lower t-level cut $L(N_A; s)$ are DII of K, for any $s, t \in [0, 1]$.

Conversely assume that the upper s-level cut $U(M_A; s)$ and the lower t-level cut $L(N_A; s)$ are DII of K, for any $s, t \in [0, 1]$. Here we will show that both DIFII-1 and DIFII-2 are true.

Suppose that DIFII-1 is not true, then there exist $p^* \in K$ such that $M_A(0) \leq M_A(p^*)$ and if we take

$$s^* = \frac{M_A(p^*) + M_A(0)}{2} \text{ this gives us } M_A(0) < s^*. \quad (1)$$

So, $0 < s^* < M_A(p^*) \leq 1$ then $M_A(p^*) > s^* \Rightarrow p^* \in U(M_A; s^*)$ and those $U(M_A; s) \neq \Phi$, since $U(M_A; s)$ is DII of K, then we have $0 \in U(M_A; s)$ this gives $M_A(0) \geq s^*$ and this is a contradiction with equation (1), so DIFII-1 is true.

Suppose that DIFII-1 is not true, then there exist $p^* \in K$ such that $N_A(0) > N_A(p^*)$ and if we take

$$t^* = \frac{N_A(p^*) + N_A(0)}{2} \text{ this gives us } N_A(0) > t^*. \quad (2)$$

So, $0 \geq t^* < N_A(p^*) \geq 1$ then $N_A(p^*) < t^* \Rightarrow p^* \in L(N_A; t)$ and those $L(N_A; t) \neq \Phi$.

As $L(N_A; t)$ is DII of K, then we have $0 \in L(N_A; t)$ this gives $N_A(0) \leq t^*$ and this is a contradiction with equation (2), so DIFII-1 is true. Now suppose DIFII-2 is not true then there exists $p^*, q^*, r^* \in K$ such that: $M_A(\Delta(p^*)) < \min\{M_A(\Delta((p^* \otimes (q^* \otimes p^*) \otimes r^*)), M_A(\Delta(r^*))\}$. Put

$$s^* = \frac{M_A(p^*) + \min\{M_A(\Delta((p^* \otimes (q^* \otimes p^*) \otimes r^*)), M_A(\Delta(r^*))\}}{2}.$$

This give us

$$M_A(\Delta(p^*)) < s^* \quad (3)$$

and

$$0 \leq s^* < \min\{M_A(\Delta((p^* \otimes (q^* \otimes p^*) \otimes r^*)), M_A(\Delta(r^*))\} \leq 1.$$

Thus $\min\{M_A(\Delta((p^* \otimes (q^* \otimes p^*) \otimes r^*)), M_A(\Delta(r^*))\} > s^*$.

This means $M_A(\Delta((p^* \otimes (q^* \otimes p^*) \otimes r^*)) \geq s^*$, $\mu_A(\Delta(r^*)) \geq s^*$.

Implies that $\Delta((p^* \otimes (q^* \otimes p^*)) \otimes r^*)), \Delta(r^*) \in U(M_A; s)$. Since $U(M_A; s)$ is DII of K, it follows that $\Delta(p^*) \in U(M_A; s)$ and we get $M_A(\Delta(p^*)) \geq s^*$ and this is a contradiction with equation (3) and so DIFII-2 is true. Now assume DIFII-2 is not true then there exists $p^*, q^*, r^* \in K$ such that: $N_A(\Delta(p^*)) > \max\{N_A(\Delta((p^* \otimes (q^* \otimes p^*)) \otimes r^*)), N_A(\Delta(r^*))\}$. Put $t^* = \dfrac{N_A(p^*) + \max\{N_A(\Delta((p^* \otimes (q^* \otimes p^*) \otimes r^*)), N_A(\Delta(r^*))\}}{2}$.

This gives us

$$N_A(d(p^*)) > t^* \tag{4}$$

and $0 \geq t^* > \max\{N_A(\Delta((p^* \otimes (q^* \otimes p^*)) \otimes r^*)), N_A(\Delta(r^*))\} \geq 1$. This means that $\max\{N_A(\Delta((p^* \otimes (q^* \otimes p^*) \otimes r^*)), N_A(\Delta(r^*))\} \leq t^*$. Implies that $N_A(\Delta((p^* \otimes (q^* \otimes p^*) \otimes r^*)) \leq t^*$ and $\lambda_A(\Delta((r^*)) \leq t^*$.

