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CHARACTERIZATIONS FOR A SINGLETON SET OF ORDERED PAIR OF NON-NEGATIVE REAL NUMBERS TO BE AN EDGE DEGREE SET OF SOME INTUITIONISTIC FUZZY GRAPHS

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Abstract

In this paper, some properties of edge degree set are studied. A necessary and sufficient condition for a singleton set of ordered pair of real numbers to be an edge degree set of an intuitionistic fuzzy graph is obtained. Also a necessary and sufficient condition for a singleton set of ordered pair of real numbers to be an edge degree set of an intuitionistic fuzzy graph on path and cycle. They provide conditions for realization of a singleton set of ordered pair of non-negative real numbers as an edge degree set of intuitionistic fuzzy graphs on path, cycle.

1. Introduction

Rosenfeld introduced the concept of fuzzy graphs in 1975 [11]. Bhattacharya [3] gave some remarks on fuzzy graphs. K. R. Bhutani introduced the concepts of weak isomorphism, co-weak isomorphism and isomorphism between fuzzy graphs [4]. K. Radha and A. Rosemine, introduced degree sequence of fuzzy graph [8]. K. T. Atanassov [2] introduced

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the concept of intuitionisitic fuzzy sets as a generalization of fuzzy sets. Intuitionistic Fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry, economics and washing machine. R. Parvathi and M. G. Karunambigai discussed some concepts in intuitionistic fuzzy graphs [5]. R. Parvathi and M. G. Karunambigai and R. Buvaneswari introduced constant intuitionistic fuzzy graphs [4]. In this paper, a necessary and sufficient condition for a singleton set of ordered pair of real numbers to be an edge degree set and hence conditions for realization K. Radha and P. Pandian of a singleton set of ordered pair of non-negative real numbers as an edge degree set of an intuitionistic fuzzy graph on path, cycle are obtained.

Definition 1.1 [5]. $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ is an intuitionistic fuzzy graph (IFG) on $G^* = (V, E)$ where

(i) $V = \{v_1, v_2, v_3, \ldots, v_n\}$, $\mu_1 : V \to [0, 1]$ and $\gamma_1 : V \to [0, 1]$ denote the degree of membership value and the degree of non-membership value of the elements $v_i \in V$ respectively such that $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$, for every $v_i \in V$, $i = 1, 2, 3, \ldots, n$.

(ii) $E \subseteq V \times V$, $\mu_2 : V \times V \to [0, 1]$ and $\gamma_2 : V \times V \to [0, 1]$ are such that $\mu_2(v_i, v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j)$ and $\gamma_2(v_i, v_j) \leq \gamma_1(v_i) \vee \gamma_1(v_j)$ and $0 \leq \mu_2(v_i, v_j) \leq \gamma_2(v_i, v_j) \leq 1$ for every $v_iv_j \in E$, $i, j = 1, 2, 3, \ldots, n$.

Definition 1.2 [6]. Let $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ be an IFG on $G^* = (V, E)$. Then degree of a vertex $v_i \in G$ is defined by $d_G(v_i) = (d_G^{\mu_1}(v_i), d_G^{\gamma_1}(v_i))$ or by $(d_G^{\mu_1}(v_i), d_G^{\gamma_1}(v_i))$ where $d^{\mu_1}(v_i) = \Sigma\mu_2(v_i, v_j)$ and $d^{\mu_1}(v_i) = \Sigma\gamma_2(v_i, v_j)$, where the summation runs over all $v_iv_j \in E$.

