TRIANGULAR INTUITIONISTIC FUZZY GRACEFUL LABELING IN SOME PATH RELATED GRAPHS

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Abstract

In this article, we present a new labeling concept called Triangular Intuitionistic Fuzzy Graceful Labeling. We conclude that the intuitionistic fuzzy union of two paths $P_m, P_n$, $(m, n \leq 22)$ supports triangular intuitionistic fuzzy graceful labeling.

1. Triangular Intuitionistic Fuzzy Labeling Graph

Definition 1.1. In a graph $G$ if the labels assigned to the vertices are triangular intuitionistic fuzzy number then it is said to be a triangular intuitionistic fuzzy labeling.

Triangular intuitionistic fuzzy number (TIFN) is an intuitionistic fuzzy number which is a subset of $[0, 1]$ denoted by TIFN that is $TIFN \subseteq [0, 1]$.

Example 1.2. Let $G_1$ be an intuitionistic fuzzy graph with $(\mu_A, v_A)$, the vertex membership and non-membership of $G_1$ and $(\mu_B, v_B)$, the edge membership and non-membership of $G_1$ respectively where the injective map $\mu_A : V \rightarrow TIFN$ such that,

$$\mu_A(v_i) = \frac{t^2}{100}, \forall v_i \in V, 1 \leq i \leq 4.$$
Also the injective map \( v_A : V \to TIFN \) such that,

\[
v_A(v_i) = \frac{i^2}{100}, \quad \forall v_i \in V, \ 1 \leq i \leq 4.
\]

The edges of membership functions are defined by a function \( \mu_B : V \times V \to TIFN \) such that,

\[
\mu_B(v_i, v_{i+1}) = | \mu_A(v_{i+1}) + \mu_A(v_i) |, \quad \forall v_i \in V, \ 1 \leq i \leq 4
\]

and the edges of non-membership functions are defined by a function \( v_B : V \times V \to TIFN \) such that,

\[
v_B(v_i, v_{i+1}) = | v_A(v_{i+1}) - v_A(v_i) |, \quad \forall v_i \in V, \ 1 \leq i \leq 4.
\]

From the above assumption, we have

\[
\mu_A(v_i) < \mu_A(v_{i+1}) \text{ and } v_A(v_i) < v_A(v_{i+1})
\]
\[
\mu_B(v_{i+1}, v_i) < \mu_B(v_{i+2}, v_{i+1}) \text{ and } v_B(v_{i+1}, v_i) < v_B(v_{i+2}, v_{i+1}).
\]

Clearly \( G_1 \) is a Triangular intuitionistic fuzzy labeling graph.

**Theorem 3.1.** The path graph \( P_m, \ (m \leq 22) \) admits triangular intuitionistic fuzzy graceful labeling.

**Proof.** Let \( P_m \) be a path graph with \( (\mu_A, v_A) \), the vertex membership and non-membership and \( (\mu_B, v_B) \), the edge membership and non-membership respectively.

Define an injective map \( \mu_A : V \to TIFN \) such that,

\[
\mu_A(v_i) = \frac{m^2 + \left( i^2 - \frac{1}{100} \right)}{1000}, \quad \forall v_i \in V, \ 1 \leq i \leq m.
\]
Also the injective map \( v_A : V \rightarrow TIFN \) such that,
\[
v_A(v_i) = \frac{1000 - (m^2 + i^2)}{1000}, \quad \forall v_i \in V, 1 \leq i \leq m.
\]

The edge membership functions of \( P_m \), are defined by a function \( \mu_B : V \times V \rightarrow TIFN \) such that,
\[
\mu_B(v_i, v_{i+1}) = |\mu_A(v_{i+1}) - \mu_A(v_i)|, \quad \forall v_i \in V, 1 \leq i \leq m
\]
and the edge non-membership functions of \( P_m \), are defined by a function \( v_B : V \times V \rightarrow TIFN \) such that,
\[
v_B(v_i, v_{i+1}) = |v_A(v_{i+1}) + v_A(v_i)|, \quad \forall v_i \in V, 1 \leq i \leq m
\]

Clearly \((\mu_A, v_A)\) and \((\mu_B, v_B)\) is distinct for all vertices and edges of \( P_m \) respectively. Hence is a Triangular intuitionistic fuzzy graceful graph.

