Advances and Applications in Mathematical Sciences
Volume 20, Issue 6, April 2021, Pages 1077-1084

TRIANGULAR INTUITIONISTIC FUZZY GRACEFUL LABELING IN SOME PATH RELATED GRAPHS

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Abstract

In this article, we present a new labeling concept called Triangular Intuitionistic Fuzzy Graceful Labeling. We conclude that the intuitionistic fuzzy union of two paths P_m, P_n, $(m, n \leq 22)$ supports triangular intuitionistic fuzzy graceful labeling.

1. Triangular Intuitionistic Fuzzy Labeling Graph

Definition 1.1. In a graph G if the labels assigned to the vertices are triangular intuitionistic fuzzy number then it is said to be a triangular intuitionistic fuzzy labeling.

Triangular intuitionistic fuzzy number (TIFN) is an intuitionistic fuzzy number which is a subset of $[0, 1]$ denoted by TIFN that is $TIFN \subseteq [0, 1]$.

Example 1.2. Let G_1 be an intuitionistic fuzzy graph with (μ_A, v_A), the vertex membership and non-membership of G_1 and (μ_B, v_B), the edge membership and non-membership of G_1 respectively where the injective map $\mu_A : V \to TIFN$ such that,

$$\mu_A(v_i) = \frac{t^2}{100}, \forall v_i \in V, 1 \leq i \leq 4.$$

2010 Mathematics Subject Classification: 03F55, 05C72, 05C78.
Keywords: intuitionistic fuzzy number, triangular intuitionistic fuzzy number, triangular intuitionistic fuzzy labeling, triangular intuitionistic fuzzy graceful labeling.
Received January 22, 2020; Accepted May 13, 2020

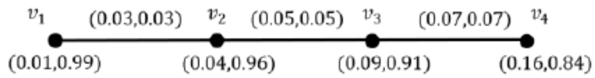


Figure 3.1

Also the injective map $\upsilon_A : V \to TIFN$ such that,

$$\upsilon_A(v_i) = \frac{i^2}{100}, \forall v_i \in V, 1 \le i \le 4.$$

The edges of membership functions are defined by a function $\mu_B : V \times V \to TIFN$ such that,

$$\mu_B(v_i, v_{i+1}) = | \mu_A(v_{i+1}) + \mu_A(v_i) |, \forall v_i \in V, 1 \le i \le 4$$

and the edges of non-membership functions are defined by a function $\upsilon_B : V \times V \to TIFN$ such that,

$$\upsilon_B(v_i, v_{i+1}) = | \upsilon_A(v_{i+1}) - \upsilon_A(v_i) |, \forall v_i \in V, 1 \le i \le 4.$$

From the above assumption, we have

$$\mu_A(v_i) < \mu_A(v_{i+1}) \text{ and } \upsilon_A(v_i) < \upsilon_A(v_{i+1})$$

$$\mu_B(v_{i+1}, v_i) < \mu_B(v_{i+2}, v_{i+1}) \text{ and } \upsilon_B(v_{i+1}, v_i) < \upsilon_B(v_{i+2}, v_{i+1}).$$

Clearly G_1 is a Triangular intuitionistic fuzzy labeling graph.

Theorem 3.1. *The path graph* $P_m, (m \le 22)$ *admits triangular intuitionistic fuzzy graceful labeling*.

Proof. Let P_m be a path graph with (μ_A, υ_A), the vertex membership and non-membership and (μ_B, υ_B), the edge membership and non-membership respectively.

Define an injective map $\mu_A : V \to TIFN$ such that,

$$\mu_A(v_i) = \frac{m^2 + \left(i^2 - \frac{1}{100}\right)}{1000}, \forall v_i \in V, 1 \le i \le m.$$

Also the injective map $\upsilon_A : V \to TIFN$ such that,

$$\upsilon_A(v_i) = \frac{1000 - (m^2 + i^2)}{1000}, \forall v_i \in V, 1 \leq i \leq m.$$

The edge membership functions of P_m, are defined by a function $\mu_B : V \times V \to TIFN$ such that,

$$\mu_B(v_i, v_{i+1}) = | \mu_A(v_{i+1}) - \mu_A(v_i) |, \forall v_i \in V, 1 \leq i \leq m$$

and the edge non-membership functions of P_m, are defined by a function $\upsilon_B : V \times V \to TIFN$ such that,

$$\upsilon_B(v_i, v_{i+1}) = | \upsilon_A(v_{i+1}) + \upsilon_A(v_i) |, \forall v_i \in V, 1 \leq i \leq m$$

Clearly (μ_A, υ_A) and (μ_B, υ_B) is distinct for all vertices and edges of P_m respectively. Hence is a Triangular intuitionistic fuzzy graceful graph.