The degree of an edge $uv \in E$ is defined by $d(uv) = (d^{\mu_1}(uv), d^{\gamma_2}(uv))$ where $d^{\mu_2}(v_i) = d^{\mu_1}(u) + d^{\mu_1}(v) - 2\mu_2(uv)$ and $d^{\gamma_2}(uv) = d^{\gamma_2}(u) + d^{\gamma_1}(v) - 2\mu_2(uv)$. For the sake of simplicity, we use the notations $d^{\mu}(u)$ and $d^{\mu}(e)$ for the membership degree of vertex and edge respectively and $d^{\gamma}(u)$ and

$d^{\gamma}(e)$ for the nonmembership degree of vertex and edge respectively [6]. If each edge of an intuitionistic fuzzy graph G has the same degree (k_1, k_2), then G is said to be an edge regular intuitionistic fuzzy graph.

Definition 1.4 [7]**.** A path P_n in IFG is a sequence of distinct vertices $v_1, v_2, \ldots, v_n$ such that for each edge v_iv_j, at least one of $\mu_2(v_iv_j), \gamma_2(v_iv_j)$ is non-negative. The length of a path $P = v_1v_2, \ldots, v_{n+1}(n > 0)$ is n. A path $P = v_1v_2, \ldots, v_{n+1}(n > 0)$ is called a cycle if $v_1 = v_{n+1}$ and $n \geq 3$.

Definition 1.5 [7]**.** An intuitionistic fuzzy graph G on $G^* : (V, E)$ is (k_1, k_2)-regular if $d_G(v_i) = (k_1, k_2)$ for all $v_i \in V$. Also G is said to be a regular intuitionistic fuzzy graph of degree (k_1, k_2).

Definition 1.6 [10]**.** We use the following order relation to compare two ordered pairs of real numbers:

(i) $(u, v) = (x, y)$ if and only if $u = x$ and $v = y$.

(ii) $(u, v) > (x, y)$ if and only if either $u > x$ or $u = x$ and $v > y$.

Definition 1.7 [10]**.** Let $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ be an IFG on $G^* = (V, E)$. A sequence of ordered pairs of positive real numbers $((d_{11}, d_{12}), (d_{21}, d_{22}), \ldots, (d_{n1}, d_{n2}))$ with $((d_{11}, d_{12}) \geq (d_{21}, d_{22}) \geq \ldots \geq (d_{n1}, d_{n2}))$, where $d_{i1} = d^{\mu}(v_i)$ and $d_{2i} = d^{\gamma}(v_i)$, is the degree sequence of G.

Definition 1.8 [11]**.** Let $G : (\mu_1, \gamma_1; \mu_2, \gamma_2)$ be an IFG on $G^* : (V, E)$. A sequence of ordered pairs of positive real numbers $(d_1, d_2, d_3, \ldots, d_n)$ with $d_1 \geq d_2 \geq d_3 \geq \ldots \geq d_n$, where $d_i = d(e_i) = (d^{\mu}(e_i), d^{\gamma}(e_i))$ is the edge degree sequence of the IFG G.

2. Realization of a Set of Ordered Pairs as Edge Degree Set of IFGs

Definition 2.1. The set of distinct ordered pair of positive real numbers occurring in an edge degree sequence of an intuitionistic fuzzy graph is called its edge degree set.

Definition 2.2. The set of distinct ordered pair of positive real numbers is called an edge degree set if it is the edge degree set of some IFG. The IFG is said to realize the edge degree set.

Theorem 2.3. *The edge degree set of an intuitionistic fuzzy graph is a singleton set* $\{(d_1, d_2)\}$ *of ordered pair of non-negative real numbers if and only if it is* (d_1, d_2)*-edge regular.*

Proof of Theorem 2.3. The edge degree set is a singleton set $\{(d_1, d_2)\}$ if and only if edge degree of each edge is (d_1, d_2) which happens if and only if the intuitionistic fuzzy graph is (d_1, d_2)-edge regular.

Theorem 2.4. *Any singleton set of ordered pair of non-negative real numbers is the edge degree set of an intuitionistic fuzzy graph.*

Proof of Theorem 2.4. Let (d_1, d_2) be any ordered pair of non-negative real numbers.

Let n be any positive integer such that $0 < d_1/2n \leq 1,\ 0 < d_2/2n \leq 1$.