**Example 1.4.** Let \( P_m \) be a path graph with,
\[
\mu_A(v_i) = \frac{m^2 + \left( i^2 - \frac{1}{1000} \right)}{1000}, \quad \forall v_i \in V, 1 \leq i \leq 3
\]
and
\[
v_A(v_i) = \frac{1000 - (m^2 + i^2)}{1000}, \quad \forall v_i \in V, 1 \leq i \leq 3.
\]

Let \( m = 22 \).

For \( i = 1 \),
\[
\mu_A(v_1) = 0.48499 \quad \text{and} \quad v_A(v_1) = 0.515
\]

For \( i = 2 \),
\[
\mu_A(v_2) = 0.48799 \quad \text{and} \quad v_A(v_2) = 0.512
\]

For \( i = 3 \),
\[
\mu_A(v_3) = 0.49299 \quad \text{and} \quad v_A(v_3) = 0.507
\]

and the edge function are given by
\[
\mu_B(v_i, v_{i+1}) = |\mu_A(v_{i+1}) - \mu_A(v_i)|, \quad \forall v_i \in V, 1 \leq i \leq 3
\]
\[ v_B(v_i, v_{i+1}) = |v_A(v_{i+1}) - v_A(v_i)|, \forall v_i \in V, 1 \leq i \leq 3. \]

For \( i = 1 \),
\[ \mu_B(v_1, v_2) = 0.003 \text{ and } v_B(v_1, v_2) = 0.003. \]

For \( i = 2 \),
\[ \mu_B(v_2, v_3) = 0.005 \text{ and } v_B(v_2, v_3) = 0.005 \]

\[ v_1 (0.003,0.003) \quad v_2 (0.005,0.005) \quad v_3 (0.48499,0.515) \quad (0.48799,0.512) \quad (0.49299,0.507) \]

\( P_m \)

**Figure 3.2.**

Clearly \((\mu_A, v_A)\) and \((\mu_B, v_B)\) is distinct for all vertices and edges of \( P_m \) respectively. Hence \( P_m \) is a Triangular intuitionistic fuzzy graceful graph.

**Theorem 1.5.** The union of two IF triangular labeled path graphs is also an IF triangular labeled path graph.

**Proof.** Let \( P_m, P_n, (m, n \leq 22) \) be two IF triangular labeled path graphs.

Consider the path \( P_m \) with vertex membership function \( \mu_{A_1} : V_1 \to TIFN \) such that,
\[
\mu_{A_1}(v_i) = \frac{m^2 + \left( i^2 - \frac{1}{100} \right)}{1000}, \forall v_i \in V_1, 1 \leq i \leq m
\]

and vertex non-membership function \( v_{A_1} : V_1 \to TIFN \) such that,
\[
v_{A_1}(v_i) = \frac{1000 - \left( m^2 + i^2 \right)}{1000}, \forall v_i \in V_1, 1 \leq i \leq m.
\]

The edge membership functions of \( P_m \), are defined by a function \( \mu_{B_1} : V_1 \times V_1 \to TIFN \) such that,
\[
\mu_{B_1}(v_i, v_{i+1}) = |\mu_{A_1}(v_{i+1}) - \mu_{A_1}(v_i)|, \forall v_i \in V_1, 1 \leq i \leq m
\]
and the edge non-membership functions of \( P_m \), are defined by a function 
\[
\nu_{B_1} : V_1 \times V_2 \rightarrow \text{TIFN} \quad \text{such that,} \\
\nu_{B_1}(v_i, v_{i+1}) = |\nu_{A_1}(v_{i+1}) - \nu_{A_1}(v_i)|, \forall v_i \in V_1, 1 \leq i \leq m.
\]

