Advances and Applications in Mathematical Sciences
Volume 21, Issue 11, September 2022, Pages 6437-6454

AN EFFORT FOR CONTROLLING THE MOSAIC DISEASE OF JATROPHA CURCAS PLANT

ROSHMI DAS[1] and ASHIS KUMAR SARKAR[2]

Department of Mathematics
Jadavpur University Kolkata-700032, India
E-mail: aksarkar.jumath@gmail.com

Abstract

Jatropha Curcas is very much essential plant for ecological as well as environmental purpose. The seeds of the plant contain 37% of oil that can be used to obtain a better quality of biodiesel which is very useful as an alternative fuel. But such an important plant is affected by the Mosaic virus (Begomovirus) through the vector whitefly (Bemisia tabaci) which causes mosaic disease. In this paper we propose a model for the dynamics of this disease and its possible control via insecticide spraying. The result shows that the system possesses a steady state for some parameter values, Hopf bifurcation for some other parameter values and unstable condition for some other parameter values. Pontryagin minimum principle is applied to minimize the cost of spraying.

1. Introduction

The genus Jatropha of family Euphorbiaceae has more than 400 species distributed worldwide and among them Jatropha Curcas is recorded from India. It is commonly known as physic or purging nut. The seeds of this plant produce biodiesel which is an efficient substitute fuel for diesel engine. It is also an essential ingredient in various soap, dye and wood industries [6]. Jatropha Curcas is semi evergreen shrub or small tree with large green to pale green leaves.

It grows between (3-5) meter in height but grows upto (8-10) meter under favourable conditions. It is a multipurpose and drought resistant crop which is grown in marginal lands with lesser input. The tree can be grown in dry

2020 Mathematics Subject Classification: 92B05.

Keywords: Jatropha curcas plant, Whitefly (Bemisia tabaci), Mosaic virus (Begomovirus), Mosaic disease, Optimal control.

[1]Corresponding author; E-mail: dasroshmi02@gmail.com

Received September 17, 2021; Accepted January 12, 2022

and infertile conditions and cultivated in rough, sandy and salty soils. Fruits are produced in winter but can produce several crops during the year if the soil moisture is good and temperature is sufficiently high [8].

Begomovirus affects the Jatropha curcas plant by the vector whitefly which results mosaic disease. The symptoms that occur for this disease are severe mosaic, mottling, blistering of leaves, yellowing of leaves, reduced leaf size, stunting of the diseased plant. In this disease the mosaic virus passes from an infected whitefly to a susceptible plant and vice-versa [2][3]. Plant density is a key factor for the spreading of the virus. Normally the whitefly needs 3 hours feeding time to procure the virus and a latent phase of 8 years and also requires 10 minutes time to contaminate the young leaves [5][9]. Symptoms occur after 3-5 weeks. In this paper our objective is to understand the dynamics of the model regarding Jatropha Curcas plant and whitefly interaction and also the way of controlling such disease.

2. Statement of the Model

In our model the mosaic virus (Begomovirus) which is responsible for the disease is taken implicitly by considering its vector whiteflies. The vector can be controlled by removing infected plant biomass, spraying insecticide etc. Here growth of the plant is considered in logistic form and the attack pattern is taken as type-II function. Here 'x' denotes the Jatropha Curcas plant population and 'v' denotes the whitefly population. 'k' is the carrying capacity, 'r' is the growth rate of the plant.

The loss of whiteflies occur in the following ways.

d = natural mortality of whitefly.

e = natural mortality of the host plant.

f = by their killing the host plant.

Based on the above assumptions the model takes the form.

3. Model

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - \frac{cxv}{a + x}$$

$$\frac{dv}{dt} = v\left[\frac{cx}{a+x} - (e+d+f)\right] \tag{1}$$

with the initial conditions,

$$x(0) = x_0 > 0,\ v(0) = v_0 > 0$$

Here x_0 is the initial plant population density and v_0 is the initial whitefly density.

