Advances and Applications in Mathematical Sciences
Volume 20, Issue 7, May 2021, Pages 1329-1337

MATHEMATICAL MODEL FOR NEWTONIAN AND NON-NEWTONIAN FLOW THROUGH TAPERED ARTERIES IN PRESENCE OF POROUS EFFECTS

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Abstract

The objective of this paper is to study the effect of porous medium on a Newtonian and non-Newtonian flow through tapered tubes. In this paper, we have taken a steady and laminar flow of Newtonian and non-Newtonian fluids, Here Navier-Stokes's equations are used in the formulation of the model. The Newtonian and non-Newtonian fluids were examined in the effects of porous medium. The experimental data were compared with theoretical predictions. Aforesaid flow quantities are significantly higher for flow in tubes with variable permeability than for flow in tubes with constant permeability. The present work is validated from the previously published literature.

1. Introduction

The mathematical analysis of time independent flow of Newtonian and non-Newtonian fluid models has become a topic of increasing interest among the researchers, since it has wide applications in many branches of engineering and medical sciences such as polymer processing industry, environmental science, magneto hydrodynamics, and bio-fluid dynamics. Blood flow is now well known to the physiologists as one of the major

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2010 Mathematics Subject Classification: 68T05, 68T07.

Keywords: mathematical model, Newtonian and non-Newtonian flows, Navier-Stokes's equations, porous medium, permeability parameter.

Received September 7, 2020; Accepted February 18, 2021

mechanisms due to its applications in arterial mechanics. In particular, blood flow mechanism in arteries is an important field of research because arterial diseases are a major cause of death in most of western countries [Song et al. [2]]. The study of flow through tapered conduits is important not only for an understanding of the blood flow behavior in arteries but also for the design of prosthetic blood vessels. Localized narrowing in a blood vessel is commonly referred to as stenosis. This vascular disease frequently occurs, in mammalian arteries. The intimal thickening of stenotic artery was understood as an early process in the beginning of atherosclerosis. In this study, pressure drop, pressure gradient and flux were measured in rigid walled model of tapered grafts under steady flow conditions. Newtonian and non- Newtonian fluids were examined and the experimental data were compared with theoretical prediction. Pressure drop flow rate data were obtained both Newtonian and non-Newtonian flows. Walawender et al. [3] worked on experimental studies on the blood flow through tapered tube. Oka [1] gave an idea of pressure development in a non-Newtonian flow through tapered tube, while Walawender and Chen [5] discussed about blood flow in tapered tubes. Wang et al. [6] worked on numerical study of pulsating flow through a tapered artery with stenosis. Mandal [7] discussed about an unsteady analysis of non-Newtonian blood flow through tapered arteries with a stenos is. Kumar et al. [8] worked on the numerical study axi symmetric blood flow in a constricted rigid tube, while Shankar and Hemlatha [9] investigated a nonlinear mathematical model for blood flow through tapered tubes. Kumar et al. [11] investigated computational technique for flow in blood vessels with porous effects. Kumar et al. [12] considered a finite element Galerkin's approach for a computational study of arterial flow. Saket and Kumar [13] investigated reliability of convective diffusion process in porous blood vessels. Jaiswal and Yadav [14] developed two-phase model of blood flow through a porous layered artery in the presence of a uniform magnetic field. Gupta [16] investigated performance model and analysis of blood flow in small vessels with magnetic effects. Liu and Liu [15] made a non-Newtonian fluid model is used to investigate the 2D pulsatile blood flow through a tapered artery with stenosis. In this paper we consider the previous work of Kumar et al. [11] with porous medium. The aim of present investigation is to study the effect of porous medium on a mathematical model for Newtonian and non-Newtonian flows through tapered tube.

2. Mathematical Model

The physical situation of the geometry of the tapered artery may be shown as given in figure (1) below:

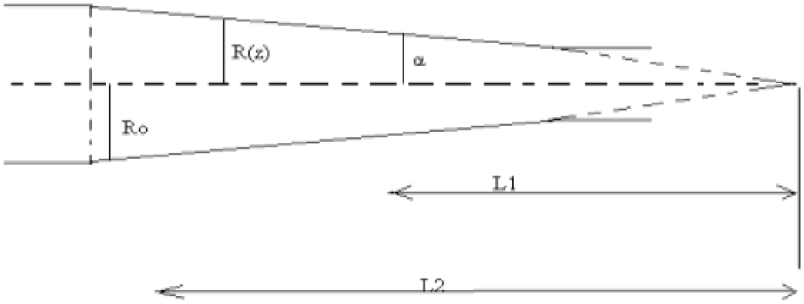

Figure 1. Physical situation of geometry of tapered artery.

