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OPEN SUPPORT INDEPENDENCE NUMBER OF SOME STANDARD GRAPHS UNDER ADDITION AND MULTIPLICATION

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Abstract

In this paper, open support independence number of a set and open support independence number of a graph under addition and multiplication are introduced. Open support independence number of the path, star, complete graph, wheel, cycle, complete bipartite graph and bistar under addition and multiplication are studied.

I. Introduction

Graphs considered in this paper are finite, undirected and without loops or multiple edges. Let $G = (V, E)$ be a graph with p vertices and q edges. Terms not defined here are used in the sense of Harary [4]. For each vertex

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$v \in V$, the open neighborhood of v is the set $N(v)$ containing all the vertices u adjacent to v [6]. The degree of a vertex $v \in (G)$ is the number of edges of G incident with v and is denoted by $\deg_G(v)$ or $\deg v$. In a graph G, an independent set is a subset S of $V(G)$ such that no two vertices in S are adjacent. A maximum independent set is an independent set of maximum size [5].

Recently the concept of open support of a graph under addition was introduced by Balamurugan et al. [1] and further studied in [2]. Open support of a graph under multiplication was introduced in [3]. Motivated by these definitions, the concept of open support independence number of a graph under addition and multiplication are introduced.

In this paper, open support independence number of a set under addition, open support independence number of a graph G under addition, open support independence number of a set under multiplication and open support independence number of a graph G under multiplication are introduced. Open support independence number of some standard graphs under addition and multiplication are studied. The following definitions are necessary for the present study.

Definition 1.1. Let $G = (V, E)$ be a graph. A subset S of V is called an independent set of G of no two vertices in S are adjacent in G.

Definition 1.2. An independent set 'S' is maximum in G if G has no independent set S' with $|S'| > |S|$.

Definition 1.3. The number of vertices in a maximum independent set of G is called the independence number of G and is denoted by $\alpha(G)$.

Definition 1.4 [1]**.** Let $G = (V, E)$ be a graph. An open support of a vertex v under addition is defined by $\sum_{u \in N(v)} \deg u$ and is denoted by supp (v).

Definition 1.5 [1]**.** Let $G = (V, E)$ be a graph. An open support of the graph G under addition is defined by $\sum_{u \in V(G)} \operatorname{supp}(v)$ and is denoted by supp (G).

Definition 1.6 [3]**.** Let $G = (V, E)$ be a graph. An open support of a vertex v under multiplication is defined by $\prod_{u \in N(v)} \deg u$ and is denoted by $mult(v)$.

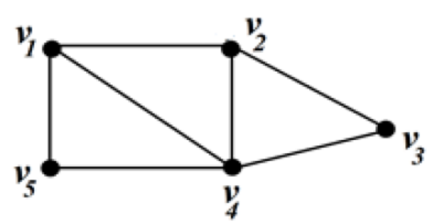
Definition 1.7 [3]**.** Let $G = (V, E)$ be a graph. An open support of the graph G under multiplication is defined by $\prod_{u \in V(G)} mult(v)$ and is denoted by $mult(G)$.

II. Main Results

Definition 2.1. Let $G = (V, E)$ be a graph. Let S denote the maximum independent set of G. Open support independence number of the set S under addition, denoted by $\operatorname{supp} S^{+}(G)$, is defined by $\operatorname{supp} S^{+}(G) = \sum_{v \in S} \operatorname{supp}(v)$. Open support independence number of G under addition, denoted by $\operatorname{supp} \alpha^{+}(G)$, is defined by $\operatorname{supp} \alpha^{+}(G) = \max\{\operatorname{supp} S_i^{+}(G); i \geq 1\}$.

Definition 2.2. Let $G = (V, E)$ be a graph. Let S denote the maximum independent set of G. Open support independence number of the set S under multiplication, denoted by $\operatorname{supp} S^{\times}(G)$, is defined by $\operatorname{supp} S^{\times}(G) = \prod_{v \in S} mult(v)$. Open support independence number of G under multiplication, denoted by $\operatorname{supp} \alpha^{\times}(G)$ is defined by $\operatorname{supp} \alpha^{\times}(G) = \max\{mult\, S_i^{\times}(G); i \geq 1\}$.

