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MEAN SQUARE DEVIATION OF TIME TO HIRING IN A SINGLE GRADE WORKFORCE STRUCTURE WITH CLUMP OF EGRESSES AND A RANDOM THRESHOLD

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Abstract

A single grade workforce structure in which egress of employees occurs in clumps at random epochs is considered. Two stochastic exemplars are constructed utilizing CUM and MAX strategies of hiring. Analytical result for mean square deviation of time to hiring is determined under the conditions (i) cumulative count for egresses is a compound Poisson process (ii) deprivation of workforce in each clump of egresses are independent exponential random variables and (iii) threshold is an exponential random variable. A quantitative illustration for analytical results with relevant findings and conclusion are presented.

1. Introduction

As a pioneer of modelling workforce designing, the author in [1] and [2] has discussed the statistical approach and renewal theory exemplars. Several stochastic exemplars utilizing CUM strategy of hiring for the problem of time to hiring in a single grade workforce structure are constructed by many researchers. Assuming that egress occurs as the result of strategic decision

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taken in the workforce structure, mean square deviation of time to hiring is determined under different conditions on deprivation of workforce, thresholds and egress times. For details, one may refer to [11], [15], [10], [17], [14], [12], [16], [7], [9], [3] and [6]. In [4], mean square deviation for time to hiring is determined for the workforce structure with egresses in clumps when the process of total number of egresses is a compound Poisson process with independent and identically distributed decapitated geometric random variables as summands and the threshold is a positive integer valued random variable. The problem of time to hiring for the workforce structure is also analyzed by several authors utilizing another strategy of hiring referred as MAX strategy of hiring in the literature. In this context, one may refer to [11], [15], [17], [13], [8], [3] and [6]. Recently, in [5] the authors have analyzed the problem in [4] utilizing the MAX strategy of hiring. Present paper is an extension of [4] and [5] when the egress in the workforce structure occurs in clumps and deprivation of workforce is continuous, by composing two exemplars. In exemplar-1, the partial sum of independent exponential deprivation of workforce is randomly indexed by a homogeneous Poisson process; threshold is an exponential random variable. Mean square deviation of time to hiring is determined utilizing CUM strategy of hiring for this exemplar. In exemplar-2, the partial sum and CUM strategy of hiring of exemplar-1 are replaced by maximum and MAX strategy of hiring. A remark on the suitability of the exemplars is given with justification.

2. Description of Exemplar-1

Consider a workforce structure consisting of a single grade in which egress of employees occurs in clumps at random epochs in $(0, \infty)$. Let $\{B(x),\ x \geq 0\}$ be a Poisson process with degree $'b'$, $b > 0$, where $B(x)$ is the count for clumps in $(0, x]$. Let U_i be the deprivation of workforce due to egresses in the i^{th} clump, $i = 1, 2, 3, \ldots$. It is presumed that U_i's are independent and $U_i \sim \exp(\theta_1)$, $\theta_1 > 0$, $i = 1, 2, 3, \ldots$. Let $f_m(\cdot)$ be probability density of $\sum_{i=1}^{m} U_i$ with Laplace transform $\overline{f_m}(\cdot)$. Let $C(x)$ be the total deprivation of workforce due to $B(x)$ clumps of egresses. Let Y be the threshold level for the cumulative deprivation of workforce due to clumps of egresses. It is presumed that $Y \sim \exp(\theta_2)$, $\theta_2 > 0$. Y, $B(x)$ and U_i, $x \geq 0$,

$i \geq 1$ are statistically independent. The CUM strategy of hiring states that hiring is done when the total deprivation of workforce due to clumps of egresses exceeds the threshold. Let R_{CUM} be the time to hiring with expectation $M(R_{CUM})$, mean square deviation $V(R_{CUM})$, and average aggregate deprivation of work force due to clump of egresses in the interval of time to hiring $M(C(R_{CUM}))$.

3. Main Result for Examplar-1

By CUM strategy of hiring

$$P(R_{CUM} > x) = P(C(x) \leq Y)$$

Since, $C(x) = \sum_{i=1}^{B(x)} U_i$, by conditioning upon $B(x)$ and using the independence of $B(x)$ over U_i and Y for all $x \geq 0$, $i = 1, 2, 3, \ldots$ we get

$$P(R_{CUM} > x) = \sum_{m=0}^{\infty} P\left(\sum_{i=1}^{m} U_i \leq Y\right) P(B(x) = m) \tag{1}$$

Note that $\sum_{i=1}^{0} U_i \leq Y$ is an empty set. Therefore without loss of generality, m can vary from 0 to ∞.