Here, we first assume that the fluid under consideration is incompressible and Newtonian. Let the flow is fully developed laminar axially symmetric flow. We will employ circular cylindrical co-ordinates (r, θ, z) to state the problem mathematically and choose the z axis along the axis of the tube. The flow of fluid is governed by the Navier-stokes equations, which are given as follows:

$$\frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + V_z \frac{\partial V_r}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial r} + v\left(\frac{\partial^2 V_r}{\partial r^2} + \frac{\partial^2 V_r}{\partial z^2} + \frac{1}{r} \frac{\partial V_r}{\partial r} - \frac{V_r}{r^2}\right) - \frac{\mu}{k} V_r \quad (1)$$

$$\frac{\partial V_z}{\partial t} + V_z \frac{\partial V_z}{\partial r} + V_z \frac{\partial V_z}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial z} + v\left(\frac{\partial^2 V_z}{\partial r^2} + \frac{1}{r} \frac{\partial V_z}{\partial r} + \frac{\partial^2 V_z}{\partial z^2}\right) - \frac{\mu}{k} V_z \quad (2)$$

The equation of continuity is given as follows:

$$\frac{1}{r} \frac{\partial}{\partial r}(r \cdot V_r) + \frac{\partial V_z}{\partial z} = 0. \quad (3)$$

Oka and Murata [1] presented general theory of non-Newtonian steady flow through tapered tubes but their solution was restricted to small angles of taper. They observed an expression for flow rate through any cross section.

The standard general formula of volumetric flow rate through a slightly tapering tube for any time independent fluid characterized by $F(t)$ is given by:

$$Q = \frac{\pi R^3(z)}{\tau_w^3(z)} \int_0^{\tau_\omega} \tau^2 F(t) dt. \tag{4}$$

Where τ_w is the wall shear stress which is function of z and τ is the shear stress and R is the radius of tapered tube at a point z.

The expressions for τ_w and τ are given by

$$\tau_w = -\frac{1}{2} R(z)\left(\frac{dp}{dz}\right) \text{ and } \tau = \mu\left(\frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r}\right) \tag{5}$$

For Newtonian fluid, we have

$$F(t) = \frac{\tau}{\sigma} = e = -\frac{\partial V}{\partial r} \tag{6}$$

Putting above value in relation (4) and then integrating we obtain volumetric flow Q as:

$$Q = \frac{\pi R^4(z)}{8\mu}\left(\frac{dp}{dz}\right) \tag{7}$$

The pressure drop ΔP is given as follows:

$$\Delta P = \frac{8\mu Q}{\pi} \int_0^z R^{-4}(z) dz \tag{8}$$

According to figure (1) we see that $R(z) = R_0 - z \tan \alpha$ and for small values of α, we obtain $R(z) = R_0 - z\alpha$, where $R_0 = \alpha L_1$.

In this case, we have

$$\Delta P = \frac{8\mu Q}{3\pi\alpha^4} (L_2^{-3} - L_1^{-3}). \tag{9}$$

Now for non-Newtonian fluid which obeys the power low equation, we obtain,

$$F(t) = \frac{du}{dr} = -k\tau^3 \tag{10}$$

Putting all the above values in equation (4), we have:

$$Q = \frac{\pi K}{n+3} R^{n+3}\left(\frac{dp}{dz}\right)^{n}. \tag{11}$$

In this case we also obtain:

$$\Delta P = \frac{2n}{3\alpha^{(n+3/n)}}\left[\frac{Q(n+3)}{\pi K}\right]\left[L_2^{-3/n} - L_1^{-3/n}\right]. \tag{12}$$

3. Results and Discussion

The results have been numerically workout for various combinations of the parameters involved in the solutions. In this model tapered tube cast in square section moulds and taper angles (α) are taken as 0.5, 0.74, 1.0 and 1.25 and these values are called as nominal values of α.

Table 1. Showing the dimension of Tapered Tubes.