For mathematical simplicity we consider the following transformation,

$$x = aX,\ v = av,\ t = \frac{\tau}{r}.$$

The transformed equations are,

$$\frac{dx}{d\tau} = X(\alpha - X) - \frac{\beta XV}{1+X}$$

$$\frac{dV}{d\tau} = V\left[\frac{\beta X}{1+X} - \gamma\right] \tag{2}$$

where $\alpha = \frac{k}{a}$, $\beta = \frac{ck}{ra}$, $\gamma = \frac{(d+e+f)}{ra}$.

3.1 Solution properties

3.1.1 Lemma 1. *The solutions of* (2) *are positive.*

Proof. Since $x(0) = x_0 > 0$ and $v(0) = v_0 > 0$, we have $X(0) = X_0 > 0$ and $V(0) = V_0 > 0$. Suppose $X(\tau)$ is not positive for all $\tau \geq 0$. Since $X_0 > 0$ then there exist τ_0 with $X(\tau_0) = 0$ and $X(\tau) > 0$ for $0 \leq \tau \leq \tau_0$. For $X(\tau) > 0$

$$\frac{\dot{X}(\tau)}{X(\tau)} = \alpha - X - \frac{\beta V}{1+X} > -X - \frac{\beta V}{1+X}$$

$$X(\tau_0) > X_0 \exp\left[-\frac{\tau_0^2}{2} - \int_0^{\tau_0} \beta V/(1/X) d\tau\right] > 0$$

This is a contradiction and hence $X(\tau)$ is positive for all $\tau \geq 0$. Similarly it can be shown that $V(\tau)$ is also positive for all $\tau \geq 0$.

3.2 Equilibria. The equilibrium points are obtained by setting $\frac{dX}{d\tau} = 0$ and $\frac{dV}{d\tau} = 0$ and solving the equations $X(\alpha - X) - \frac{\beta XV}{1+X} = 0$ and $\frac{\beta XV}{1+X} - \gamma V = 0$.

We have seen that there are three equilibrium points i.e. $E_0(0, 0)$, $E_1(\alpha, 0)$ which is the whitefly free equilibrium, $E_2(X^*, V^*)$ which is the interior equilibrium. From the two equations we obtain

$$X^* = \frac{\gamma}{\beta - \gamma}$$

$$V^* = \frac{\alpha\beta - \alpha\gamma - \gamma}{(\beta - \gamma)^2}$$

Clearly $E_2(X^*, V^*)$ is feasible if $\alpha > \frac{\gamma}{\beta - \gamma} > 0$.

3.3 Stability. From the variational matrix we obtain the behavior of different equilibrium points of the system.

The equilibrium $E_0(0, 0)$ is saddle as its eigen values are α and-γ.

The eigen values of $E_1(\alpha, 0)$ are $-\alpha, \frac{\alpha\beta}{\alpha+1} - \gamma$

Therefore if $\frac{\alpha\beta}{1+\alpha} - \gamma > 0$ then $E_1(\alpha, 0)$ becomes saddle, in this case $E_2(X^*, V^*)$ exists. But if $\frac{\alpha\beta}{1+\alpha} - \gamma < 0$ then $E_1(\alpha, 0)$ becomes stable and in this case $E_2(X^*, V^*)$ does not exists. The characteristic equation for $E_2(X^*, V^*)$ is a quadratic equation which is as follows,

$$\lambda^2 + \lambda\left(-\alpha + 2X^* + \frac{\beta V^*}{(1+X^*)^2}\right) + \frac{\beta^2 X^* V^*}{(1+X)^3} = 0$$

which can be written as

$$\lambda^2 + A\lambda + B = 0$$

where,

$A = X^* \frac{\beta X^* V^*}{(1+X^*)^2}$ and $B = \frac{\beta^2 X^* V^*}{(1+X^*)^3} > 0$ A can be > 0 equal to 0 or < 0.