Implies that $\Delta((p^* \otimes (q^* \otimes p^*) \otimes r^*)), \Delta(r^*) \in L(N_A; t)$. Since $L(N_A; t)$ is DII of K, it follows that $d(p^*) \in L(N_A; t)$ and we get $N_A(\Delta(p^*)) \leq t^*$ this is a contradiction with equation (4) and so DIFII-2 is true and satisfied, therefore A is DIFII of K.

Notation: For any intuitionistic fuzzy sets $A = (K, M_A, N_A)$ and $B = (K, M_B, N_B)$, we write $A \subseteq B$, that is, $M_A \leq M_B$ and $N_A \geq N_B \Leftrightarrow M_A(p) \leq M_B(p)$ and $N_A(p) \geq N_B(p)$ for all $p \in K$.

Proposition 4.10. *Let I and A be ideals of K with $I \subseteq A$. If I is a DII, then so is A.*

Theorem 4.11 (Extension theorem of DIFII)**.** *Let $A = (K, M_A, N_A)$ and*

$B = (K, M_B, N_B)$, are DIFI of K such that $A \subseteq B$ and $A(0) = B(0)$. If A is DIFII of K, then so is B.

Proof. To prove that B is a DIFII of K, it suffices to show that, for any $s, t \in [0, 1]$, $U(M_B; s)$ and $L(N_B; t)$ are either empty (or) an DII of K. If the level subset $U(M_B; s)$ is non empty, then $U(M_A; s) \neq \Phi$ and $U(M_A; s) \subseteq U(M_B; s)$.

In fact, if $p \in U(M_A; s) \Rightarrow M_A(p) \geq s \Rightarrow M_B(p) \geq M_A(p) \geq s$, implies that $p \in U(M_B; s)$ and also if the level subset $L(N_B; t)$ is non empty, then $L(N_A; t) \neq \Phi$ and $L(N_A; t) \subseteq L(N_B; t)$. In fact, if $p \in L(N_A; t) \Rightarrow N_A(p) \leq t$, implies that $N_B(\ ) \leq N_A(p) \leq t \Rightarrow p \in L(N_B; t)$. By hypothesis, $A = (K, M_A, N_A)$ is an DIFII of K. By Theorem 4.9, for any $s, t \in [0,1]$, $U(M_A; s)$ and $L(N_A; t)$ are derivation implicative ideals of K. By Proposition 4.10, for any $s, t \in [0, 1]$, $U(M_B; s)$ and $L(N_B; t)$ are derivation implicative ideals of K. Hence, by Theorem 4.9, $B = (K, M_B, N_B)$ is DIFII of K.

Theorem 4.12. *Let $A = (K, M_A, N_A)$ be DIFI of K,* then *$A = (K, M_A, N_A)$ is DIFII of K if and only if it satisfies the conditions $M_A(\Delta(p)) \geq M_A(\Delta((p \otimes (q \otimes p))))$ and $N_A(\Delta(p)) \leq N_A(\Delta((p \otimes (q \otimes p))))$ for all $p, q, r \in K$.*

Proof. Let $A = (K, M_A, N_A)$ is DIFII of K. Put $r = 0$ in (DIFII-2) and (DIFII-3), we get

$$M_A(\Delta(p)) \geq \min\{M_A(\Delta((p \otimes (q \otimes p)) \otimes r)), M_A(\Delta(r))\}$$

$$\geq \min\{M_A(\Delta((p \otimes (q \otimes p)) \otimes 0)), M_A(\Delta(0))\}$$

$$\geq \min\{M_A(\Delta((p \otimes (q \otimes p)))), M_A(\Delta(0))\} = M_A(\Delta((p \otimes (q \otimes p))))$$

and

$$N_A(\Delta(p)) \leq \max\{N_A(\Delta((p \otimes (q \otimes p)) \otimes r)), N_A(\Delta(r))\}$$

$$\leq \max\{N_A(\Delta((p \otimes (q \otimes p)) \otimes 0)), N_A(\Delta(0))\}$$

$$\leq \max \{N_A(\Delta((p \otimes (q \otimes p)))), N_A(\Delta(0))\} = N_A(\Delta((p \otimes (q \otimes p))))$$