Now consider a complete graph K_{n+2}. Assign $\mu_2(e) = d_1/2n$ and $\gamma_2(e) = d_2/2n$ as the membership value and the non-membership value respectively for every edge e in K_{n+2}. Then $d^{\mu}(uv) = d^{\mu}(u) + d^{\mu}(v) - 2\mu_2(uv)$ $= (d_1/2n)(n+1) + (d_2/2n)(n+1) - 2(d_1/2n) = d_1$. Similarly $d^{\gamma}(uv) = d_2$. Therefore $d(e) = (d^{\mu}(uv), d^{\gamma}(uv)) = (d_1, d_2)$. This is true for every edge e. Take $(\mu_1(v), \gamma_1(v)) = (1, 1)$ for every vertex v in K_{n+2}. Hence K_{n+2} realizes $\{(d_1, d_2)\}$ as its edge degree set.

Theorem 2.5. *Any finite set* $\{(d_1, d_2'), (d_1, d_2'), \ldots, (d_k, d_k')\}$ *of ordered pairs of positive real numbers is an edge degree set of some intuitionistic fuzzy graph.*

Proof of Theorem 2.5. By previous theorem, there exists an intuitionistic fuzzy graph G_i realizing $\{(d_i, d_i')\}$ as the edge degree set for all $i = 1, 2, 3, \ldots, k$ Then the intuitionistic fuzzy graph $G = G_1 \cup G_2 \cup \ldots \cup G_k$ realizes $\{(d_1, d_2'), (d_1, d_2'), \ldots, (d_k, d_k')\}$ as the edge degree set. The intuitionistic fuzzy graph obtained in this theorem is disconnected.

3. Characterization for a Singleton Set to be Edge Degree Set of an IFG on Cycle

Theorem 3.1. *Let* d_1 *and* d_2 *be any two positive real numbers. Then* $\{(d_1, d_2)\}$ *is an edge degree set of an intuitionistic fuzzy graph on a cycle (or an intuitionistic fuzzy graph on a cycle realizes* $\{(d_1, d_2)\}$ *as an edge degree set) if and only if* $0, d_1 \leq 2$ *and* $0, d_2 \leq 2$.

Proof of Theorem 3.1. Let C_n be a cycle $v_1v_2v_3 \ldots v_nv_1$ on n vertices where $n \geq 3$ is any positive integer.

If $\{(d_1, d_2)\}$ is the degree set of an intuitionistic fuzzy graph $(\mu_1, \gamma_1; \mu_2, \gamma_2)$ on C_n, then $0, d_1 \leq 2$ and $0, d_2 \leq 2$. Conversely assume that $0 < d_1 \leq 2$ and $0 < d_2 \leq 2$. Then $0 < d_i/2 \leq 1, i = 1, 2$. Assign $(\mu_2(v_iv_{i+1}), \gamma_2(v_iv_{i+1})) = (d_1/2, d_2/2), \forall\ i = 1, 2, \ldots, n$. Assign any value satisfying the condition of intuitionistic fuzzy graph as $\mu_1(v_i)$ and $\gamma_1(v_i)$ for all i. Then $d^{\mu}(v_iv_{i+1}) = d_1$ and $d^{\gamma}(v_iv_{i+1}) = d_2$. Hence $\{(d_1, d_2)\}$ is the edge degree set of the intuitionistic fuzzy graph $(\mu_1, \gamma_1; \mu_2, \gamma_2)$ on C_n.

Corollary 3.2. *Let* d_1 *and* d_2 *be any two positive real numbers. Then an intuitionistic fuzzy graph on a cycle is* (d_1, d_2)*-edge regular if and only if* $0 < d_1 \leq 2$ *and* $0 < d_2 \leq 2$.