Therefore if $A > 0$ then $1 - \frac{\beta V^*}{(1+X^*)^2} > 0$ i.e. $\frac{\beta + \gamma}{\beta - \gamma} > \alpha$

If $A = 0$, then $\frac{\beta + \gamma}{\beta - \gamma} = \alpha$ and if $A < 0$, then $\frac{\beta + \gamma}{\beta - \gamma} < \alpha$

3.3.1 Theorem 1. *If* $\frac{\beta + \gamma}{\beta - \gamma} > \alpha$ *and* $\alpha > \frac{\gamma}{\beta - \gamma}$ *then the system* (2) *is globally asymptotically stable.*

Proof. If possible, let Γ be any periodic orbit around $E_2(X^*, V^*)$ in the positive XV-plane. Then,

$$\Delta = \int_\Gamma div(\dot{X}, \dot{V}) d\tau$$

$$= \int_\Gamma \left[\alpha - 2X - \frac{\beta V}{(1+X)^2} + \frac{\beta X}{(1+X)} - \gamma\right] d\tau$$

$$= \int_\Gamma \left[\frac{\beta V}{(1+X)} + X + \frac{\beta V}{(1+X)^2}\right] d\tau$$

$$= \int_\Gamma \left[\frac{\gamma(\alpha\beta - \alpha\gamma - \gamma - \gamma - \beta)}{\beta(\beta - \gamma)}\right] d\tau$$

Under the given assumption $E_2(X^*, V^*)$ is locally stable. Thus $\Delta < 0$. Then by Poincare criteria any periodic orbit Γ in the positive XV plane is stable, leads to a contradiction. Therefore, there is no periodic orbit around $E_2(X^*, V^*)$ in the positive XV plane and thus $E_2(X^*, V^*)$ is a global attractor. This completes the proof of the theorem.

3.3.2 Theorem 2. *If* $\frac{\beta + \gamma}{\beta - \gamma} = \alpha$ *and* $\alpha > \frac{\gamma}{\beta - \gamma}$ *then the system* (2) *leads* *to small amplitude Hopf bifurcating periodic solutions near* E_2.

Proof. To prove this theorem we have to satisfy all the conditions for Hopf bifurcation. If $\frac{\beta+\gamma}{\beta-\gamma}=\alpha$ then the two roots of the characteristic equation $\lambda^2+A\lambda+B=0$ are purely imaginary namely $\pm i\eta$;

where $\eta^2=\frac{\beta^2 XV}{4(1+X)^3}$.

The necessary and sufficient condition for Hopf bifurcation to occur is that there exist a $\alpha=\alpha^*$ such that

(i) $\frac{\beta+\gamma}{\beta-\gamma}=\alpha$ and

(ii) $\left.\frac{d(\text{Real}\lambda)}{d\alpha}\right|_{\alpha=\alpha^*}\neq 0$

Now we have

$$\left.\frac{d\left(-X+\frac{\beta VX}{(1+X)^2}\right)}{d\alpha}\right|_{\alpha=\frac{\beta+\gamma}{\beta-\gamma}}=\frac{\gamma}{\beta}\neq 0$$

Hence all the conditions for a Hopf bifurcation are satisfied. Then there exists small amplitude Hopf bifurcating periodic solutions near E_2. This completes the proof of the theorem.