$M_A(\Delta(p)) \geq M_A(\Delta((p \otimes (q \otimes p))))$ and $N_A(\Delta(p)) \leq N_A(\Delta((p \otimes (q \otimes p))))$, for all $p, q, r \in K$. Conversely assume that, $M_A(\Delta(p)) \geq M_A(\Delta((p \otimes (q \otimes p))))$ and $N_A(\Delta(p)) \leq N_A(\Delta((p \otimes (q \otimes p))))$, for all $p, q, r \in K$.

Since $A = (K, M_A, N_A)$ is DIFI of K,

$$M_A(\Delta(p)) \geq M_A(\Delta((p \otimes (q \otimes p)))) \geq \min \{M_A(\Delta((p \otimes (q \otimes p)) \otimes r)), M_A(\Delta(r))\}$$

and

$$N_A(\Delta(p)) \leq N_A(\Delta((p \otimes (q \otimes p)))) \leq \max \{N_A(\Delta((p \otimes (q \otimes p)) \otimes r)), N_A(\Delta(r))\}$$

for all $p, q, r \in K$.

Therefore $A = (K, M_A, N_A)$ is DIFII of K.

Theorem 4.13. *Let $A = (K, M_A, N_A)$ be DIFI of K. Then the following are equivalent*

(i) *A is DIFII ideal of K.*

(ii) $M_A(\Delta(p)) \geq M_A(\Delta((p \otimes (q \otimes p))))$ *and* $N_A(\Delta(p)) \leq N_A(\Delta((p \otimes (q \otimes p))))$ *for all* $p, q \in K$.

(iii) $M_A(\Delta(p)) = M_A(\Delta((p \otimes (q \otimes p))))$ *and* $N_A(\Delta(p)) = N_A(\Delta((p \otimes (q \otimes p))))$, *for all* $p, q \in K$.

Proof. (i) $\Rightarrow$ (ii) That is, Let A be DIFII of K.

Put $r = 0$ in (DIFII-2) and (DIFII-3), we get

$$M_A(\Delta(p)) \geq \min \{M_A(\Delta((p \otimes (q \otimes p)) \otimes r)), M_A(\Delta(r))\}$$

$$\geq \min \{M_A(\Delta((p \otimes (q \otimes p)) \otimes 0)), M_A(\Delta(0))\}$$

$$\geq \min \{M_A(\Delta((p \otimes (q \otimes p)))), M_A(\Delta(0))\} = M_A(\Delta((p \otimes (q \otimes p))))$$

and

$N_A(\Delta(p)) \leq \max\{N_A(\Delta((p \otimes (q \otimes p)) \otimes r)), N_A(\Delta(r))\}$

$\leq \max\{N_A(\Delta((p \otimes (q \otimes p)) \otimes 0)), N_A(\Delta(0))\}$

$\leq \max\{N_A(\Delta((p \otimes (q \otimes p)))), N_A(\Delta(0))\} = N_A(\Delta((p \otimes (q \otimes p))))$

$M_A(\Delta(p)) \geq M_A(\Delta((p \otimes (q \otimes p))))$ and $N_A(\Delta(p)) \leq N_A(\Delta((p \otimes (q \otimes p))))$, for all $p, q \in K$. Hence the condition (ii) holds.

(iii) $\Rightarrow$ (ii) Observe that in K, $\Delta(p \otimes (p \otimes q)) \leq p \otimes (p \otimes q) \leq p$ by (ii).

Applying Theorem 3.5, we have $M_A(\Delta(p)) \geq M_A(\Delta((p \otimes (q \otimes p))))$ and $N_A(\Delta(p)) \leq N_A(\Delta((p \otimes (q \otimes p))))$ for all $p, q \in K$.

It follows from (ii) that $M_A(\Delta(p)) = M_A(\Delta((p \otimes (q \otimes p))))$ and $N_A(\Delta(p)) = N_A(\Delta((p \otimes (q \otimes p))))$ for all $p, q \in K$. Hence the condition (iii) holds.

