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ACYCLIC COLORING OF EXTENDED DUPLICATE GRAPH OF FIRECRACKER $(F_{n,2})$ GRAPH FAMILIES

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Abstract

In this paper we present acyclic coloring algorithms to color the vertices of middle graph and total graph of extended Duplicate graph of Firecracker $(F_{n,2})$ graph. Also we obtain the chromatic number of the same.

Introduction

A proper coloring of a graph G is the coloring of the vertices of G such that no two neighbors in G are assigned the same color. Throughout this paper, by a graph we mean a finite, undirected, simple graph and the term coloring is used to denote vertex coloring of graphs.

A acyclic coloring of a graph G is the proper vertex coloring such that the subgraph induced by any two color classes does not contains a cycle. The notion of acyclic chromatic number was introduced by B. Grunbaum [6] in 1973. The acyclic chromatic number of a graph $G = G(V, E)$ is the minimum number of colors which are necessary to color G acyclically and is denoted by $a(G)$ [2, 3].

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An (n, k) firecracker is a graph obtained by the concatenation of n, k- stars by linking one leaf from each [4, 7, 8].

The concept of extended duplicate graph was introduced by Thirusangu et al. [9]. A duplicate graph of G is denoted by $DG = (V_1, E_1)$ where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and $f : V \to V'$ is bjective (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge set E_1 of DG is the edge v_1v_2 is in E if and only if both v_1v_2' and $v_1'v_2$ are edges in E_1. The extended duplicate graph of DG, denoted by EDG, is defined as, add an edge between any two vertex from V to any other vertex in V', except the terminal vertices of V and V'. For convenience, we take $v_2 \in V$ and $v_2' \in V'$ and thus the edge v_2v_2' is formed [10].

The middle graph of G, denoted by $M(G)$ was introduced in 1981 and is defined as follows [1]. The vertex set of $M(G)$ is $V(G) \cup E(G)$. Any two vertices x, y in $M(G)$ are adjacent in $M(G)$ if one of the following case holds.

(i) x and y are adjacent edges in G

(ii) x and y are incident in G.

The total graph of G, denoted by $T(G)$, is defined as follows [1, 5]. The vertex set of $T(G)$ is $V(G) \cup E(G)$. Any two vertices x, y in the vertex set of $T(G)$ are adjacent in $T(G)$ if one of the following cases holds.

(i) x, y are in $V(G)$ and x is adjacent to y in G.

(ii) x, y are in $E(G)$ and x, y are adjacent in G.

(iii) x is in $V(G)$, y is in $E(G)$ and x, y are incident in G.

The extended duplicate graph of Firecracker $(F_{n,2})$ graph is denoted by $EDG((F_{n,2}))$ with 4n vertices $\{v_1, v_2, \cdots, v_{2n}, v_1', v_2', v_3', \cdots, v_{2n}'\}$.

Acyclic Coloring of Middle graph of Extended duplicate graph of Firecracker $(F_{n,2})$ graph $(M[EDG(F_{n,2})])$

Coloring Algorithm 1:

Input: $M[EDG((F_{n,2}))]$, $n \geq 2$

$V \leftarrow \{v_1, v_2, v_3, \cdots, v_{2n}, v_1^{'}, v_2^{'}, v_3^{'}, \cdots, v_{2n}^{'}, x_1, x_2, x_3, \cdots, x_{4n-1}\}$

{

for ($k = 1$ to $2n$))

}

$v_k, v_k' \leftarrow 1;$

}

if $n \equiv 1 (\text{mod } 2)$

{

for (($k = 1$ to $\left\lfloor \frac{n}{2} \right\rfloor$)

{

$x_{2k-1}, x_{2n+2(k-1)} \leftarrow 2;$

}

$x_{2n-1}, x_{4n-2} \leftarrow 2;$

}

else

for (($k = 1$ to $\left\lfloor \frac{n}{2} \right\rfloor$)