3.3.3 Theorem 3. *If* $\frac{\beta+\gamma}{\beta-\gamma}<\alpha$ *and* $\alpha>\frac{\gamma}{\beta-\gamma}$ *then there exists at least one stable limit cycle around* $E_2(X^*, V^*)$ *in the positive XV plane.*

Proof. If possible, let Γ be any periodic orbit around E_2 in the positive XV plane. Then

$$=\int_\Gamma div(\dot{X},\dot{V})d\tau$$

$$= \int_{\Gamma} \left(\alpha - 2X - \frac{\beta V}{(1+X)^2} + \frac{\beta X}{(1+X)} - \gamma \right) d\tau$$

$$= \int_{\Gamma} \left(\frac{\gamma(\alpha\beta - \alpha\gamma - \gamma - \beta)}{\beta(\beta - \gamma)} \right) d\tau$$

So, we can conclude that $\Delta > 0$ if $\frac{\beta + \gamma}{\beta - \gamma} < \alpha.$ Hence by Poincare criteria any periodic orbit is stable. Therefore there exists at least one stable limit cycle around $E_2(X^*, V^*)$ in the positive XV plane.

4. Persistence and Permanence of the System

The idea of persistence was first came to the light by Freedman and Waltman. From the biological point of view persistence implies that all the populations are present and none of them will become extinct. Persistence and permanence are very useful to decide the questions of survival and extinction of n-species whose growth equations are governed by the differential equations

$$\dot{x}_i = x_i f_i(x_1, x_2, \ldots, x_n) \tag{3}$$

4.1 Some definitions

(1) The system is said to be weakly persistent if $\limsup x_i(t) > 0$ for all orbits in $\operatorname{int} \mathbb{R}_+^n$ and strongly persistent if $\liminf x_i(t) > 0.$

(2) The system is said to be permanent if there exists a compact set $B \subset \operatorname{int} \mathbb{R}_+^n$ such that all orbits in $\operatorname{int} \mathbb{R}_+^n$ end up in B.

(3) The system is uniformly persistence if there exist $\delta > 0$ such that for each compact set $x_i, \liminf x_i(t) \geq \delta > 0$ for all $(x_1(t), x_2(t), \ldots, x_n(t)) = X(t) \in \operatorname{int} \mathbb{R}_+^n.$

(4) An equilibrium fixed point x^* is said to be saturated equilibrium if $x_i^* = 0$ then $f_i(x_1^*, x_2^*, \ldots x_n^*) \leq 0.$

With the concept of saturated equilibria and by the method of average Lyapunov function we have the following theorem for permanent coexistence of both the species of the system [4].

4.2 Theorem. *The system is permanent iff* $\alpha > \frac{\gamma}{\beta - \gamma}$.

Proof. The index theorem states that the system with dissipativeness assumption has at least one saturated equilibrium. If all these saturated equilibria are regular, then the sum of their indices is $+1$. From the theorem 1 the system is dissipative and so there exists at least one saturated equilibrium and the sum of their indices is $+1$ if they are regular. The permanence of the system implies that none of the boundary fixed points are saturated. Hence the interior fixed point exists and must be saturated. Hence all the eigen values are negative or have negative real parts.

We now construct the average Lyapunov function to prove the sufficient condition. In our model, we consider the average Lyapunov function as $\sigma(X) = X^{r_1} \cdot V^{r_2}$ where $r_i > 0\, i = 1, 2$.

$$\text{Let, } \psi(X) = \frac{\dot{\sigma}(X)}{\sigma(X)}$$

$$= r_1 \frac{\dot{X}}{X} + r_2 \frac{\dot{V}}{V}$$

$$= r_1\left[\alpha - X - \frac{\beta V}{(1 + X)}\right] + r_2\left[\frac{\beta X}{(1 + X)} - \gamma\right]$$

If $\psi(X) > 0$ for the ω-limit sets of trajectories initiated in $\mathbb{R}^3_+$, then the trajectories more away from the boundary and the system (2) is permanent. It is evident that there is no periodic trajectory. Hence if there exist $r_1 > 0$ such that $\Psi(E_1) > 0$, then (2) is permanent.

Therefore for $E_0(0, 0)$, $\psi(X) = \alpha r_1 - \gamma r_2 > 0$

$$E_1(\alpha, 0),\ \psi(X) = \left(\frac{\alpha\beta}{1 + \alpha} - \gamma\right) r_2 > 0$$

The inequalities are evidently satisfied for at least one positive

$r = (r_1, r_2)$ if $\alpha > \frac{\gamma}{\beta - \gamma}$. Hence the system is uniformly persistent (or permanent) if $\alpha > \frac{\gamma}{\beta - \gamma}$.