Example 2.3. Consider the following graph G.



In G, $\deg v_1 = 3$, $\deg v_2 = 3$, $\deg v_3 = 2$, $\deg v_4 = 4$ and $\deg v_5 = 2$.

Maximum independent set of G are $\{S_1, S_2, S_3\}$, where $S_1 = \{v_1, v_3\}$, $S_2 = \{v_2, v_5\}$ and $S_3 = \{v_4, v_5\}$.

Open support independence number of G under addition. Consider the set S_1

$$\operatorname{supp}(v_1) = \sum_{u \in N(v_1)} \deg u = \deg v_2 + \deg v_4 + \deg v_5 = 9$$

$$\operatorname{supp}(v_3) = \sum_{u \in N(v_3)} \deg u = \deg v_2 + \deg v_4 = 7$$

Hence $\operatorname{supp} S_1^+(G) = \sum_{v \in S_1} \operatorname{supp}(v) = 16$

Consider the set S_2

$$\operatorname{supp}(v_2) = \sum_{u \in N(v_2)} \deg u = \deg v_1 + \deg v_3 + \deg v_4 = 9$$

$$\operatorname{supp}(v_5) = \sum_{u \in N(v_5)} \deg u = \deg v_1 + \deg v_4 = 7$$

Hence $\operatorname{supp} S_2^+(G) = \sum_{v \in S_2} \operatorname{supp}(v) = 16$

Consider the set S_3

$$\operatorname{supp}(v_3) = \sum_{u \in N(v_3)} \deg u = \deg v_2 + \deg v_4 = 7$$

$$\operatorname{supp}(v_5) = \sum_{u \in N(v_5)} \deg u = \deg v_1 + \deg v_4 = 7$$

Hence $\operatorname{supp} S_3^+(G) = \sum_{v \in S_3} \operatorname{supp}(v) = 14$

Therefore $\operatorname{supp} \alpha^+(G) = \max\{\operatorname{supp} S_1^+(G), \operatorname{supp} S_2^+(G), \operatorname{supp} S_3^+(G)\}$

$$= 16.$$

Open support independence number of G under multiplication.

Consider the set S_1

$$mult(v_1) = \prod_{u \in N(v_1)} \deg u = \deg v_2 \times \deg v_4 \times \deg v_5 = 24$$

$$mult(v_3) = \prod_{u \in N(v_3)} \deg u = \deg v_2 \times \deg v_4 = 12$$

Hence $\operatorname{supp} S_1^{\times}(G) = \prod_{v \in S_1} mult(v) = 288$

Consider the set S_2

$$mult(v_2) = \prod_{u \in N(v_2)} \deg u = \deg v_1 \times \deg v_3 \times \deg v_4 = 24$$

$$mult(v_5) = \prod_{u \in N(v_5)} \deg u = \deg v_1 \times \deg v_4 = 12$$

Hence $\operatorname{supp} S_2^{\times}(G) = \prod_{v \in S_2} mult(v) = 288$

Consider the set S_3

$$mult(v_3) = \prod_{u \in N(v_1)} \deg u = \deg v_2 \times \deg v_4 = 12$$

$$mult(v_5) = \prod_{u \in N(v_5)} \deg u = \deg v_1 \times \deg v_4 = 12$$

Hence $\operatorname{supp} S_3^{\times}(G) = \prod_{v \in S_3} mult(v) = 144$

Therefore $\operatorname{supp} \alpha^{\times}(G) = \max \{\operatorname{supp} S_1^{\times}(G), \operatorname{supp} S_2^{\times}(G), \operatorname{supp} S_3^{\times}(G)\}$

$$= 288$$

Theorem 2.4. *Let $G = K_{1,n}$ where $n \geq 1$ be a star. Then $\operatorname{supp} \alpha^{+}(G) = n^2$ and $\operatorname{supp} \alpha^{\times}(G) = n^n$.*

Proof. Let $G = K_{1,n}$ where $n \geq 1$. Let $v, v_1, v_2, \ldots, v_n$ be the vertices of G where v is the central vertex and $v_1, v_2, \ldots, v_n$ are the pendant vertices. Then $\deg v = n$ and $\deg v_i = 1; 1 \leq i \leq n$. $S = \{v_1, v_2, \ldots, v_n\}$ is the unique maximum independent set of G.