Since U_i's are independent of Y, by conditioning upon $\sum_{i=1}^{0} U_i$ and using the assumption that Y is an exponential random variable with parameter $\theta_2 (\theta_2 > 0)$, $P(\sum_{i=1}^{m} U_i \leq Y)$ in (1) can be written as

$$P\left(\sum_{i=1}^{m} U_i \leq Y\right) = \overline{f_m}(\theta_2) \tag{2}$$

By hypothesis, $f_m(\cdot)$ is a gamma density function with scale parameter θ_1 and shape parameter m. Applying convolution theorem for Laplace transform we get

$$\overline{f_m}(\theta_2) = \left(\frac{\theta_1}{\theta_1 + \theta_2}\right)^m \tag{3}$$

From (2) and (3) we get

$$P\left(\sum_{i=1}^{m} U_i \leq Y\right) = \left(\frac{\theta_1}{\theta_1 + \theta_2}\right)^m \tag{4}$$

From (1) and (4) we get

$$P(R_{CUM} > x) = \sum_{m=0}^{\infty}\left(\frac{\theta_1}{\theta_1 + \theta_2}\right)^m P(B(x) = m)$$

which is the probability generating function of the Poisson process $B(x)$ evaluated at $\left(\frac{\theta_1}{\theta_1 + \theta_2}\right)$.

Therefore

$$P(R_{CUM} > x) = e^{-\left(\frac{b\theta_2}{\theta_1 + \theta_2}\right)x} \tag{5}$$

From (5) we find that the distribution of R_{CUM} is exponential with parameter $\frac{b\theta_2}{\theta_1 + \theta_2}$.

Therefore

$$M(R_{CUM}) = \left(\frac{\theta_1 + \theta_2}{b\theta_2}\right) \tag{6}$$

and

$$V(R_{CUM}) = \left(\frac{\theta_1 + \theta_2}{b\theta_2}\right)^2 \tag{7}$$

(6) and (7) give the expectation and mean square deviation of time to hiring.

Result for $M(C(R_{CUM}))$ is derived below:

Since $C(R_{CUM}) = \sum_{i=1}^{B(R_{CUM})} U_i$, by Wald's lemma, the average aggregate deprivation of workforce up to time to hiring is given by

$$M(C(R_{CUM})) = M(U_i)M(B(R_{CUM})), 1, 2, 3, \ldots, \tag{8}$$

Since R_{CUM} is independent of $B(x)$ and $M[B(x)] = bx$, by conditioning upon R_{CUM} it can be shown that

$$M(B(R_{CUM})) = bM(R_{CUM}) \tag{9}$$

From (6), (8) and (9) we get

$$M(C(R_{CUM})) = \left(\frac{\theta_1 + \theta_2}{\theta_1\theta_2}\right) \tag{10}$$

(10) gives the average aggregate deprivation of workforce up to hiring time.

4. Description of Exemplar-2

In this section, we study the work in exemplar-1 utilizing the MAX strategy of hiring. The MAX strategy of hiring states that hiring is done when the maximum deprivation of workforce due to clumps of egresses exceeds the threshold.

5. Main Result of Exemplar-2

By MAX strategy of hiring

$$P(R_{MAX} > x) = P(\max_{1 \le i \le B(x)} U_i \le Y)$$

By conditioning upon $B(x)$ and using the independence of $B(x)$ over U_i and Y for all $x \ge 0$, $i = 1, 2, 3, \ldots$ we get

$$P(R_{MAX} > x) = \sum_{m=0}^{\infty} P(\max_{1\le i\le m} U_i \le Y)P(B(x) = m) \tag{11}$$

By conditioning upon Y and using the independence of U_i over Y, $i = 1, 2, 3, \ldots$ we get

$$P(\max_{1\le i\le m} U_i \le Y) = \int_0^{\infty} P(\max_{1\le i\le m} U_i \le y)\, dP(Y \le y)$$

i.e. $P(\max_{1\le i\le m} U_i \le Y) = \theta_2 \int_0^\infty [F(y)]^m e^{-\theta_2 y} dy$ (12)

where $F(y) = P[U_i \le y] = 1 - e^{-\theta_2 y}$, 1, 2, 3, …, by hypothesis.