This completes the proof of the theorem.

5. The Optimal Control Problem

We now reformulate the model as an optimal control problem to minimize the costs of insecticide spraying. The migration of infected whiteflies are not considered. Assuming that all the infected vectors are under the control of insecticide spraying. We now introduce the control variable $u(t)$ such that $0 \leq u(t) \leq 1$ defined on $[t_0, t_f]$ where t_0 and t_f are the starting and finishing time of control respectively [1] [7] [9].

Now the model takes the form,

$$\frac{dX}{d\tau} = X(\alpha - X) - (1 - u(t))\frac{\beta XV}{1 + X}$$

$$\frac{dV}{d\tau} = V\left[(1 - u(t))\frac{\beta X}{1 + X} - \gamma\right] \tag{4}$$

If we consider $u(t) = 0$ then there is no reduction in the contact rate between the infected whiteflies and the plants.

If we consider $u(t) = 1$ then there is no such contact rate between them. $u(t)$ plays the key role to express the reduction of contact rate between them by the spraying of insecticide.

We define the objective functional to minimize the cost of insecticide spraying as follows:

$$j(u(t)) = \int t_0^{t_f} [Pu^2 - QX^2] d\tau \text{ where } P \geq 0 \text{ and } Q \geq 0$$

Here the first term represents the costs of spraying insecticide and labor charge and the last term represents the extra revenues obtained by the larger population of healthy Jatropha Curcas plants.

Now we are going to find the optimal control.

5.1 Theorem. *The objective cost function* $J(u^*)$ *is minimum for the optimal control* u^* *corresponding to the interior equilibrium* $E_2(X^*, V^*)$ *and also there are adjoint variables* ξ_1, ξ_2 *satisfying the system of equations*,

$$\frac{d\xi_1}{dt} = 2QX - \xi_1\left[\alpha - 2X - (1 - u(t))\frac{\beta V}{(1 + X)^2}\right] - \xi_2\left[(1 - u(t))\frac{\beta V}{(1 + X)^2}\right]$$

$$\frac{d\xi_2}{dt} = \xi_1(1 - u(t))\frac{\beta X}{1 + X} - \xi_2\left[(1 - u(t))\frac{\beta X}{1 + X} - \gamma\right] \quad (5)$$

with the boundary condition $\xi_i(t_f) = 0\,(i = 1, 2)$. The optimal control can be given as,

$$u^*(t) = \max\left\{0, \min\left\{1, \frac{\beta XV(\xi_2 - \xi_1)}{1P(1 + X)}\right\}\right\} \quad (6)$$

Proof. Applying the Pontryagin Minimum Principle the optimal control variable $u^*(t)$ satisfies

$$\frac{\partial H}{\partial u^*(t)} = 0$$

Which implies

$$2Pu^* + \xi_1\frac{\beta XV}{1 + X} - \xi_2\frac{\beta XV}{1 + X} = 0$$

$$\Rightarrow u^* = \frac{(\xi_2 - \xi_1)\beta XV}{2P(1 + X)}$$

we first construct the Hamiltonian as follows:

$$H = Pu^2 - QX^2 + \xi_1\left[X(\alpha - X) - (1 - u(t))\frac{\beta XV}{1 + X}\right]$$

$$+ \xi_2\left[(1 - u(t))\frac{\beta XV}{1 + X} - \gamma V\right]$$

For the boundedness of the optimal control we have

$$u^*(t) = \begin{cases} 0 & \frac{\beta XV(\xi_2 - \xi_1)}{2P(1+X)} \le 0 \\ \frac{\beta XV(\xi_2 - \xi_1)}{2P(1+X)} & 0 < \frac{\beta XV(\xi_2 - \xi_1)}{2P(1+X)} < 1 \\ 1 & \frac{\beta XV(\xi_2 - \xi_1)}{2P(1+X)} \ge 1 \end{cases}$$

According to Pontryagin Minimum Principle adjoint variables satisfy the following equations:

$$\frac{d\xi_i}{dt} = \frac{\partial H}{\partial X_i} \tag{7}$$

where $i = 1, 2X_i = X, V$.

