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NATURAL DIFFERENCE LABELING ON CENTIPEDE GRAPH, STAR GRAPH $S_{m,n,r}$ AND TWIG GRAPH

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Abstract

A graph G is said to admit natural difference labeling, if its vertices can be labeled by non-negative integers such that the induced edge labels obtained by the absolute value of the difference of the labels of the end vertices are the first n natural numbers. A graph which admits natural difference labeling is called a natural difference graph. In this paper, Centipede graph, Star graph $S_{m,n,r}$ and Twig graph admit natural difference labeling is proved.

1. Introduction

Graph Labeling is the assignment of labels to vertices or edges or both subject to certain conditions. Graph Labeling has immense applications in various fields like communication network addressing, data base management, astronomy and so on. A dynamic survey on Graph Labeling is updated regularly by Joseph A. Gallian. In [5] T. K. Mathew Varkey and B. S. Sunoj have proved Absolute Difference of Cubic and Square Sum Labeling of a class of trees. The concept of Natural Difference Labeling is as in [6]. A brief summary of the definitions which are useful for our present investigations.

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Definition 1 (Natural Difference Labeling)**.** A natural difference labeling of a graph G is a one to one function $f : V(G) \to I$ (where I is the set of all non-negative integers) that induces a bijection $f^* : E(G) \to N$ (where N is the set of all natural numbers) of the edges of G defined by $f^*(uv) = | f(u) - f(v) | \ \forall\, e = uv \in E(G)$. The edge labels are the first n natural numbers. The graph which admits such labeling is called natural difference graph.

Definition 2 (Centipede graph)**.** The Centipede graph (Centipede tree) is the tree on $2n$ nodes obtained by joining the bottoms of n copies of the path graph P_2 laid in a row with $2n-1$ edges and it is denoted by C_n.

Note. Centipede graph is isomorphic to the firecracker graph.

Definition 3 (Graph $S_{m,n,r}$)**.** The graph $S_{m,n,r}$ is a star graph on a path of r vertices where the left end of the graph is a star graph on m vertices and the right end of the graph is a star graph on n vertices.

Definition 4 (Twig graph)**.** A graph obtained from a path by attaching exactly two pendant edges to each interior vertex of the path is called a twig graph and it is denoted by $T(n)$.

2. Main Results

Here we prove that Centipede graph, Graph $S_{m,n,r}$ and Twig graph admit Natural Difference Labeling.

Theorem 1. *Centipede graph admits Natural Difference Labeling.*

Proof. Let $v_0, v_1, v_2, \ldots v_{2n-1}$ be the vertices of the centipede graph with $2n-1$ edges. Let

$$V(C_n) = \{v_i : 0 \le i \le 2n-1\}$$

$$E(C_n) = \{v_i v_{i+1} : 0 \le i \le 2n-1\}$$

Define f by

$$f(v_i) = \frac{i(i+1)}{2}\, 0 \le i \le 3$$

and for $3 < i \leq 2n - 1$, i even, j takes values from $2, 4, 6, \ldots$ i.e.

$$f(v_i) = f(v_j) + i(j = 2, 4, 6, \ldots)$$

and when i odd, j takes values from $4, 6, \ldots$ i.e.

$$f(v_i) = f(v_j) + i(j = 4, 6, \ldots)$$

We see that the induced edge labels obtained by the difference of the labels of the end vertices i.e.

$$f^*(v_{i-1}v_i) = i(0 \leq i \leq 2n - 1)$$

are the first $2n - 1$ natural numbers. Thus Centipede graph admits natural difference labeling. See Figure 1

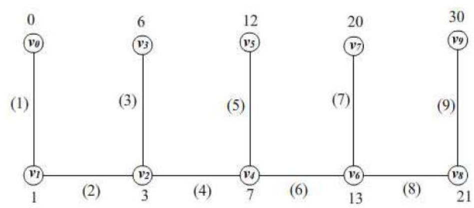

Figure 1. Natural Difference Labeling of C_5.