Consider the path \( P_n \) with vertex membership function 
\[
\mu_{A_2} : V_2 \times V_2 \rightarrow \text{TIFN} \quad \text{such that,} \\
\mu_{A_2}(v_i) = \frac{i^2}{10000}, \forall v_i \in V_2, 1 \leq i \leq n
\]
and vertex non-membership function \( \nu_{A_2} : V_2 \rightarrow \text{TIFN} \) such that,
\[
\nu_{A_2}(v_i) = \frac{(1000 - i^2)}{10000}, \forall v_i \in V_2, 1 \leq i \leq n.
\]
The edge membership functions of \( P_n \), are defined by a function 
\[
\mu_{B_2} : V_2 \times V_2 \rightarrow \text{TIFN} \quad \text{such that,} \\
\mu_{B_2}(v_i, v_{i+1}) = |\mu_{A_2}(v_{i+1}) - \mu_{A_2}(v_i)|, \forall v_i \in V_2, 1 \leq i \leq n
\]
and the edge non-membership functions of \( P_n \), are defined by a function 
\[
\nu_{B_2} : V_2 \times V_2 \rightarrow \text{TIFN} \quad \text{such that,} \\
\nu_{B_2}(v_i, v_{i+1}) = |\nu_{A_2}(v_{i+1}) - \nu_{A_2}(v_i)|, \forall v_i \in V_2, 1 \leq i \leq n
\]
By definition,
\[
P_m \cup P_n = \{A_1 \cup A_2, B_1 \cup B_2\}
\]
Now,
\[
(\mu_{A_1} \cup \mu_{A_2})(v_i) = \max \{\mu_{A_1}(v_i), \mu_{A_2}(v_i)\}
\]
\[
(v_{A_1} \cup v_{A_2})(v_i) = \max \{v_{A_1}(v_i), v_{A_2}(v_i)\}
\]
\[
(\mu_{B_1} \cup \mu_{B_2})(v_i, v_{i+1}) = \max \{\mu_{B_1}(v_i, v_{i+1}), \mu_{B_2}(v_i, v_{i+1})\}
\]
\[
(v_{B_1} \cup v_{B_2})(v_i, v_{i+1}) = \max \{v_{B_1}(v_i, v_{i+1}), v_{B_2}(v_i, v_{i+1})\}
\]
Therefore \( P_m \cup P_n \) is triangular IF labeled path graph.
Example 1.6. Clearly $P_m \cup P_n$ is a triangular intuitionistic fuzzy labeled path graph.

**Theorem 1.7.** The union of two intuitionistic fuzzy triangular graceful labeled path graphs is also an intuitionistic fuzzy triangular graceful labeled path graph.

**Proof.** Let $P_m, P_n, (m, n \leq 22)$ be two intuitionistic fuzzy triangular graceful labeled path graphs.

From known theorem and by definition,

$$P_m \cup P_n = \{A_1 \cup A_2, B_1 \cup B_2\}.$$

Now,

$$
(\mu_{A_1} \cup \mu_{A_2})(v_i) = \max \{\mu_{A_1}(v_i), \mu_{A_2}(v_i)\} \\
(v_{A_1} \cup v_{A_2})(v_i) = \max \{v_{A_1}(v_i), v_{A_2}(v_i)\} \\
(\mu_{B_1} \cup \mu_{B_2})(v_i, v_{i+1}) = \max \{\mu_{B_1}(v_i, v_{i+1}), \mu_{B_2}(v_i, v_{i+1})\} \\
(v_{B_1} \cup v_{B_2})(v_i, v_{i+1}) = \max \{v_{B_1}(v_i, v_{i+1}), v_{B_2}(v_i, v_{i+1})\}.
$$
Clearly \((\mu_{A_1}, v_{A_1})\) and \((\mu_{A_2}, v_{A_2})\) are distinct for all vertices and are distinct for all edges respectively.

Hence \(P_m \cup P_n\) is triangular intuitionistic fuzzy graceful labeled path graph.

**Example 1.8.**

**Figure 3.4(a)**

Clearly \((\mu_{A_1}, v_{A_1})\) and \((\mu_{A_2}, v_{A_2})\) are distinct for all vertices and \((\mu_{B_1}, v_{B_1})\) and \((\mu_{B_2}, v_{B_2})\) are distinct for all edges respectively. Hence \(P_m \cup P_n\) is triangular intuitionistic fuzzy graceful labeled path graph.

2. Conclusion

In this article, a new labeling concept called triangular intuitionistic fuzzy graceful labeling is developed and has discussed some operations like intuitionistic fuzzy union for triangular intuitionistic fuzzy labeled path graphs. Also proved that the intuitionistic fuzzy union of two paths \(P_m, P_n, (m, n \leq 22)\) admits triangular intuitionistic fuzzy graceful labeling. Much more work could be done to investigate the labeling on different types of graphs.
References