Since $\sum_{m=0}^{\infty} P(B(x) = m)s^m = e^{-bx[1-s]}$, by hypothesis, from (11) and (12) we get

$$P(R_{MAX} > x) = \theta_2 \int_0^\infty e^{-\theta_2 y} e^{-bx(e^{-\theta_1 y})} dy. \tag{13}$$

We now determine $M(R_{MAX})$ and $V(R_{MAX})$ from (13).

We know that

$$M(R_{MAX}^n) = n\int_0^\infty x^{n-1} P(R_{MAX} > x)dx,\ n = 1, 2, 3, \ldots \tag{14}$$

From (13) and (14) it can be shown that

$$M(R_{MAX}) = \frac{\theta_2}{b(\theta_2 - \theta_1)},\ \text{if } \theta_2 > \theta_1. \tag{15}$$

and

$$M(R_{MAX}^2) = \frac{2\theta_2}{b^2(\theta_2 - 2\theta_1)},\ \text{if } \theta_2 > 2\theta_1. \tag{16}$$

Since $(2\theta_1^2 + \theta_2^2 - 2\theta_1\theta_2) = [\theta_1^2 + (\theta_2 - \theta_1)^2] > 0$ for all values θ_1 and θ_2, from (15) and (16) it can be shown that

$$V(R_{MAX}) = \frac{\theta_2(2\theta_1^2 + \theta_2^2 - 2\theta_1\theta_2)}{b^2(\theta_2 + 2\theta_1)(\theta_2 - \theta_1)^2},\ \text{if } \theta_2 > 2\theta_1. \tag{17}$$

(15) and (17) give the expectation and mean square deviation of time to hiring for this Exemplar.

6. Quantitative Illustration and Findings for Exemplars-1 and 2

b, θ_1 and θ_2 are the parameters for expectation and mean square deviation. It is palpable that (i) $M(R_{CUM})$ increases with $'\theta_1'$ keeping $'\theta_2'$ and $'b'$ fixed for exemplar-1 (ii) $M(R_{CUM})$ decreases when $'\theta_2'$ increases, keeping $'\theta_1'$ and $'b'$ fixed for exemplar-1 and (iii) Expectation and mean square deviation decrease when 'b' alone increases for both the exemplars.

For exemplar-1, table-1 given below indicates the impact on $M(R_{CUM})$ and $V(R_{CUM})$ when $b = 0.1$ and θ_1 and θ_2 vary together.

Table 1.

θ_1	θ_2	$M(R_{CUM})$	$V(R_{CUM})$
5	16	13.125	172.266
6	17	13.529	183.045
7	18	13.889	192.901
8	19	14.211	201.939

For exemplar-2, table-2 given below indicates the effect of θ_1 and θ_2 on $M(R_{CUM})$ and $V(R_{CUM})$ when $b = 0.1$.

Table 2.

θ_1	θ_2	$M(R_{MAX})$	$V(R_{MAX})$
4	15	13.636	242.621
5	15	15.000	375.000
4	16	13.333	222.222
4	17	13.077	206.772
5	16	14.545	321.763
6	17	15.455	441.157
7	18	16.364	632.231

Findings

(1) From Table-1, it is observed that $M(R_{CUM})$ and $V(R_{CUM})$ both decrease when θ_1 and θ_2 decrease simultaneously.

(2) From Table-2, it is observed that $M(R_{MAX})$ and $V(R_{MAX})$ both increase when (i) θ_1 increases and θ_2 fixed and (ii) θ_1 and θ_2 increase simultaneously.

(3) From Table-2, it is observed that $M(R_{MAX})$ and $V(R_{MAX})$ both decrease when θ_2 increases and θ_1 fixed.

Remark. Since U_i is positive for at least one i, $\sum_{i=1}^{m} U_i$ is always greater than $\max_{1\leq i\leq m} U_i$, the time to hiring is advanced in exemplar-1, but it is delayed in exemplar-2. This justifies the following observation on the choice of the suitability of these exemplars.

(i) Exemplar-1 is more suitable than exemplar-2 if the plans in the work force structure is to prepone the time to hiring.

(ii) Exemplar-2 is suitable than exemplar-1 if the plans in the work force structure is to postpone the time to hiring.

7. Conclusion

The present work contributes to the existing literature in the sense that the exemplars-1 and 2 discussed when egress of employees occurs in clumps in the workforce structure and threshold is an exponential random variable, utilizing CUM and MAX strategies of hiring. Consistency of results on averages with logical inference is an important finding.

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