NORMAL α	ACTUAL α	R_e	$L_1(cm)$	$L_2(cm)$
0.50	0.49	4.62	53.08	37.12
0.74	0.72	4.52	36.15	20.19
1.0	0.93	4.35	26.01	12.17
1.25	1.23	4.27	20.51	10.51

It is observed from figure (2) in the porous medium effect of pressure drop, flux and pressure gradient with rigid tapered grafts (for Newtonian and non- Newtonian fluid). The results show that if we take Newtonian fluid, then flux, pressure drop and pressure gradient will increase. We also observed from figure (3) that in presence of porous medium, these increments are due to small if we take non-Newtonian fluid tapered grafts then the taper tubes do not change the flow patterns but only changes the value. In figure (4) we have plotted graph between velocity distribution and radial distance for variable permeability parameter and shown the pattern of the velocity distribution.

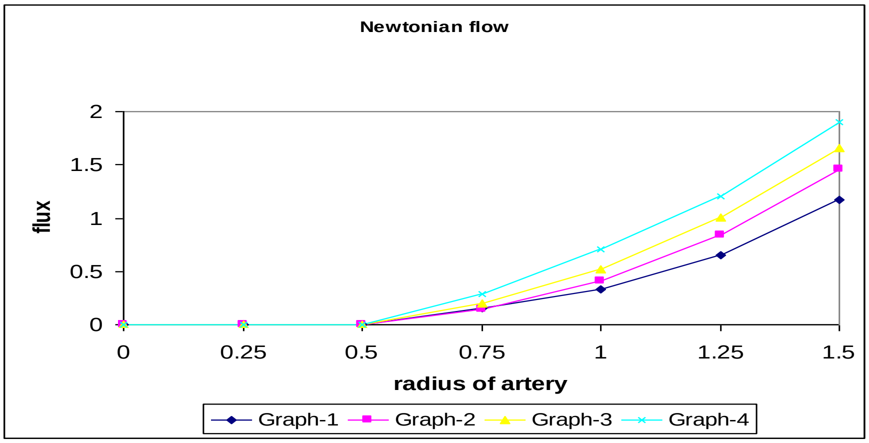

Figure 2. Plot of volumetric flow rate (i.e., flux) against radius of tapered artery for Newtonian flow.

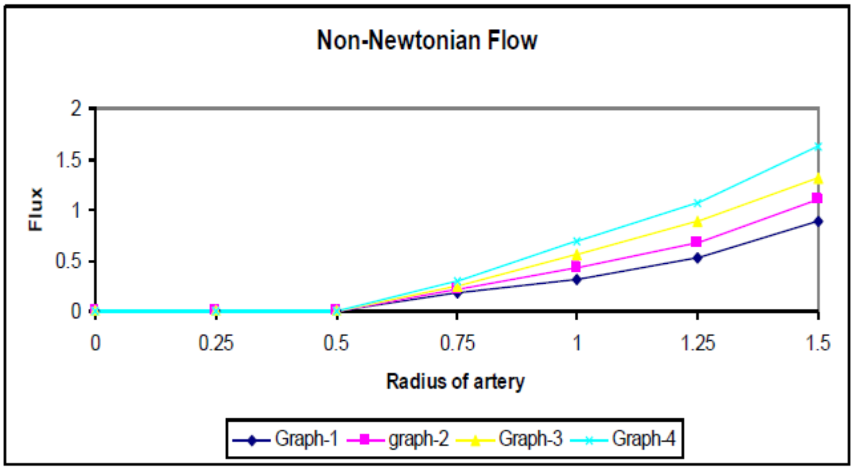

Figure 3. Volumetric flow rate (i.e., flux) against radius of tapered artery for Non-Newtonian flow.

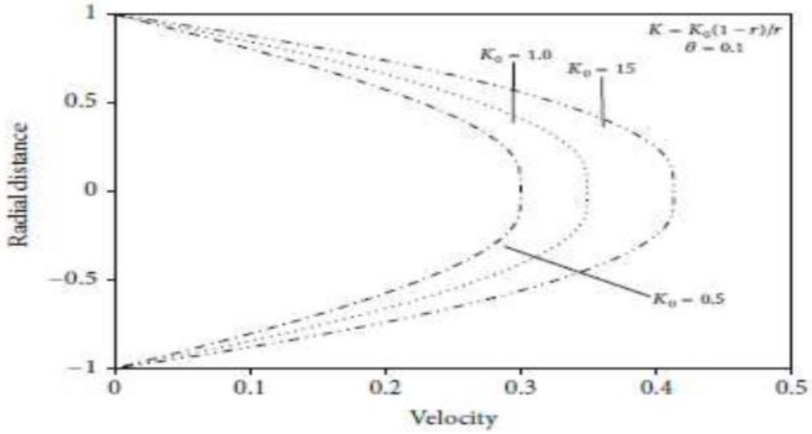

Figure 4. Velocity distribution for variable permeability parameter.