{

$x_{2k-1}, x_{2n+2(k-1)}\} \leftarrow 2;$

}

}

if $n \equiv 1 (\bmod 2)$

{

for $(k = 1$ to $\left\lfloor \frac{n}{2} \right\rfloor)$

{

$x_{2k}, x_{2n+2k-1} \leftarrow 3;$

}

$x_n, x_{3n-1} \leftarrow 3;$

else

for $(k = 1$ to $\left\lfloor \frac{n}{2} \right\rfloor)$

{

$x_{2k}, x_{2n+2k-1} \leftarrow 3;$

}

$x_n, x_{2n-1}, x_{3n-1}, x_{4n-2} \leftarrow 3;$

}

$x_{4n-1} \leftarrow 4;$

}

Output: Vertex Colored $M[EDG((F_{n,2}))]$

Theorem 1. *The acyclic chromatic number of Middle graph of Extended Duplicate graph of Firecracker* $(F_{n,2})$ *graph is given by* $a(M[EDG(F_{n,2})]) = 4, n \geq 2$.

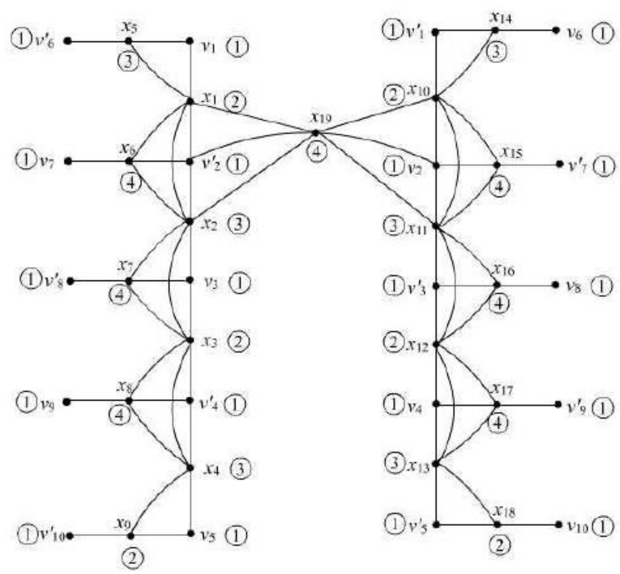
Proof. Color the vertices of $M[EDG(F_{n,2})]$ as given in the algorithm 1.

The color class of 1 is $\{v_i, v_i'; 1 \leq i \leq 2n\}$. The color class of 2 is $\{x_{2k-1}, x_{2n+2(k-1)}, x_{2n-1}, x_{4n-2}; 1 \leq k \leq \left\lfloor \frac{n}{2} \right\rfloor\}$ when n is odd and

$\{x_{2k-1},\, x_{2n+2(k-1)}; 1 \leq k \leq \left\lfloor \frac{n}{2} \right\rfloor\}$ when n is even. The color class of 3 is $\{x_{2k},\, x_{2n+2k-1},\, x_n,\, x_{3n-1}; 1 \leq k \leq \left\lfloor \frac{n}{2} \right\rfloor\}$ when n is odd and $\{x_{2k},\, x_{2n+2k-1},\, x_n,\, x_{2n-1},\, x_{3n-1},\, x_{4n-2}; 1 \leq k \leq \left\lfloor \frac{n}{2} \right\rfloor\}$ when n is even. The color class of 4 is x_{4n-1}

Case (i)

Consider the color classes of 1 and 2. The induced subgraph of the color classes of 1 and 2 is a collection of paths P_3 and isolated vertices, therefore, it is an acyclic graph.



a(M[EDG($F_{5,2}$)]) = 4

Figure 1.

Case (ii). Consider the color classes of 1 and 3. The induced subgraph of the color classes of 1 and 3 is a collection of paths P_3 and isolated vertices, therefore, it is an acyclic graph.

Case (iii). Consider the color classes of 1 and 4. The induced subgraph of the color classes of 1 and 4 is a collection of paths P_2 and isolated vertices, therefore, it is an acyclic graph.