$$\text{supp}(v_1) = \sum_{u \in N(v_1)} \deg u = \deg v = n$$

Similarly $\text{supp}(v_2) = \text{supp}(v_3) = \ldots = \text{supp}(v_n) = n.$

Hence $\text{supp}\, \alpha^{+}(G) = \sum_{u \in S} \text{supp}(u)$

$= n(n)$

$= n^2.$

Similarly $\text{supp}\, \alpha^{\times}(G) = \prod_{u \in S} mult(u)$

$= n^n.$

Theorem 2.5. *Let $G = P_n$ where $n > 2$ be a path on n vertices. Then*

$$\text{supp}\, \alpha^{+}(G) = \begin{cases} 2n-2 & \text{if } n \text{ is odd} \\ 2n-3 & \text{if } n \text{ is even} \end{cases} \text{ and } \text{supp}\, \alpha^{\times}(G) = \begin{cases} 2^{n-1} & \text{if } n \text{ is odd} \\ 2^{n-2} & \text{if } n \text{ is even} \end{cases}$$

Proof. Let $G = P_n$ where $n > 2$. Let $v_1, v_2, \ldots, v_n$ be the vertices of G.

Then $\deg v_1 = \deg v_n = 1$ and $\deg v_i = 2$ for all $i = 2, 3, \ldots, n-1$.

Case (i). Suppose n is odd.

$S = \{v_1, v_3, v_5, \ldots, v_n\}$ is the unique maximum independent set of G.

$$\text{supp}(v_1) = \sum_{v \in N(v_1)} \deg v = \deg v_2 = 2$$

Similarly $\text{supp}(v_n) = 2$

$$\text{supp}(v_3) = \sum_{v \in N(v_3)} \deg v = \deg v_2 + \deg v_4 = 4$$

Similarly $\text{supp}(v_5) = \text{supp}(v_7) = \ldots = \text{supp}(v_{n-2}) = 4$

Hence $\text{supp}\, \alpha^{+}(G) = \sum_{v \in S} \text{supp}(v)$

$$= 2 + 4\left(\frac{n-3}{2}\right) + 2$$

$$= 2n - 2$$

Similarly $\operatorname{supp} \alpha^{\times}(G) = \prod_{u \in S} mult(u)$

$$= 2 \times 4\left(\frac{n-3}{2}\right) \times 2$$

$$= 2^{n-1}.$$

Case (ii). Suppose n is even. There are two subcases.

Subcase (a). In this case, we consider the two maximum independent sets with only one pendant vertex.

Without loss of generality let $S_1 = \{v_1, v_3, v_5, \ldots, v_{n-1}\}$.

Proof is similar for the other set.

$$\operatorname{supp}(v_1) = \sum_{v \in N(v_1)} \deg v = \deg v_2 = 2$$

$$\operatorname{supp}(v_3) = \sum_{v \in N(v_3)} \deg v = \deg v_2 + \deg v_4 = 4$$

Similarly $\operatorname{supp}(v_5) = \operatorname{supp}(v_7) = \ldots = \operatorname{supp}(v_{n-3}) = 4$

$$\operatorname{supp}(v_{n-1}) = \sum_{v \in N(v_{n-1})} \deg v = \deg v_{n-2} + \deg v_n = 3$$

Hence $\operatorname{supp} \alpha^{+}(G) = \sum_{v \in S} \operatorname{supp}(v)$

$$= 2 + 4\left(\frac{n-4}{2}\right) + 3$$

$$= 2n - 3$$

Similarly $\operatorname{supp} \alpha^{\times}(G) = \prod_{u \in S_1} mult(u)$

$$mult\,(v_{n-1}) = \prod\nolimits_{u \in N(v_{n-1})} \deg u = \deg v_{n-2} \times \deg v_n = 2$$

$$= 2 \times 4^{\left(\frac{n-4}{2}\right)} \times 2$$

$$= 2^{n-2}$$

Subcase (b). In this case, we consider all the maximum independent sets with both the pendant vertices. Without loss of generality let $S_2 = \{v_1, v_3, v_5, \ldots, v_{n-3}, v_n\}$. Proof is similar for the other set. As before, $\operatorname{supp}(v_1) = \operatorname{supp}(v_n) = 2$ and $\operatorname{supp}(v_3) = \operatorname{supp}(v_5) = \ldots = \operatorname{supp}(v_{n-3}) = 4$.

