Advances and Applications in Mathematical Sciences
Volume 21, Issue 4, February 2022, Pages 1965-1973

MEAN SQUARE CORDIAL LABELING OF MULTIPLE SHELL AND SHELL FLOWER GRAPH

S. DHANALAKSHMI and N. PARVATHI

Department of Mathematics
Faculty of Engineering and Technology
SRM IST, Ramapuram
Chennai-600089, India

Department of Mathematics
Faculty of Engineering
and Technology SRM IST
Kattankulathur, Chennai -603203, India

Abstract

Graph theory plays an important role in a variety of real-world systems. Graph concepts such as labeling and colouring are used to depict a variety of processes and relationships in material, social, biological, physical, and information systems. In heterogeneous fields, graph labeling is utilised. Some applications, such as communication network addressing, fault tolerant system design, and automatic channel allocation, make use of this technology. For network designing, the structure of shell graph plays a prominent role which is taken here for discussion under one of the variations in cordial labeling. Cordial Labeling is used in Automated Routing algorithms, Communications-related Adhoc Networks and variety of other applications. These networks and routing paths are best represented by a complicated graph made up of vertices and edges. Mean square cordial labeling is one such variation in cordial labeling and this labeling technique is applied here for some classes of shell graphs. The design of shell topology is unaffected by attacks and failures to solve complicated problems in networks. Hence the study of mean square cordial labeling technique for such stable networks is of high importance in computer field. Here we investigated that multiple shell graph and shell-flower graph are mean square cordial graphs.

2020 Mathematics Subject Classification: 05C78.

Keywords: Mean square cordial labeling, Shell graph, Shell bow graph, Multiple shell, Shell-Flower graph.

*Corresponding author; E-mail: dhanalas1@srmist.edu.in

Received November 4, 2021; Accepted December 10, 2021

1. Introduction

Graph labeling is an interesting topic of graph theory with diversified applications. Rosa proposed the notion of graph labeling in 1967. Graph labeling is the process of assigning integers to edges, vertices, or both, based on specified criteria. Various graph labeling [1] has been researched in the literature over the last three decades (Gallian, 2010). Cordial graph labeling was introduced by Cahit [2] as a weaker form of both graceful graphs and harmonious graphs. Researchers explored their ideas of cordial labeling under many variations such as prime cordial, mean cordial, product cordial, divisor cordial etc. Cordial labeling of different types of shell graph were studied by A. Meena et al. [3]. Jeba et al. [4] produced some interesting results for duplication of prism graph in edge product cordial labeling. An intensive survey of prime and divisor cordial was carried out by A. Parthiban et al. [5]. One such variation of cordial labeling is mean square cordial labeling, introduced by A. Nellai Murugan. He analysed new results in the same labeling technique for some special graphs [6]. Dhanalakshmi et al. [7-11] examined the same labeling for some snake graphs, star related graphs, shell 3 graphs, cyclic and acyclic graphs. In this paper we investigated that multiple shell graph and shell-flower graph are mean square cordial graphs.

2. Preliminaries

Definition 1. A Mean Square Cordial Labeling of a Graph $G(V, E)$ with p vertices and q edges is a surjection from V to $\{0, 1\}$ such that each edge uv is assigned the label $(\lceil (f(u)^2 + f(v)^2)/2 \rceil)$ where $\lceil x \rceil$ ceil (x) is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1.

Definition 2. A shell graph $C(k, k-3)$ is the graph obtained by taking $(k-3)$ concurrent chords in a cycle C_k. Note that shell graph is same as the Fan graph $F_{n-1} = P_{n-1} + K_1$.

Definition 3. A multiple shell is defined to be a collection of edge disjoint shells that have common apex vertex. Hence a double shell consists of two edge disjoint shells with a common apex vertex.

Definition 4. A double shell consists of two edge disjoint shells with a common apex. A shell bow graph is defined as a double shell in which each shell has any order.