Case (iv). Consider the color classes of 2 and 3. The induced subgraph of the color classes of 2 and 3 is a collection P_{2n+1} path when n is odd and P_{n+1} path when n is even therefore, it is an acyclic graph.

Case (v). Consider the color classes of 2 and 4. The induced subgraph of the color classes of 2 and 4 is a collection of path $\{x_{n+1}\ x_1, x_{4n-1}\ x_{2n}\ x_{3n}\}$ and isolated vertices therefore, it is an acyclic graph.

Case (vi). Consider the color classes of 3 and 4. The induced subgraph of the color classes of 3 and 4 is a collection of path $\{x_{n+1}\ x_2\ x_{4n-1}\ x_{2n+1}\ x_{3n}\}$ and isolated vertices therefore, it is an acyclic graph.

Thus, the induced subgraph of any two color classes is acyclic and therefore the coloring given in the algorithm 1, is an acyclic coloring.

Hence $a(M[EDG(F_{n\,2})]) = 4n \geq 2$.

Acyclic Coloring of Total graph of Extended duplicate graph of Firecracker $(F_{n,2})$ graph $(T[EDG(F_{n,2})])$.

Coloring Algorithm 2.

Input: $T[EDG(F_{n,2})]$, $n \geq 2$

$V \leftarrow \{v_1, v_2, v_3, \cdots, v_{2n}, v'_1, v'_2, v'_3, \cdots, v'_{2n}, x_1, x_2, x_3, \cdots, x_{4n-1}\}$

{

for $\{k = 1$ to $n)$

{

if $n \equiv 1 (\text{mod } 2)$

{

$v_{2k-1}, v2_k \leftarrow 1;$

}

else

{

$v_{2k-1}, v'_{n+2k}, v_{2k}, v'_{n+2k-1} \leftarrow 1;$

}

}

if $n \equiv 1 (\text{mod } 2)$

for $(k = 1 \text{ to } n)$

{

$v'_2, v'_{2k-1} \leftarrow 2;$

}

else

for $(k = 1 \text{ to } \left\lfloor \frac{n}{2} \right\rfloor)$

{

$v'_{n+2k-1}, v'_{2k}, v_{n+2k}, v'_{2k-1} \leftarrow 2;$

}

if $n \equiv 1 (\text{mod } 2)$

{

for $(k = 1 \text{ to } \left\lfloor \frac{n}{2} \right\rfloor)$

{

$x_{2k-1}, x_{2n+2(k-1)} \leftarrow 3;$

}

$x_{2n-1}, x_{4n-2} \leftarrow 3;$

else

for $(k = 1 \text{ to } \left\lfloor \frac{n}{2} \right\rfloor)$

{

$x_{2k-1}, x_{2n+2(k-1)} \leftarrow 3;$

}

if $n \equiv 1(\text{mod } 2)$

{

for $(k = 1 \text{ to } \left\lfloor \frac{n}{2} \right\rfloor)$

{

$x_{2k}, x_{2n+(2k-1)} \leftarrow 4;$

}

$x_n, x_{3n-1} \leftarrow 4;$

else

for $(k = 1 \text{ to } \left\lfloor \frac{n}{2} \right\rfloor)$

{

$x_{2k}, x_{2n+2(k-1)} \leftarrow 4;$

$x_{n-1}, x_{4n-2} \leftarrow 4;$

}

for $(k = 1 \text{ to } n - 2)$

{

$x_{n+k}, x_{3n+(k-1)} \leftarrow 5;$

}

$x_{4n-1} \leftarrow 5;$

}

Output: Vertex Colored $T[EDG((F_{n,2}))]$.

Theorem 2. *The acyclic chromatic number of Total graph of Extended Duplicate graph of Fire cracker graph* $F_{n,2}$ *is given by* $a(T[EDG(S_n)]) = 5$, $n \geq 2$.

Proof. Color the vertices of $T[EDG(F_{n,2})]$ as given in the algorithm 2.

Case (i). Consider the color classes of 1 and 2. The induced subgraph of the color classes 1 and 2 is the collection of tree therefore, it is an acyclic graph.