Example 1.4. Let P_m be a path graph with,

$$\mu_A(v_i) = \frac{m^2 + \left(i^2 - \frac{1}{1000}\right)}{1000}, \forall v_i \in V, 1 \leq i \leq 3$$

and

$$\upsilon_A(v_i) = \frac{1000 - (m^2 + i^2)}{1000}, \forall v_i \in V, 1 \leq i \leq 3.$$

Let $m = 22$.

For $i = 1$,

$\mu_A(v_1) = 0.48499$ and $\upsilon_A(v_1) = 0.515$

For $i = 2$,

$$\mu_A(v_2) = 0.48799 \text{ and } \upsilon_A(v_2) = 0.512$$

For $i = 3$,

$$\mu_A(v_3) = 0.49299 \text{ and } \upsilon_A(v_3) = 0.507$$

and the edge function are given by

$$\mu_B(v_i, v_{i+1}) = | \mu_A(v_{i+1}) - \mu_A(v_i) |, \forall v_i \in V, 1 \leq i \leq 3$$

and

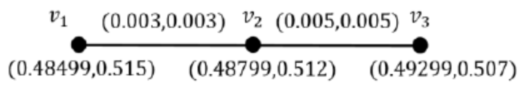
$$v_B(v_i, v_{i+1}) = |\, v_A(v_{i+1}) - v_A(v_i) \,|, \ \forall v_i \in V, 1 \le i \le 3.$$

For $i = 1$,

$$\mu_B(v_1, v_2) = 0.003 \text{ and } v_B(v_1, v_2) = 0.003.$$

For $i = 2$,

$$\mu_B(v_2, v_2) = 0.005 \text{ and } v_B(v_2, v_3) = 0.005$$



$\boldsymbol{P_m}$

Figure 3.2.

Clearly (μ_A, ν_A) and (μ_B, ν_B) is distinct for all vertices and edges of P_m respectively. Hence P_m is a Triangular intuitionistic fuzzy graceful graph.

Theorem 1.5. *The union of two IF triangular labeled path graphs is also an IF triangular labeled path graph.*

Proof. Let $P_m, P_n, (m, n \le 22)$ be two IF triangular labeled path graphs.

Consider the path P_m with vertex membership function $\mu_{A_1} : V_1 \to TIFN$ such that,

$$\mu_{A_1}(v_i) = \frac{m^2 + \left(i^2 - \frac{1}{100}\right)}{1000}, \ \forall v_i \in V_1, 1 \le i \le m$$

and vertex non-membership function $v_{A_1} : V_1 \to TIFN$ such that,

$$v_{A_1}(v_i) = \frac{1000 - (m^2 + i^2)}{1000}, \ \forall v_i \in V_1, 1 \le i \le m.$$

The edge membership functions of P_m, are defined by a function $\mu_{B_1} : V_1 \times V_1 \to TIFN$ such that,

$$\mu_{B_1}(v_i, v_{i+1}) = |\, \mu_{A_1}(v_{i+1}) - \mu_{A_1}(v_i) \,|, \ \forall v_i \in V_1, 1 \le i \le m$$

and the edge non-membership functions of P_m, are defined by a function $\upsilon_{B_1} : V_1 \times V_2 \rightarrow TIFN$ such that,

$$\upsilon_{B_1}(v_i, v_{i+1}) = | \upsilon_{A_1}(v_{i+1}) - \upsilon_{A_1}(v_i) |, \forall v_i \in V_1, 1 \leq i \leq m.$$

Consider the path P_n with vertex membership function $\mu_{A_2} : V_2 \times V_2 \rightarrow TIFN$ such that,

$$\mu_{A_2}(v_i) = \frac{i^2}{10000}, \forall v_i \in V_2, 1 \leq i \leq n$$

and vertex non-membership function $\upsilon_{A_2} : V_2 \rightarrow TIFN$ such that,

$$\upsilon_{A_2}(v_i) = \frac{(1000 - i^2)}{10000}, \forall v_i \in V_2, 1 \leq i \leq n.$$

The edge membership functions of P_n, are defined by a function $\mu_{B_2} : V_2 \times V_2 \rightarrow TIFN$ such that,

$$\mu_{B_2}(v_i, v_{i+1}) = | \mu_{A_2}(v_{i+1}) - \mu_{A_2}(v_i) |, \forall v_i \in V_2, 1 \leq i \leq n$$

and the edge non-membership functions of P_n, are defined by a function $\upsilon_{B_2} : V_2 \times V_2 \rightarrow TIFN$ such that,

$$\upsilon_{B_2}(v_i, v_{i+1}) = | \upsilon_{A_2}(v_{i+1}) - \upsilon_{A_2}(v_i) |, \forall v_i \in V_2, 1 \leq i \leq n$$