That is $X_1 = X, X_2 = V$ and the equations can be determined by using (7). This completes the proof of the theorem.

6. Numerical Simulation and Discussions

To verify the theoretical results numerical simulations have been carried out using MATLAB-2016a. Here we have used MATLAB routine ODE23. Distinctive permissible estimations of the system parameters have been taken to ensure our theoretical results.

Keeping in mind the feasibility criteria we have chosen the value of α by using the following conditions:

(1) $\frac{\beta + \gamma}{\beta - \gamma} > \alpha$

(2) $\frac{\beta + \gamma}{\beta - \gamma} = \alpha$

(3) $\frac{\beta + \gamma}{\beta - \gamma} < \alpha$

For the set of parameter values $\alpha = 2, \beta = 0.2, \gamma = 0.1$ satisfying the condition $\frac{\beta + \gamma}{\beta - \gamma} > \alpha$ the equilibrium point becomes (1, 10). The corresponding figure shows the local as well as global stability. Phase portrait is also represented by the same parameter values and it also reflects the same results.

For the set of parameter values $\alpha = 3, \beta = 0.2, \gamma = 0.1$ satisfying the condition $\frac{\beta + \gamma}{\beta - \gamma} < \alpha$ the equilibrium point becomes (1, 20). The corresponding figure shows the small amplitude Hopf bifurcation around the equilibrium point. Phase portrait also justify the same results.

Similarly for the set of parameter values of $\alpha = 3.5, \beta = 0.2, \gamma = 0.1$ satisfying the condition $\frac{\beta + \gamma}{\beta - \gamma} < \alpha$ the equilibrium point becomes (1, 25). The corresponding figure shows the large oscillation which leads to unstable condition. Phase portrait is also represented by the same parameter values and it also reflects the same results.

We used control theory by using a control parameter $u(t)$ in the basic model. It is observed that in the presence of control the growth of plants stabilized but the growth of infected whitefly declines.

In the realistic situation we observe the same phenomena.

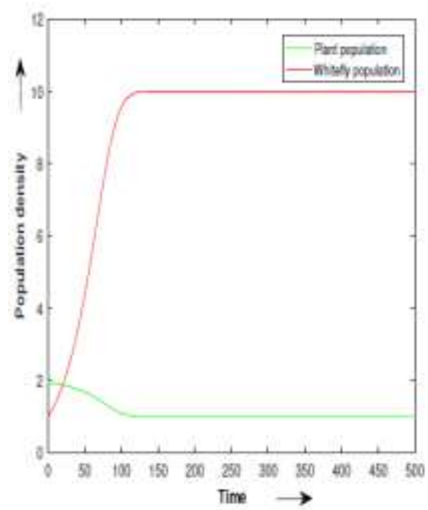

Figure 1. Variation of plant-herbivore densities with time for $\alpha = 2, \beta = 0.2, \gamma = 0.1$. Here we observe local stability for the population with increasing time.

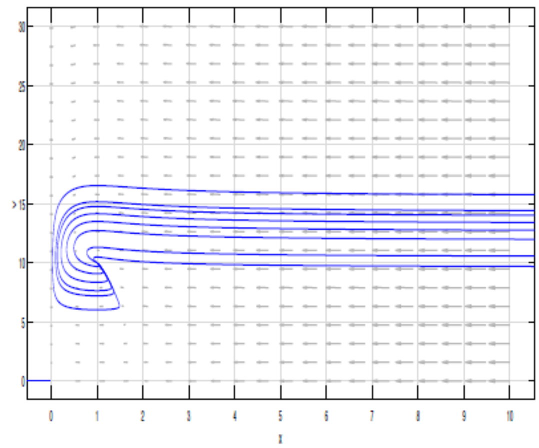

Figure 2. Variation of plant-herbivore densities $\alpha = 2, \beta = 0.2, \gamma = 0.1$. This shows the phase portrait in the XV plane which is globally asymptotically stable state of the model.