(iii) $\Rightarrow$ (i). Suppose the condition (iii) holds. Since $A = (K, M_A, N_A)$ be DIFI of K, we have $M_A(\Delta((p \otimes (q \otimes p)))) \geq \min\{M_A(\Delta((p \otimes (q \otimes p)) \otimes r)), M_A(\Delta(r))\}$ and $N_A(\Delta((p \otimes (q \otimes p)))) \leq \max\{N_A(\Delta((p \otimes (q \otimes p)) \otimes r)), N_A(\Delta(r))\}$ for all $p, q, r \in K$. Combining (iii) we obtain

$M_A(\Delta(p)) \geq \min\{M_A(\Delta((p \otimes (q \otimes p)) \otimes r)), M_A(\Delta(r))\}$ and

$N_A(\Delta(p)) \leq \max\{N_A(\Delta((p \otimes (q \otimes p)) \otimes r)), N_A(\Delta(r))\}$ for all $p, q, r \in K$.

Obviously A satisfies $M_A(\Delta(0)) \geq M_A(\Delta(p))$ and $N_A(\Delta(0)) \leq N_A(\Delta(p))$ for all $p \in K$. Therefore $A = (K, M_A, N_A)$ is DIFII of K. Hence, the condition (i) holds.

Theorem. 4.14. *An DIFS $A = (K, M_A, N_A)$ of K is DIFII if and only if $(p \otimes (q \otimes p)) \otimes r \leq u$ $M_A(\Delta(p)) \geq \min\{M_A(\Delta(r)), M_A(\Delta(u))\}$ and $N_A(\Delta(p)) \leq \max\{N_A(\Delta(r)), N_A(\Delta(u))\}$ $M_A(p) \geq \min\{M_A(r), M_A(u)\}$ and $N_A(p) \leq \max\{N_A(r), N_A(u)\}$, for all $p, q, r, u \in K$.*

Proof. Assume that $A = (K, M_A, N_A)$ is a DIFII of K.

Let $p, q, r, u \in K$ be such that $(p \otimes (q \otimes p)) \otimes r \leq u$.

Since, $A = (K, M_A, N_A)$ is DIFII of K by Theorem 4.3, it follows from Theorem 3.3 that

$$M_A(\Delta((p \otimes (q \otimes p)))) \geq \min\{M_A(\Delta(r)), M_A(\Delta(u))\} \text{ and}$$

$$N_A(\Delta((p \otimes (q \otimes p)))) \leq \max\{N_A(\Delta(r)), N_A(\Delta(u))\}.$$

Making use of the Theorem 4.12, we have (iii)

$$M_A(\Delta(p)) \geq \min\{M_A(\Delta(r)), M_A(\Delta(u))\} \text{ and}$$
$$N_A(\Delta(p)) \leq \max\{N_A(\Delta(r)), N_A(\Delta(u))\}.$$

Conversely, assume that $A = (K, M_A, N_A)$ satisfies $(p \otimes (q \otimes p)) \otimes r \leq u$ $M_A(\Delta(p)) \geq \min\{M_A(\Delta(r)), M_A(\Delta(u))\}$ and $N_A(\Delta(p)) \leq \max\{N_A(\Delta(r)), N_A(\Delta(u))\}$ for all $p, q, r, u \in K$.

Obviously, $A = (K, M_A, N_A)$ satisfies $M_A(0) \geq M_A(p)$ and $N_A(0) \leq N_A(p)$. Since $\Delta((p \otimes (q \otimes p))) \otimes (((p \otimes (q \otimes p))) \otimes r) \leq \Delta(r)$, it follows from hypothesis $M_A(\Delta(p)) \geq \min\{M_A(\Delta((p \otimes (q \otimes p))) \otimes r), M_A(\Delta(r))\}$ and $N_A(\Delta(p)) \leq \max\{N_A(\Delta((p \otimes (q \otimes p))) \otimes r), N_A(\Delta(r))\}$ for all $p, q, r \in K$. Thus $A = (K, M_A, N_A)$ is DIFII of K.

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