Theorem 2. *The graph* $S_{m,n,r}$ *admits Natural Difference Labeling.*

Proof. Let $P_r : u_0u_1, \ldots, u_{r-1}$ be a path of length $r - 1(r \geq 1)$. Let $v_1, v_2, \ldots, v_m$ be the vertices adjacent to u_0 and $w_1, w_2, \ldots, w_n$ be the vertices adjacent to u_{r-1}. Define a function f by

$f(u_0) = 0$

$f(v_j) = j(1 \leq j \leq m)$

$f(u_i) = m + s + f(u_{i-1})(1 \leq i \leq r)(s = 1, 2, \ldots)$

$f(w_i) = f(u_{r-1}) + f^*(u_{r-1}w_i)(1 \leq i \leq n)$

and the induced edge labeling f^* is defined by

$f^*(u_ov_i) = i(1 \leq i \leq m)$

$f^*(u_{i-1}u_i) = m + i\,(1 \le i \le r-1)$

$f^*(u_{r-1}w_i) = f^*(u_{r-2}u_{r-1}) + t\,(1 \le i \le n)\,(t = 1, 2, \ldots)$

We see that the induced edge labels obtained by the difference of the labels of the end vertices are the first $(m + n + r) - 1$ natural numbers. Thus the graph $S_{m,n,r}$ admits natural difference labeling. See Figure 2. □

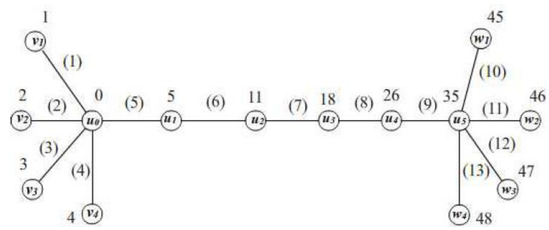

Figure 2. Natural Difference Labeling of $S_{4,4,6}$.

Theorem 3. *Twig graph admits Natural Difference Labeling.*

Proof. Let $u_0, u_1, \ldots u_{n-1}$ denote the vertices of the twig graph along the path. Let $v_1, v_2, \ldots v_{n-2}$ denote the vertices of the upper part of the twig graph and $w_1, w_2, \ldots w_{n-2}$ denote the vertices of the lower part of the twig graph. Define a function *f* by

$f(u_0) = 0$

$f(u_j) = f(u_{j-1}) + i + k\,(j = 1, 2, \ldots)(i = 1, 3, 5, \ldots)(k = 0, 1, 2, \ldots)$

$f(v_k) = f(u_j) + 3j - 1\,(j = 1, 2, \ldots)(k = 1, 2, \ldots)$

$f(w_k) = f(u_j) + 2j + k\,(j = 1, 2, \ldots)(k = 1, 2, \ldots)$

and the induced labeling f^* is defined by

$$f^*(u_{i-1}u_i) = 3i - 2\,(1 \le i \le n-1)$$

$$f^*(u_iv_i) = 3i - 1\,(1 \le i \le n-2)$$

$$f^*(u_iw_i) = 3i\,(1 \le i \le n-2)$$

For $n = 3$ and 4, we see that the induced edge labels obtained by the difference of the labels of the end vertices are the first $n + 1$ and $n + 2$ natural numbers. For $n > 4$ we see that the induced edge labels obtained by the difference of the labels of the end vertices are the first $2n + i$ natural numbers where $i = 0, 1, 2, \ldots$. Thus Twig graph admits natural difference labeling. See Figure 3. □

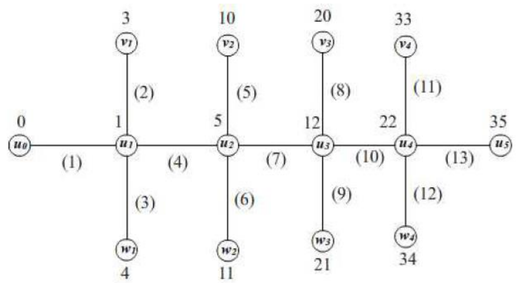

Figure 3. Natural Difference Labeling of $T(6)$.

3. Conclusion

Centipede graph, Graph $S_{m,n,r}$ and Twig graph admit Natural Difference Labeling are proved in this paper. Natural Difference Labeling on other graphs is under investigation.

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