By definition,

$$P_m \cup P_n = \{A_1 \cup A_2, B_1 \cup B_2\}$$

Now,

$$(\mu_{A_1} \cup \mu_{A_2})(v_i) = \max \{\mu_{A_1}(v_i), \mu_{A_2}(v_i)\}$$

$$(\upsilon_{A_1} \cup \upsilon_{A_2})(v_i) = \max \{\upsilon_{A_1}(v_i), \upsilon_{A_2}(v_i)\}$$

$$(\mu_{B_1} \cup \mu_{B_2})(v_i, v_{i+1}) = \max \{\mu_{B_1}(v_i, v_{i+1}), \mu_{B_2}(v_i, v_{i+1})\}$$

$$(\upsilon_{B_1} \cup \upsilon_{B_2})(v_i, v_{i+1}) = \max \{\upsilon_{B_1}(v_i, v_{i+1}), \upsilon_{B_2}(v_i, v_{i+1})\}.$$

Therefore $P_m \cup P_n$ is triangular IF labeled path graph.

Example 1.6.

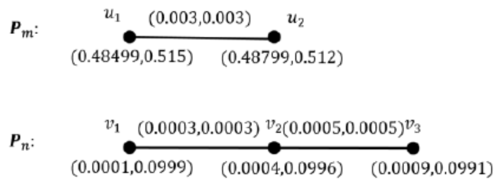


Figure 3.3(a)

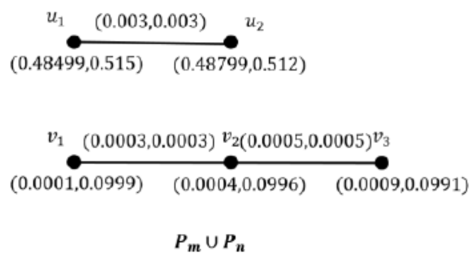


Figure 3.3(b)

Clearly $P_m \cup P_n$ is a triangular intuitionistic fuzzy labeled path graph.

Theorem 1.7. *The union of two intuitionistic fuzzy triangular graceful labeled path graphs is also an intuitionistic fuzzy triangular graceful labeled path graph.*

Proof. Let $P_m, P_n, (m, n \leq 22)$ be two intuitionistic fuzzy triangular graceful labeled path graphs.

From known theorem and by definition,

$$P_m \cup P_n = \{A_1 \cup A_2, B_1 \cup B_2\}.$$

Now,

$$(\mu_{A_1} \cup \mu_{A_2})(v_i) = \max\{\mu_{A_1}(v_i), \mu_{A_2}(v_i)\}$$

$$(v_{A_1} \cup v_{A_2})(v_i) = \max\{v_{A_1}(v_i), v_{A_2}(v_i)\}$$

$$(\mu_{B_1} \cup \mu_{B_2})(v_i, v_{i+1}) = \max\{\mu_{B_1}(v_i, v_{i+1}), \mu_{B_2}(v_i, v_{i+1})\}$$

$$(v_{B_1} \cup v_{B_2})(v_i, v_{i+1}) = \max\{v_{B_1}(v_i, v_{i+1}), v_{B_2}(v_i, v_{i+1})\}.$$

Clearly (μ_{A_1}, v_{A_1}) and (μ_{A_2}, v_{A_2}) are distinct for all vertices and and are distinct for all edges respectively.

Hence $P_m \cup P_n$ is triangular intuitionistic fuzzy graceful labeled path graph.

Example 1.8.

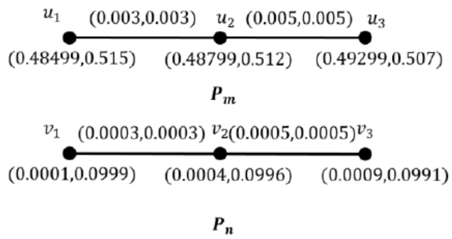


Figure 3.4(a)

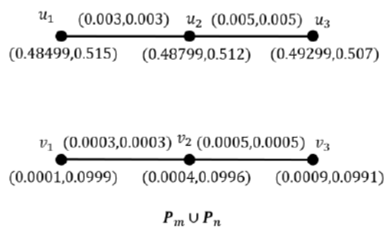


Figure 3.4(b)

Clearly (μ_{A_1}, v_{A_1}) and (μ_{A_2}, v_{A_2}) are distinct for all vertices and (μ_{B_1}, v_{B_1}) and (μ_{B_2}, v_{B_2}) are distinct for all edges respectively. Hence $P_m \cup P_n$ is triangular intuitionistic fuzzy graceful labeled path graph.

2. Conclusion

In this article, a new labeling concept called triangular intuitionistic fuzzy graceful labeling is developed and has discussed some operations like intuitionistic fuzzy union for triangular intuitionistic fuzzy labeled path graphs. Also proved that the intuitionistic fuzzy union of two paths P_m, P_n, $(m, n \leq 22)$ admits triangular intuitionistic fuzzy graceful labeling. Much more work could be done to investigate the labeling on different types of graphs.

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