Hence $\operatorname{supp} S_2^{+}(G) = \sum_{v \in S_2} \operatorname{supp}(v)$

$$= 2 + 4\left(\frac{n-4}{2}\right) + 2$$

$$= 2n - 4$$

Therefore $\operatorname{supp} \alpha^{+}(G) = \max\{\operatorname{supp} S_i^{+}(G); i \geq 1\}$

$$\operatorname{supp} \alpha^{+}(G) = \max\{\operatorname{supp} S_1^{+}(G), \operatorname{supp} S_2^{+}(G)\}$$

$$= \max\{2n - 3, 2n - 4\}$$

$$= 2n - 3$$

Similarly $\operatorname{supp} S_2^{+}(G) = \prod_{u \in S_2} mult\,(u)$

$$= 2 \times 4^{\left(\frac{n-4}{2}\right)} \times 2$$

$$= 2^{n-2}$$

Therefore $\operatorname{supp} \alpha^{\times}(G) = 2^{n-2}$.

Theorem 2.6. *Let $G = C_n$ where $n \geq 3$ be a cycle on n vertices. Then*

$$\operatorname{supp} \alpha^{+}(G) = \begin{cases} 2n - 2 & \textit{if } n \textit{ is odd} \\ 2n & \textit{if } n \textit{ is even} \end{cases} \textit{ and } \operatorname{supp} \alpha^{\times}(G) = \begin{cases} 2^{n-1} & \textit{if } n \textit{ is odd} \\ 2^{n} & \textit{if } n \textit{ is even} \end{cases}.$$

Proof. Let $G = C_n$ where $n \geq 3$. Let $v_1, v_2, \ldots, v_n$ be the vertices of G.

Then $\deg v_i = 2$ for all $i = 1, 2, 3, \ldots, n$.

Case (i). Let n be odd.

Consider the maximum independent set $S = \{v_1, v_3, v_5, \ldots, v_{n-2}\}$.

Proof is similar for the other set.

$$\operatorname{supp}(v_1) = \sum_{v \in N(v_1)} \deg v = \deg v_2 + \deg v_n = 4$$

Similarly $\operatorname{supp}(v_3) = \operatorname{supp}(v_5) = \ldots = \operatorname{supp}(v_{n-2}) = 4$

Hence $\operatorname{supp} \alpha^{+}(G) = 4\left(\frac{n-1}{2}\right)$

$= 2n - 2$

Similarly $\operatorname{supp} \alpha^{\times}(G) = \prod_{v \in S} mult(v)$

$= 2^{n-1}$.

Case (ii). Let n be even.

$S_1 = \{v_1, v_3, v_5, \ldots, v_{n-1}\}$ and $S_2 = \{v_2, v_4, v_6, \ldots, v_n\}$ are two maximum independent sets of G. Consider the set S_1.

Proof is similar for the set S_2.

As before $\operatorname{supp}(v_1) = \operatorname{supp}(v_3) = \operatorname{supp}(v_5) = \ldots = \operatorname{supp}(v_{n-1}) = 4$

Hence $\operatorname{supp} \alpha^{+}(G) = 4\left(\frac{n}{2}\right)$

$= 2n$.

Similarly $\operatorname{supp} \alpha^{\times}(G) = \prod_{v \in S_1} mult(v)$

$= 2^{n}$.

Theorem 2.7. *Let $G = K_n$ where $n \geq 1$ be the complete graph on n*

vertices. Then $\operatorname{supp} \alpha^{+}(G) = (n-1)^2$ *and* $\operatorname{supp} \alpha^{\times}(G) = (n-1)^{n-1}$.

Proof. Let $G = K_n$ where $n \geq 1$. Let $v_1, v_2, \ldots, v_n$ be the vertices of G. Then $\deg v_i = n-1$ for all $i = 1, 2, 3, \ldots, n$. $S_i = \{v_i\}; 1 \leq i \leq n$ are maximum independent sets of G.