4. Conclusion

The effect of porous medium on Newtonian and non-Newtonian flows through tapered tubes receives attention regarding flux. In both the cases of constant and variable permeability, the flux increases with the increase in the radial distance as well as permeability factor. The shear stress and plug core radius are considerably higher in the variable permeability case than those of the constant permeability.

NOMENCLATURE:

V_r = Radial fluid velocity

V_z = Axial fluid velocity

P = Pressure

ρ = Fluid density

$\upsilon = \frac{\mu}{\rho}$ = Kinematic fluid viscosity

τ = Shear stress

τ_w = Wall shear stress

R = Radius of tapered tube

ΔP = Pressure drop

Q = Volumetric flow rate, i.e., flux

$\frac{\mu}{k}$ = Permeability factor of porous medium.

References

[1] S. Oka and T. Murata, Theory of the steady slow motion of non-Newtonian fluids through a tapered tube, Japanese Journal of Applied Physics 8 (1969), 5-8.

[2] F. Song, Y. Xu and H. Li, Blood flow in capillaries by using porous media model, Journal of Central South University of Technology 14(1) (2007), 46-49.

[3] W. P. Walawender, C. Tine and L. C. Cerny, Experimental studies on the blood flow throughout tapered tubes, International Journal Engg. Science 10 (1972), 1123-1135.

[4] S. Oka, Pressure development in a non-Newtonian flow through a tapered tube, Biorheology 10 (1973), 207-212.

[5] W. P. Walawender and T. Y. Chen, Blood flow in tapered tubes, Microvasc. Res. 9 (1975), 190-205.

[6] X. J. Wang, G. T. Liu, B. Q. Ali and L. G. Liu, Numerical study of pulsating flow through a tapered artery with stenosis, Chinese J. Physics 42(4-I) (2004), 20-31.

[7] P. K. Mandal, A unsteady analysis of non-Newtonian blood flow through tapered arteries with stenos is, Journal of Non Linear Mechanics 40 (2005), 151-164.

[8] S. Kumar and S. Kumar, Numerical study of the ax symmetric blood flow in a constricted rigid tube, International Review of Pure and Applied Mathematics 2(2) (2006), 99-109.

[9] D. S. Sankar and K. Hemalatha, Non-Linear mathematical models for blood flow through tapered tubes, Applied Mathematics and Computation 188 (2007), 567-582.

[10] Anil Kumar, Mathematical Modeling and Analysis of Blood Flow through Artery of Varying Cross section area and its Applications 5(1) (2017), 1-4. Biomedical Science and Engineering, USA.

[11] Anil Kumar, C. L. Varshney and G. C. Sharma, Computational technique for flow in blood vessels with porous effects, International Journal of Applied Mathematics and Mechanics, (Springer Publishing) China 26(1) (2005), 63-72.

[12] Anil Kumar, G. C. Sharma and M. Jain, Finite element Galerkin's approach for a computational study of arterial flow, International Journal of Applied Mathematics and Mechanics 22(9) (2001), 1012-1018.

[13] R. K. Saket and Anil Kumar, Reliability of convective diffusion process in porous blood vessels, International Journal of Chemical Product and Process Modeling, Vol.(3) Article (25) ISSN: 1934-2659, Canada 2008.

[14] Jaiswal and P. K. Yadav, A micropolar-Newtonian blood flow model through a porous

layered artery in the presence of a magnetic field, Physics of Fluids 31 (2019), 071901.

[15] Liu Yadong and Liu Wenjun, Blood flow analysis in tapered stenosed arteries with the influence of heat and mass transfer, Journal of Applied Mathematics and Computing (2020), 1-19, https://doi.org/10.1007/s12190-020-01328-5.

[16] Anil Kumar Gupta, Performance Model and Analysis of Blood Flow in Small Vessels with Magnetic Effects, International Journal of Engineering, IJE Transactions A: Basics 25(2) (2012), 190-196, Iran, Impact Factor 1.0986, ISSN 1728-1431,