Definition 5. A shell-flower graph is formed by 'k' copies of the union of the shell $C(n, n-3)$ and K_2 where one end vertex of K_2 is joined to the apex of the shell. We denote this graph by $[C(n, n-3)\ U\ K_2]^k$ where the superscript k denotes the k copies of $[C(n, n-3)\ U\ K_2]$.

3. Main Results

Theorem 3.1. *Multiple shell admits mean square cordial labeling when* $n \neq 1(\text{mod } 2)$, $k \neq 0(\text{mod } 2)$.

Proof. Consider the multiple shell graph as G. Let $V(G) = \{u_0, v_i : i$ varies from 1 to $n-1\}$ and $E(G) = \{[(u_0 v_i) : i$ varies from 1 to $n-1] \cup [(v_i v_{i+1}) : i$ varies from 1 to $n-2]\}$ where u_0 is the apex vertex and v_i be the vertices of the cycles of a shell respectively (excludes the apex vertex).

Here $|V| = nk$ and $|E| = (2n-3)k$

Define f maps from $V(G)$ to $\{0, 1\}$

To prove G admits mean square cordial labelling, we discussed three cases namely (i) $n, k \equiv 0(\text{mod } 2)$ (ii) $n, k \equiv 1(\text{mod } 2)$ and (iii) even k and odd n.

Case 1. $n, k \equiv 0(\text{mod } 2)$

$f(u_0) = 0$

$f(v_i) = 0, 1 \leq i \leq \frac{(n-1)k}{2}$

$\qquad 1, \frac{(n-1)k}{2} + 1 \leq i \leq (n-1)k$

Hence the labeling of edges are

$$f(u_0v_i) = 0,\ 1 \le i \le \frac{(n-1)k}{2}$$

$$1,\ \frac{(n-1)k}{2} + 1 \le i \le (n-1)k$$

$$f(v_iv_{i+1}) = 0,\ 1 \le i \le n-2, \text{ for } k/2 \text{ shells}$$

$$1,\ 1 \le i \le n-2, \text{ for } (k/2)+1 \text{ to } k \text{ shells}$$

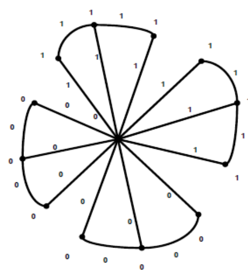

Figure 2. Multiple shell graph of $n = 4$ and $k = 4$.

Case 2. $n, k \equiv 1 (\mathrm{mod}\ 2)$

$$f(u_0) = 0$$

$$f(v_i) = 0,\ 1 \le i \le \frac{(n-1)k}{2}$$

$$1,\ \frac{(n-1)k}{2} + 1 \le i \le (n-1)k$$

Hence the labeling of edges are

$$f(u_0v_i) = 0,\ 1 \le i \le \frac{(n-1)k}{2}$$

$$1,\ \frac{(n-1)k}{2} + 1 \le i \le (n-1)k$$

$$f(v_iv_{i+1}) = 0,\ 1 \le i \le n-2, \text{ for } (k-1)/2 \text{ shells}$$

$$1,\ 1 \le i \le n-2, \text{ for } ((k-1)/2)+1 \text{ to } k-1 \text{ shells}$$

For the k^{th} shell,

$$f(v_i v_{i+1}) = 0,\ 1 \le i \le \frac{n-1}{2}$$

$$1,\ \frac{n+1}{2} \le i \le n-2$$

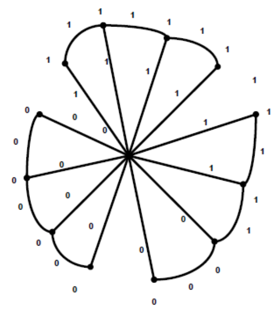

Figure 3. Multiple shell graph of $n = 3$ and $k = 3$.

Case 3. $n \equiv 1 (\bmod 2),\ k \equiv 0 (\bmod 2)$

$$f(u_0) = 0$$

$$f(v_i) = 0,\ 1 \le i \le \frac{(n-1)k}{2}$$

$$1,\ \frac{(n-1)k}{2} + 1 \le i \le (n-1)k$$

Hence the labeling of edges are

$$f(u_0 v_i) = 0,\ 1 \le i \le \frac{(n-1)k}{2}$$

$$1,\ \frac{(n-1)k}{2} + 1 \le i \le (n-1)k$$

Above vertex and edge labeling pattern satisfies the mean square cordial labeling of graph G.