Consider the set S_1. Proof is similar for the other sets.

$$\begin{aligned} \operatorname{supp}(v_1) &= \sum_{v \in N(v_1)} \deg v = \deg v_2 + \deg v_3 + \ldots + \deg v_n \\ &= (n-1)(n-1) \\ &= (n-1)^2 \end{aligned}$$

Hence $\operatorname{supp} \alpha^{+}(G) = (n-1)^2$.

Similarly $\operatorname{supp} \alpha^{\times}(G) = \prod_{v \in S_1} mult(v)$

$$= (n-1)^{n-1}.$$

Theorem 2.8. *Let* $G = K_{m,n}$ *where* $m \geq n,\ m, n \geq 1$ *be a complete bipartite graph. Then* $\operatorname{supp} \alpha^{+}(G) = m^2 n$ *and* $\operatorname{supp} \alpha^{\times}(G) = m^{mn}$.

Proof. Let $G = K_{m,n}$ where $m \geq n$ be a complete bipartite graph with the bipartition (X, Y) where $X = \{x_1, x_2, \ldots, x_m\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$. Then $\deg x_i = n; 1 \leq i \leq m$, and $\deg y_i = m; 1 \leq i \leq n$. $S = \{x_1, x_2, \ldots, x_m\}$ is the unique maximum independent set of G.

$$\begin{aligned} \operatorname{supp}(x_1) &= \sum_{u \in N(x_1)} \deg u \\ &= \sum_{j=1}^{n} \deg y_j \\ &= mn \end{aligned}$$

Similarly $\operatorname{supp}(x_2) = \operatorname{supp}(x_3) = \ldots = \operatorname{supp}(x_n) = mn$.

Hence $\operatorname{supp} \alpha^{+}(G) = \sum_{u \in S} \operatorname{supp}(u)$

$$= m(mn) = m^2 n.$$

$$mult(x_1) = \prod_{u \in N(x_1)} \deg u = \deg y_1 \times \deg y_2 \times \ldots \times \deg y_n$$

$$= m \times m \times \ldots n \text{ times} = m^n$$

Similarly $mult(x_2) = mult(x_3) = \ldots = mult(x_m) = m^n$

Therefore $\operatorname{supp} \alpha^{\times}(G) = \prod_{v \in S} mult(v)$

$$= m^n \times m^n \times \ldots m \text{ times} = m^{mn}.$$

Theorem 2.9. *Let $G = W_n$ where $n \geq 4$ be a wheel of order n. Then*

$$\operatorname{supp} \alpha^{+}(G) = \begin{cases} \left(\frac{n-1}{2}\right)(n+5) & \text{if } n \text{ is odd} \\ \left(\frac{n-2}{2}\right)(n+5) & \text{if } n \text{ is even} \end{cases} \quad \text{and}$$

$$\operatorname{supp} \alpha^{\times}(G) = \begin{cases} [9(n-1)]^{\frac{n-1}{2}} & \text{if } n \text{ is odd} \\ [9(n-1)]^{\frac{n-2}{2}} & \text{if } n \text{ is even} \end{cases}$$

Proof. Let $G = W_n$ where $n \geq 4$. Let $v_0, v_1, v_2, \ldots, v_{n-1}$ be the vertices of G, where v_0 is the central vertex. Then $\deg v_0 = n-1$, $\deg v_i = 3$ for all $i = 1, 2, 3, \ldots, n-1$.

Case (i). Let n be odd.

$\{v_1, v_3, v_5, \ldots, v_{n-2}\}$ and $\{v_2, v_4, v_6, \ldots, v_{n-1}\}$ are the two maximum independent sets of G. Consider the set $S_1 = \{v_1, v_3, v_5, \ldots, v_{n-2}\}$. Proof is similar for the other set.

$$\operatorname{supp}(v_1) = \sum_{v \in N(v_1)} \deg v = \deg v_2 + \deg v_{n-1} + \deg v_0 = n + 5$$

Similarly $\operatorname{supp}(v_3) = \operatorname{supp}(v_5) = \ldots = \operatorname{supp}(v_{n-2}) = n + 5$

Hence $\operatorname{supp}\alpha^{+}(G) = \sum_{v \in S_1} \operatorname{supp}(v)$

$$= \left(\frac{n-1}{2}\right)(n+5)$$

$$mult(v_1) = \prod_{v \in N(v_1)} \deg v = \deg v_2 \times \deg v_{n-1} \times \deg v_0$$

$$= 3 \times 3 \times (n-1) = 9(n-1)$$

Similarly $mult(v_3) = mult(v_5) = \ldots = mult(v_{n-2}) = 9(n-1)$.

$$\operatorname{supp}\alpha^{\times}(G) = \prod_{v \in S_1} mult(v)$$

$$= [9(n-1)]^{\frac{n-1}{2}}$$

Case (ii). Let n be even.