Theorem 3.3. *Let* d_1 *and* d_2 *be any two non-negative real numbers. Then* $\{(d_1, d_2)\}$ *is the edge degree set of an intuitionistic fuzzy graph* $(\mu_1, \gamma_1; \mu_2, \gamma_2)$ *on an odd cycle (or an intuitionistic fuzzy graph on an odd cycle realizes* $\{(d_1, d_2)\}$ *as an edge degree set) if and only if* μ_2 *and* γ_2 *are constant functions of constant values* $d_1/2$ *and* $d_2/2$ *respectively.*

Proof of Theorem 3.3. Let C_n be a cycle $v_1e_1v_2e_2v_3, \ldots, v_{n-1}e_{n-1}v_2\ e_nv_2$ on n vertices, where n is an odd number.

Suppose that $\{(d_1, d_2)\}$ is the edge degree set of an intuitionistic fuzzy graph $(\mu_1, \gamma_1; \mu_2, \gamma_2)$ on C_n. Then the degree of each edge is (d_1, d_2). Let

$\mu_2(e_1) = c$. Then $(e_2) = \mu_2(e_1) + \mu_2(e_3) = d_1$ gives $\mu_2(e_3) = d_1 - c$, $d^{\mu}(e_4) = \mu_2(e_3) + \mu_2(e_5) = d_1$ gives $\mu_2(e_5) = c$, Proceeding like this, since n is an odd number, $d^{\mu}(e_n) = \begin{cases} c, & \text{if } n \equiv 1 (\text{mod } 4) \\ d_1 - c, & \text{if } n \equiv -1 (\text{mod } 4) \end{cases}$

Case 1. $n \equiv 1 (\text{mod } 4)$

$d^{\mu}(e_1) = \mu_2(e_n) + \mu_2(e_2) = d_1$ gives $\mu_2(e_2) = d_{1-c}$

$d^{\mu}(e_3) = \mu_2(e_2) + \mu_2(e_4) = d_1$ gives $\mu_2(e_4) = c$

Proceeding like this, since $n - 1 \equiv 0 (\text{mod } 4)$, $\mu_2(e_{n-1}) = c$.

Case 2. $n \equiv -1 (\text{mod } 4)$

$d^{\mu}(e_1) = \mu_2(e_n) + \mu_2(e_2) = d_1$ gives $\mu_2(e_2) = c$

$d^{\mu}(e_3) = \mu_2(e_2) + \mu_2(e_4) = d_1$ gives $\mu_2(e_4) = d_1 - c$

Proceeding like this, since $n - 1 \equiv 2 (\text{mod } 4)$, $\mu_2(e_{n-1}) = c$. Hence in both cases, $\mu_2(e_{n-1}) = c$. Also $\mu_2(e_1) = c$. Therefore $d^{\mu}(e_n) = d_1$ gives $c = d_1/2$ Hence $\mu_2(e_i) = d_1/2$ for every i. Similarly $\gamma_2(e_i) = d_2/2$ for every i.

Conversely if μ_2 and γ_2 are constant functions of constant values $d_1/2$ and $d_2/2$ respectively, then the degree of each edge is (d_1, d_2) and hence the theorem.

Corollary 3.4. *Let* d_1 *and* d_2 *be any two non-negative real numbers. Then an intuitionistic fuzzy graph* $(\mu_1, \gamma_1; \mu_2, \gamma_2)$ *on an odd cycle is* (d_1, d_2)*-edge regular if and only if* μ_2 *and* γ_2 *are constant functions of constant values* $d_1/2$ *and* $d_2/2$ *respectively.*

4. Characterization for a Singleton Set to be Edge Degree Set of an IFG on a Path

Theorem 4.1. *Let* d_1 *and* d_2 *be any two non-negative real numbers.*

Then $\{(d_1, d_2)\}$ *is an edge degree set of an intuitionistic fuzzy graph on a path* P_n *(or an intuitionistic fuzzy graph on a path realizes* $\{(d_1, d_2)\}$ *as an edge degree set) if and only if* $0 \leq d_1 \leq 1$, $0 \leq d_2 \leq 1$ *and* $2 \leq n \leq 4$.

