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A STUDY OF FRACTAL NETWORK IN BRAIN TUMOR

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Abstract

The Neurological Systems are extremely complex and fractals. This paper explains about Brain Tumors arise from the central nervous system can cause health issues in the body. Fractals Networks is described by characteristics of fractals such as self-similarity. Percolation theory describes that the behavior of the network by disconnecting nodes or links. The segmenting brain tumors is a hard and complex task. The mass of a brain cancer grows exponentially, as shown by the Natural Logarithm. Minimal Spanning tree demonstrates that a tumor cell's graph is shortest path of connecting small branches of network nodes, signifying it is acyclic. The tumor is detected, segmented, and the tumor's frequency of occurrence is displayed.

1. Introduction

Graph theory is the study of graphs, which are mathematical constructs used to depict pair-wise interactions between objects in mathematics. In this case, a graph is inclusive of nodes that are bound by links. Undirected graphs and directed graphs are distinguished [1]. Fractal objects and processes are considered to be self-invariant as a result. Fractal structures and processes have no fixed lengths, and fractal processes have no fixed time scales. In fractal analysis, the Euclidean concept of length is used as a tool. The fractal dimension [4] is a constant parameter that characterizes this operation. This study discusses the stages of a brain tumor cell and also how they affect the human brain. It depicts the growth of a cancer cell.

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1.1 Graph theory. On the Seven Bridges of Konigsberg, Leonhard Euler produced the first article in the history of graph theory. The Konigsberg bridge dilemma was a long-standing puzzle about the chances of finding a passage across all seven bridges that span a bifurcate river that flows across an island just once. Such a road, according to Euler, does not exist. He proved the first theorem in graph theory using only references to the physical configuration of the bridges. Frank Harary's novel was released in [2]. A set of vertices and edges that construct a network is defined by graph theory.

1.2 Fractals. Fractals were discovered by Benoit Mandelbrot [3]. When divided into parts, he classified them as geometric shapes in which each part is a miniature representation of the total shape. He coined the name Fractal as a new scientific term for this mathematical expression. The word fractal comes from the Latin word fractus, which means to break. Benoit has been referred to as the Father of Fractal Geometry. Fractals are an indefinitely complex process that is continuous. A self-similarity process is referred to as a fractal [6].

1.3 Percolation theory. The word percolation derives from the Latin word percolare, which meaning to stretch. The simplest fundamental, but not entirely solved, model of a phase transition is percolation theory. Understanding the percolation theory problem can help you to know a lot of other physical processes [12]. Furthermore, the concept of fractals, which is closely related to the percolation theory problem, has a wide range of applications in subjects as diverse as biology, physics, and geophysics, as well as practical applications such as oil recovery. It will begin by laying a foundation in percolation theory, which will serve as a natural prelude to scaling and renormalization group theory. For e.g., When a liquid is strained through a filter, such as when making coffee, percolation occurs.

1.4 Brain tumor. A brain tumor is a cancer that forms inside the brain or skull; are sometimes benign, while others are malignant shown in Figure, 1(b). A benign brain tumor grows slowly, has well-defined boundaries, and spreads only rarely. A malignant brain tumor spreads to neighboring brain areas, grows rapidly, and has irregular boundaries. Treatment methods differ depending on the type, volume, and location of the tumor [8]. Treatment goals could be curative or focused on symptom relief. Many of the 120 different forms of brain tumors are treatable. For many people, new medicines are extending their lives and enhancing their quality of life. It sees the brain in three dimensions, in cross sections that can be taken from the side or from the top. The use of MRI to assess brain abnormalities and their consequences on the surrounding brain is extremely beneficial.

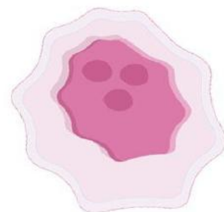


Figure 1(a). Normal Cell **Figure 1(b).** Cancer Cell.

2. Methods

The simplest model of a nerve system of brain is complex. Here, it is based on percolation that describes the behavior of the cancer cell networks. Potential energy describes the task of the cancer cell which spreads from one vertex to other vertices and also independent path. The minimal spanning tree, which is acyclic and has no loop, is used to calculate the smallest total length of connecting smaller branches to link network nodes.

Figure 2. The nerve system of brain.

2.1 Network. The topological organization of brain connections can be represented as networks of nodes and edges in neuro imaging data. The study of these networks and their structure at various scales can be done using graph theory [5], which is a broad and strong framework. When the instructor gives an instruction, the individual receives it, which improves the thought process on a specific issue and produces the desired result, it is shown in figure 3(a). The flow of the cancer cell in the brain tumor is shown in figure 3(b).

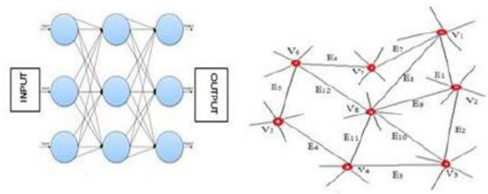


Figure 3 (a). the function of the brain Figure 3(b) the network of the cancer cell.