Case (ii). Consider the color classes of 1 and 3. The induced subgraph of the color classes of 1 and 3 is the collections of paths P_2 and isolated vertices therefore, it is an acyclic graph.

Case (iii). Consider the color classes of 1 and 4. The induced sub graph of the color classes of 1 and 4 is the collection of paths P_2 and isolated vertices therefore, it is an acyclic graph.

Case (iv). Consider the color classes of 1 and 5. The induced subgraph of the color classes of 1 and 5 is a collection of path $\{x_{4n-1}\ v_2\ x_{3n}\}$ and isolated vertices therefore, it is an acyclic graph.

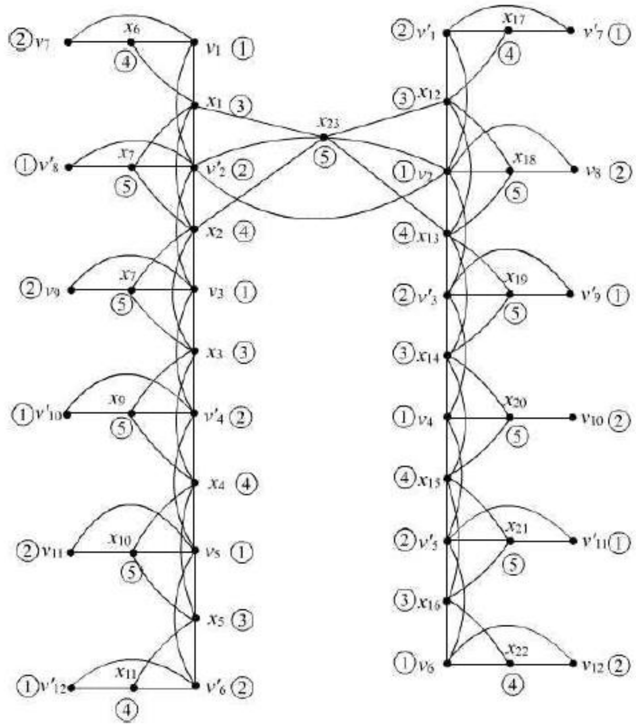
Case (v). Consider the color classes of 2 and 3. The induced subgraph of the color classes of 2 and 3 is a collection of paths P_2 and isolated vertices therefore, it is an acyclic graph.

Case (vi). Consider the color classes of 2 and 4. The induced subgraph of the color classes of 2 and 4 is a collection of paths P_2 and isolated vertices therefore, it is an acyclic graph.

Case (vii). Consider the color classes of 2 and 5. The induced subgraph of the color classes of 2 and 5 is a collection of path $\{x_{n-1}\ v'_2\ x_{4n-1}\}$ and isolated vertices therefore, it is an acyclic graph.

Case (viii). Consider the color classes of 3 and 4. The induced subgraph of the color classes of 3 and 4 is a collection of path P_{n+1} therefore, it is an acyclic graph.

Case (ix). Consider the color classes of 3 and 5. The induced subgraph of the color classes of 3 and 5 is a collection of path $\{x_{n+1}\ x_1\ x_{n-1},\ x_{2n}\ x_{3n}\}$ and isolated vertices therefore, it is an acyclic graph.



$a(T[EDG(F_{6,2})]) = 5$

Figure 2

Case (x). Consider the color classes of 4 and 5. The induced subgraph of the color classes of 4 and 5 is a collection of path $\{x_{n+2}\ x_2\ x_{4n-1}x_{2n+1}x_{3n}\}$ and isolated vertices therefore, it is an acyclic graph.

Thus, the induced subgraph of any two color classes is acyclic and therefore the coloring given in the algorithm 2 is an acyclic coloring. Hence $a(T[EDG(F_{n,2})]) = 5,\ n \geq 2$.

Conclusion

In this paper, we obtained the acyclic chromatic number of Middle graph and Total graph of Extended Duplicate graph of Firecracker graph $(F_{n,2})$.

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