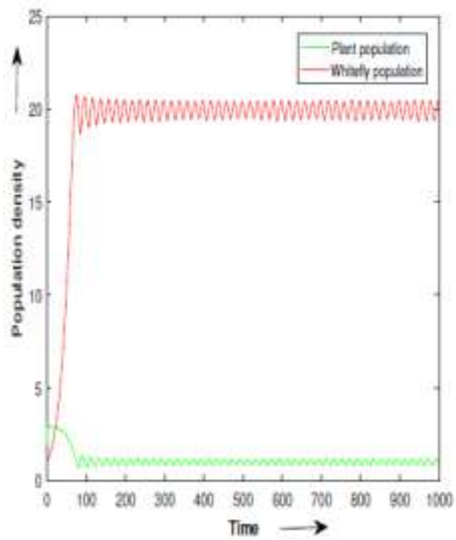

Figure 3. Small amplitude oscillation for both the population for the set of parameter values $\alpha = 3, \beta = 0.2, \gamma = 0.1$.

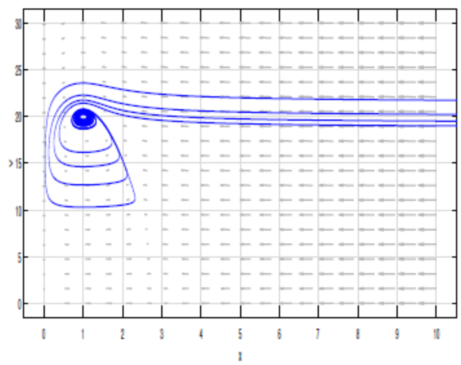

Figure 4. Variation of plant-herbivore densities in the model with $\alpha = 3, \beta = 0.2, \gamma = 0.1.$ This shows the phase portrait in the XV plane which shows Hopf bifurcation.

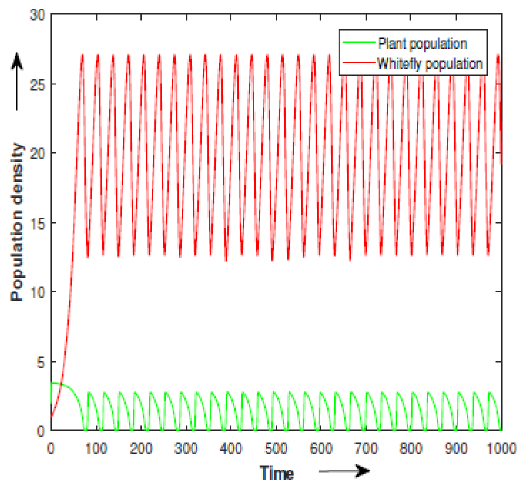

Figure 5. Large oscillations of both the population for the set of parameter values $\alpha = 3.5, \beta = 0.2, \gamma = 0.1.$ of model 2.

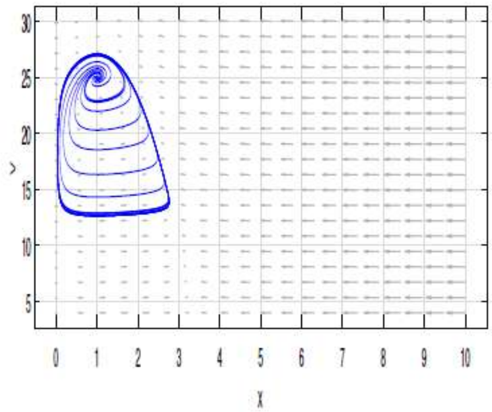

Figure 6. Limit cycle for the parameter values $\alpha = 3.5, \beta = 0.2, \gamma = 0.1$ of the model.