2.2 Complexity. The network determines that the number of vertices, edges, and proper path in a graph i.e., vertices should not intersect each other. Many natural and artificial systems are made up of several parts, each of which can be described as a graph. Here, the nerve system of Brain has several connections which is represented below, the connection of nerve system is complexity. When a person thinks about a certain function [9], the brain begins to recall and accumulate knowledge, resulting in the output is indicated in Figure 4. As a result, the cognitive process develops a level of complexity.

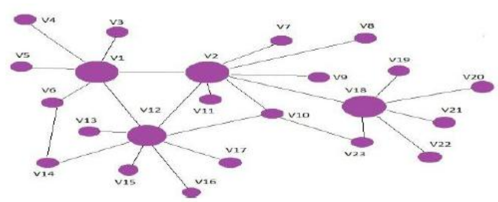


Figure 4. Complexity in Neurological system.

2.3 Self-similarity. There is a Symmetry refers to something that is the same on both sides of an axis [6]. Bilateral symmetry is the most basic type of symmetry, in which one side of a position reflects the other. When entities are repeated along a line or rotated around a point. Here, this paper clearly shows that the human brain has Self-Similarity in Figure 5. It has played a significant role in nature [7].

Figure 5. Self-Similarity in the brain.

2.4 Minimal spanning tree. A minimal spanning tree is a subset of edges in a connected weighted undirected graph that binds all of the nodes with the least amount of total edge weight possible. The minimum spanning tree uses the shortest total length of connecting small branches to connect network nodes. A spanning tree is made up of all network nodes that are connected to each other. The brain tumour cells influence the growth of new cells, causing the tumour to develop. This network is created by connecting cells, either directly or indirectly, through other cells.

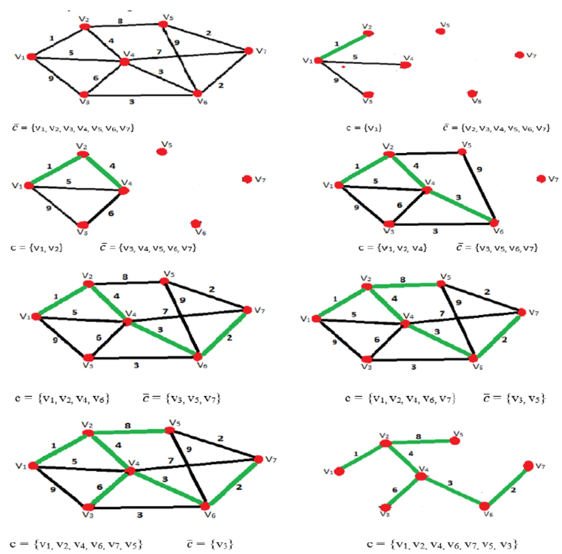


Figure 6. Spanning tree of cancer cell.

From Figure 6, the development of a tumor cell (target tissue) is the finest tree [2]. In Minimal spanning tree, the edges have been chosen by the smallest weight cell such that sorted above (distance of the cells are given randomly). The tumour cell spreads out to the normal cell, forming a spanning tree visually and by removing one from spanning tree will make a graph disconnected. The cancer affects the weakest cell, makes it dead and passes to another cell. As a result, the graph is maximally acyclic and therefore has no nodes that loop back on themselves.

2.5 Natural logarithm of brain tumor. The maximal cell concentration (ymax) and the initial cell concentration's (y0) Natural logarithm [11]. The ultimate cell growth concentration that is obtained in the tissue is the final concentration (Figure 7). Here, the quantity brain cancer is growing exponentially or exhibiting exponential growth if the rate of rise of the quantity is proportionate to the quantity at that moment in every instance. The rate of growth is a significant factor in determining the stage of a brain cancer. In this case, the provided virus development in the tissue can be characterized as,

$$y = P(t) = P_0 e^{kt} \tag{1}$$

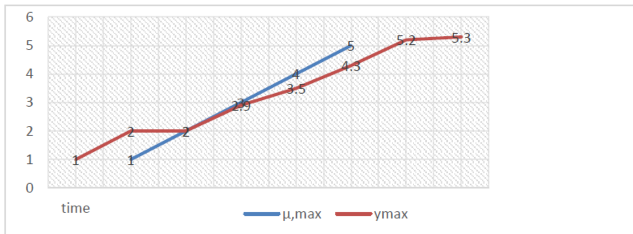


Figure 7. Growth rate of the brain.