Theorem 3.2. *Shell flower graph* $[C_{n \cdot n-3}, k_2]^k$ *admits mean square cordial labeling when* $n, k \ne 1 (\bmod 2)$.

Proof. Consider the shell-flower graph as G. Let $V(G) = \{u_0, u_j : i$ varies from 1 to k, $v_i : i$ varies from 1 to $n-1\}$ and $E(G) = \{[(u_0 u_j) : j$

varies from 1 to $k] \cup [(u_0 v_i) : i$ varies from 1 to $n-1] \cup [(v_i v_{i+1}) : i$ varies from 1 to $n-1]\}$ where u_0 is the apex vertex, u_j be the pendant vertex of k_2 and v_i be the vertices of the cycles of a shell respectively (excludes the apex vertex).

Here $|V| = (2n-2)k$ and $|E| = nk + k$

Define f maps from $V(G)$ to $\{0, 1\}$

To prove G admits mean square cordial labelling, we discussed three cases namely (i) n and k are even, (ii) even n and odd k and (iii) even k and odd n.

Case 1. $n, k \equiv 0 (\bmod 2)$

$$f(u_0) = 0$$

$$f(u_j) = 0, 1 \leq j \leq \frac{k}{2}$$

$$1, \frac{k}{2} + 1 \leq j \leq k$$

$$f(v_i) = 0, 1 \leq i \leq \frac{(n-1)k}{2} - 1$$

$$1, \frac{(n-1)k}{2} \leq i \leq (n-1)k$$

Hence the labeling of edges are

$$f(u_0 u_j) = 0, 1 \leq i \leq \frac{k}{2}$$

$$1, \frac{k}{2} + 1 \leq j \leq k$$

$$f(u_0 v_i) = 0, 1 \leq i \leq \frac{(n-1)k}{2}$$

$$1, \frac{(n-1)k}{2} + 1 \leq i \leq (n-1)k$$

$$f(v_i v_{i+1}) = 0, 1 \leq i \leq n-2, \text{ for } k/2 \text{ shells}$$

$1, 1 \leq i \leq n-2,$ for $(k/2)+1$ to k shells

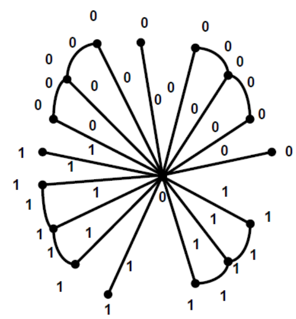

Figure 4. Shell flower graph of $n = 4$ and $k = 4$.