Proof of Theorem 4.1. Let P_n be a path $v_1 v_2 v_3, \ldots, v_n$ on n vertices. Suppose that $\{(d_1, d_2)\}$ is an edge degree set of an intuitionistic fuzzy graph $(\mu_1, \gamma_1; \mu_2, \gamma_2)$ on the path P_n. Then $n \geq 2$ and each edge of P_n has same edge degree (d_1, d_2) in $(\mu_1, \gamma_1; \mu_2, \gamma_2)$. Therefore $d^{\mu}(v_i v_{i+1}) = d_1$ and $d^{\gamma}(v_i v_{i+1}) = d_2$, $1 \leq i \leq n-1$.

If $n \geq 2$, then $d(v_1 v_2) = (0, 0)$. Therefore $d_1 = 0$ and $d_2 = 0$.

Let $n \geq 3$.

Then $d_1 = d^{\mu}(v_1 v_2) = \mu_2(v_2 v_3) \Rightarrow 0 \leq d_1 \leq 1$

$d_2 = d^{\gamma}(v_1 v_2) = \gamma_2(v_2 v_3) \Rightarrow 0 \leq d_2 \leq 1$

Suppose $n \geq 5$. Then $d_1 = d^{\mu}(v_3 v_4) \Rightarrow \mu_2(v_4 v_5) = 0$

and $d_2 = d^{\gamma}(v_3 v_4) = \gamma_2(v_4 v_5) + \gamma_2(v_4 v_5) \Rightarrow \gamma_2(v_4 v_5) = 0$

This is not possible. Hence $n \geq 4$. Conversely, assume that $0 \leq d_1 \leq 1$, $0 \leq d_2 \leq 1$ and $2 \leq n \leq 4$.

Case 1. $n = 2$.

The edge degree of the edge in any intuitionistic fuzzy graph on P_2 is $(0, 0)$.

Case 2. $n = 2$.

Consider a path $P_3 : v_1 v_2 v_3$ on three vertices. Assign $\mu_2(v_1 v_2) = \mu_2(v_2 v_3) = d_1$ and $\gamma_2(v_1 v_2) = \gamma_2(v_2 v_3) = d_2$. Also assign any value satisfying the condition of intuitionistic fuzzy graph as the membership and non-membership values of the vertices. Then $\{(d_1, d_2)\}$ is the edge degree set of the intuitionistic fuzzy graph $(\mu_1, \gamma_1; \mu_2, \gamma_2)$ on P_3.

Case 3. $n = 4$.

Consider a path $P_4 : v_1v_2v_3v_4$ on four vertices.

Assign $\mu_2(v_1v_2) = \mu_2(v_2v_4) = d_1/2$, $\mu_2(v_2v_3) = d_1$ and $\gamma_2(v_1v_2) = \gamma_2(v_3v_4) = d_2/2$, $\gamma_2(v_2v_3) = d_2$. Also assign any value satisfying the condition of intuitionistic fuzzy graph as the membership and non-membership values of the vertices. Then $\{(d_1, d_2)\}$ is the edge degree set of the intuitionistic fuzzy graph $(\mu_1, \gamma_1; \mu_2, \gamma_2)$. on P_4.

Corollary 4.2. *Let* d_1 *and* d_2 *be any two non-negative real numbers. Then an intuitionistic fuzzy graph on a path* P_n *is* (d_1, d_2)*-edge regular if and only if* $0 \leq d_1 \leq 1$, $0 \leq d_2 \leq 1$ *and* $2 \leq n \leq 4$.

5. Conclusion

In this paper, necessary and sufficient conditions for a singleton set of ordered pair of positive real numbers to be an edge degree set in some intuitionistic fuzzy graphs are derived. They will be helpful in obtaining many more properties of intuitionistic fuzzy graphs.

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