7. Conclusions

This paper deals with the interaction between the Jatropha curcas plant and the whitefly. Here we observe that depending upon the parameter values of α we get stable, unstable and bifurcating nature of the system. We also discussed about the persistence and permanence of the system. We have tried to control the mosaic disease using the pesticide. So we introduced a control parameter $u(t)$ on our basic model and observed that with the help of control the system becomes stabilized for all the pre-assumed parameter values. The results of insecticide spraying is also discussed in the numerical section. We have observed that spraying has a better effect on both the population.

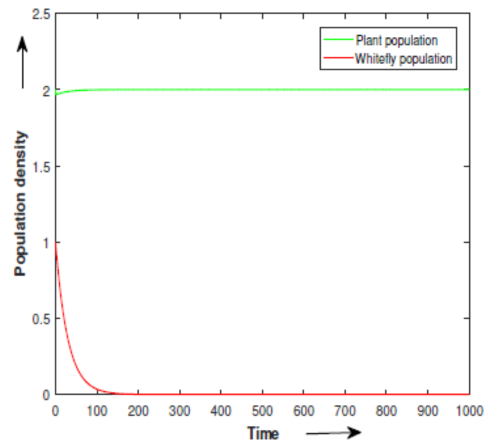

Figure 7. Effect of control on the stable state for the set of parameter values $\alpha = 2, \beta = 0.2, \gamma = 0.1$ of the model.

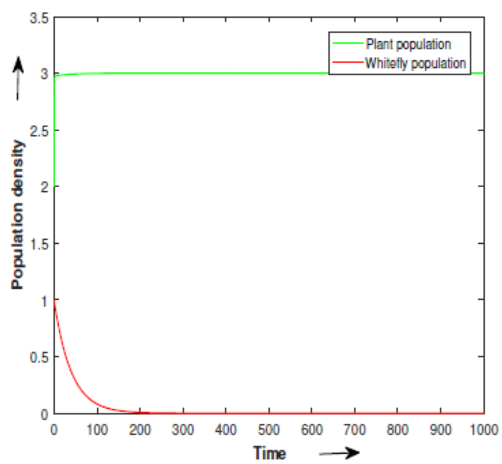

Figure 8. Effect of control on the hopf bifurcating parameter values $\alpha = 3, \beta = 0.2, \gamma = 0.1$.

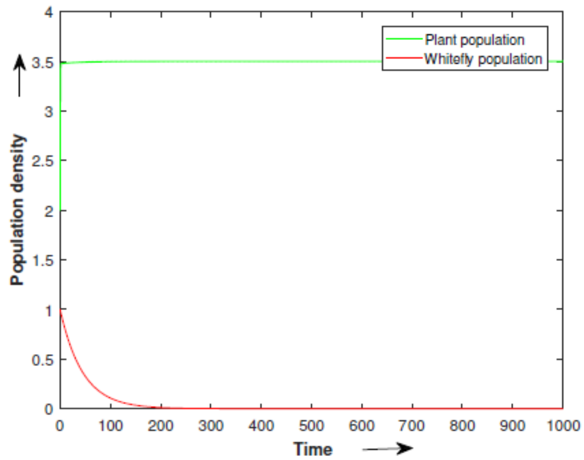

Figure 9. Effect of control on the limit cycle regarding parameter values $\alpha = 3.5$, $\beta = 0.2$, $\gamma = 0.1$.

Acknowledgments

We acknowledge with thanks the support of DST-PURSE, Phase-II program with ref. no. [P-1/RS/20/17 dated 16/20.02.2017] in the Department of Mathematics, Jadavpur University (India).

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