Case 2. $n \equiv 0 (\text{mod } 2)$, $k \equiv 1 (\text{mod } 2)$

$$f(u_0) = 0$$

$$f(u_j) = 0, 1 \leq j \leq \frac{k}{2} - 1$$

$$1, \frac{k+1}{2} + 1 \leq j \leq k$$

$$f(v_i) = 0, 1 \leq i \leq \frac{(n-1)k}{2} - 1$$

$$1, \frac{(n-1)k}{2} \leq i \leq (n-1)k$$

Hence the labeling of edges are

$$f(u_0 u_j) = 0, 1 \leq j \leq \frac{k-1}{2}$$

$$1, \frac{k+1}{2} + 1 \leq j \leq k$$

$$f(v_i) = 0, 1 \leq i \leq \frac{(n-1)k}{2} - 1$$

$$1, \frac{(n-1)k}{2} \leq i \leq (n-1)k$$

$$f(v_i v_{i+1}) = 0, 1 \leq i \leq n-2, \text{ for } (k-1)/2 \text{ shells}$$

$1, 1 \leq i \leq n-2,$ for $((k-1)/2)+1$ to $k-1$ shells

For the k^{th} shell,

$$f(v_i v_{i+1}) = 0, 1 \leq i \leq \frac{n-1}{2}$$

$$1, \frac{n+1}{2} \leq i \leq n-2$$

Case 3. $n \equiv 1 (\operatorname{mod} 2)$, $k \equiv 0 (\operatorname{mod} 2)$

$$f(u_0) = 0$$

$$f(u_j) = 0, 1 \leq j \leq \frac{k}{2}$$

$$1, \frac{k}{2}+1 \leq j \leq k$$

$$f(v_i) = 0, 1 \leq i \leq \frac{(n-1)k}{2}$$

$$1, \frac{(n-1)k}{2}+1 \leq i \leq (n-1)k$$

Hence the labeling of edges are

$$f(u_0 u_j) = 0, 1 \leq j \leq \frac{k}{2}$$

$$1, \frac{k}{2}+1 \leq j \leq k$$

$$f(u_0 v_i) = 0, 1 \leq i \leq \frac{(n-1)k}{2}$$

$$1, \frac{(n-1)k}{2}+1 \leq i \leq (n-1)k$$

$f(v_i v_{i+1}) = 0, 1 \leq i \leq n-2,$ for $k/2$ shells

$1, 1 \leq i \leq n-2,$ for $(k/2)+1$ to k shells

Above vertex and edge labeling pattern satisfies the mean square cordial labeling of graph G.

4. Conclusion

This research paper looked at the mean square cordial labeling of multiple shell graphs and shell flower graphs. The authors stated that the similar labeling trend may be observed in some more graphs. It will be quite fascinating for the researchers to investigate this labeling technique for a few additional kinds of graphs in their future work.

References

[1] G. J. Gallian, A dynamic survey of graph labeling, The electronic journal of combinotorics 16 (2010), #DS6.

[2] I. Cahit, Corrdial graphs: A weaker version of graceful and harmonious graphs, Ars. Combin. 23 (1987), 201-207. file:///C:/Users/Lenovo /Downloads/Cordial_graphs_a_weaker_version _of_grace.pdf

[3] J. Jeba Jesintha, D. Devakirubanithi and L. Monisha, Edge product cordial labeling on duplication of prism graph, Bull. Pure Appl. Sci. Sect. E. Math. Stat. 39E(2) (2020), 265-270.

[4] A. Parthiban and Vishally Sharma, A comprehensive survey on prime cordial and divisor cordial labeling of graphs Journal of Physics, Conference Series 1531 (2020).

[5] S. Meena, M. Renugha and M. Sivasakthi, Cordial labeling for different types of shell graph, International Journal of Scientific and Engineering Research 6(9) (2015).

[6] A. Nellai Murugan and S. Heerajohn Special class of mean square cordial graphs, International Journal of Applied Research 1(11) (2015), 128-131.

[7] S. Dhanalakshmi and N. Parvathi, Mean square cordial labeling related to some acyclic graphics and its rough approximations, Journal of physics: Conf series 1000 (2018) 012040. doi:10.1088/1742-6596/1000/1/01204

[8] S. Dhanalakshmi and N. Parvathi, Mean square cordial labeling related to some cyclic graphs, Indian Journal of Engineering and Technology 7(4.10) (2018), 797-800. file:///C:/Users/Lenovo/Downloads/Mean_Square_Cordial_Labeling_of_Some_Cycle_Related%20(2).pdf

[9] S. Dhanalakshmi and N. Parvathi, Mean square cordial labeling of some star related graphs, Journal of physics: Conf series 1000 (2018), 012040 doi:10.1088/1742-6596/1377/1/012027

[10] S. Dhanalakshmi and N. Parvathi, Mean square cordial labeling of some snake graphs, International Journal of Innovative Technology and Exploring Engineering 8(9S3) (2019). https://www.ijitee.org/wpcontent/uploads/papers/v8i9S3/I30900789S319.pdf

[11] S. Dhanalakshmi, Mean square cordial labeling of a Shell-butterfly graphs, International Journal of Mathematics and Computer Science 15(4) (2020), 1235-1242. http://ijmcs.future-in-tech.net