2.6 Graphical representation of brain tumor. By the Potential energy, the cancer cell is independent path so that it normally grows in a regular manner. When the control over growth is broken, however, cells divide very often and at a high rate. The tumor is a malignant brain cancer. When cancer hits a single cell, it begins to spread in n-terms. The cells are affected by a brain tumor and develops the cancer cell is shown in Figure 8,

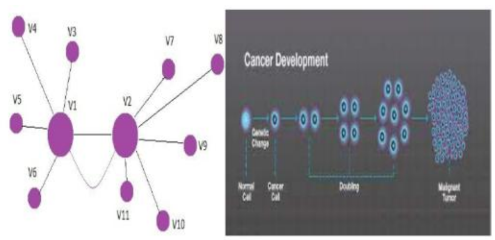


Figure 8. Development of the cancer in the Tumor.

2.7 Potential energy. Consider the single tumor cell as (v_1, v_2, v_3) in the Cartesian coordinates. In this case, the overall force of a tumor cell's operation is defined as,

$$F_a = -\frac{\partial t}{\partial v_1}, \; F_b = -\frac{\partial t}{\partial v_2}, \; F_c = -\frac{\partial t}{\partial v_3} \tag{2}$$

The tumor cell force F satisfies the, $F_{v_1} = -\frac{\partial t}{\partial v_1}$, $F_{v_2} = -\frac{\partial t}{\partial v_2}$, and $F_{v_3} = -\frac{\partial t}{\partial v_3}$ constraints.

Consider the force's work dx as it goes through the edges, which is provided by du.

It shows,

$$dx = F, du$$

$$= (F_{v_1}\vec{i} + F_{v_2}\vec{j} + F_{v_3}\vec{k}) \cdot (d_{v_1}\vec{i} + d_{v_2}\vec{j} + d_{v_3}\vec{k})$$

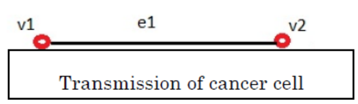
$$= F_{v_1}d_{v_1} + F_{v_2}d_{v_2} + F_{v_3}d_{v_3}$$

$$dx = -\frac{\partial t}{\partial v_1}dv_1 - \frac{\partial t}{\partial v_2}d_{v_2} - \frac{\partial t}{\partial v_3}d_{v_3}$$

$$= dt(v_1, v_2, v_3) \quad (3)$$

As a result, the force of a cancer cell goes from one vertex to other vertices of a local segment between vertices v_1 and v_2. We have,

$$D = \int_{v_2}^{v_1} F\, du \quad (4)$$



Transmission of cancer cell

$$= -\int_{v_1}^{v_2} dt$$

$$= [t]_{v_1}^{v_2}$$

$$= t_{v_1} - t_{v_2}$$

It concludes that the tumor cell's position is controlled by the initial and Transmission of cancer cell terminal vertices, but that linking these vertices creates an independent path.

2.8 Percolation theory. Percolation is a concept that describes way neural activity is routed through the brain's numerous connections [12]. A brain tumor is a collection of abnormal cells in your brain that has grown into a mass. A magnetic field and radio frequency waves are used in an MRI scan to provide a clear image of the brain's soft tissues.

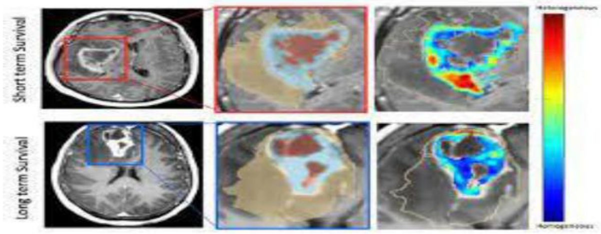


Figure 9. Cancer affected in Brain.

Brain tumors come in a wide variety of forms. Noncancerous (benign) brain tumors exist alongside cancerous brain tumors (malignant). Primary brain tumors start in the brain, but secondary (metastatic) brain cancers start elsewhere in the body and spread to the brain. The rate of growth of a brain tumor varies substantially. How a brain tumor affects the operation of your nervous system is determined by its development pace and location.

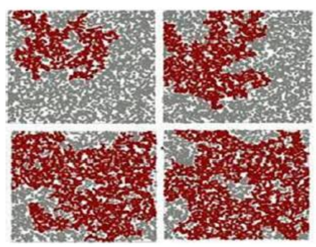


Figure 10. Percolation of Tumor in the Brain.

The occupied and blocked locations do not vary over time, and the structural pathways are not affected by location. Percolation may be useful in determining various characteristics of the complex structure to some extent [10]. Here, it is clearly shown in (figure 10) that the percolation theory appears (red dotted line).

3. Conclusion

The function of the cancer cell is depicted in this paper as a complicated network. The growth rate of cancer cells is calculated and depicted graphically by Natural Logarithm. The potential energy proves that the transmission of cancer cell develops an independent path that grows rapidly. The cancer cell is portrayed as a spanning tree and is formed into an acyclic network, according to Minimal spanning tree, which causes the disease to the weakest cell, has no loop and spreads aggressively. The area and shape of the impacted region are calculated using percolation.

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