$\{v_1, v_3, v_5, \ldots, v_{n-3}\}$ and $\{v_2, v_4, v_6, \ldots, v_{n-2}\}$ are two maximum independent sets.

Consider the maximum independent set $S_2 = \{v_1, v_3, v_5, \ldots, v_{n-3}\}$.

Proof is similar for the other set.

As before, $\operatorname{supp}(v_1) = \operatorname{supp}(v_3) = \operatorname{supp}(v_5) = \ldots = \operatorname{supp}(v_{n-3}) = n+5$

Hence $\operatorname{supp}\alpha^{+}(G) = \sum_{v \in S_2} \operatorname{supp}(v)$

$$= \left(\frac{n-2}{2}\right)(n+5)$$

Similarly $\operatorname{supp}\alpha^{\times}(G) = \prod_{v \in S_2} mult(v)$

$$= [9(n-1)]^{\frac{n-2}{2}}.$$

Theorem 2.10. *Let $G = B_{m,n}$ where $m \geq n$, $n \geq 1$ be a bistar on $m+n+2$ vertices. Then* $\operatorname{supp}\alpha^{+}(G) = \begin{cases} m(m+1) + n(n+1) \text{ if } n > 1 \\ m(m+1) + m + 2 \text{ if } n = 1 \end{cases}$ *and*

$$\text{supp}\,\alpha^{\times}(G) = \begin{cases} (m+1)^m (n+1)^n & \text{if } n > 1 \\ (m+1)^m (m+2) & \text{if } n = 1 \end{cases}.$$

Proof. Let $G = B_{m,n}$ where $m \geq n$. Let $u_1, u_2, \ldots, u_m, x, y, v_1, v_2, \ldots, v_n$ be the vertices of G. Then $\deg x = m+1, \deg y = n+1$ and $\deg u_i = \deg v_j = 1, 1 \leq i \leq m, 1 \leq j \leq n$.

Case (i). Suppose $n > 1$.

$S = \{u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_n\}$ is the unique maximum independent set of G.

$$\text{supp}\,(u_1) = \sum_{v \in N(u_1)} \deg v = m+1$$

Similarly $\text{supp}\,(u_1) = m+1 \ \forall \ 2 \leq i \leq m$

$$\text{supp}\,(v_1) = \sum_{v \in N(v_1)} \deg v = n+1$$

Similarly $\text{supp}\,(v_j) = n+1 \ \forall \ 2 \leq j \leq n$

Hence $\text{supp}\,\alpha^{+}(G) = \sum_{v \in S} \text{supp}\,(v)$

$$= \sum_{i=1}^{m} \text{supp}\,(u_i) + \sum_{j=1}^{n} \text{supp}\,(v_j)$$

$$= m(m+1) + n(n+1)$$

Similarly $\text{supp}\,\alpha^{\times}(G) = \prod_{v \in S} mult\,(v)$

$$= (m+1)^m (n+1)^n.$$

Case (ii). Suppose $n = 1$. There are two subcases.

Subcase (a). In this case we consider the maximum independent set with all the pendant vertices. Let $S_1 = \{u_1, u_2, \ldots, u_m, v_1\}$.

Now put $n = 1$ in case (i)

Hence $\operatorname{supp} S_1^+(G) = m(m+1)+2$

Similarly $\operatorname{supp} S_1^\times(G) = \prod_{v \in S_1} mult\,(v)$

$$= 2(m+1)^m$$

Subcase (b). In this case, we consider the maximum independent set $S_2 = \{u_1, u_2, \ldots, u_m, y\}$. As before, $\operatorname{supp}(u_i) = m+1 \ \forall 1 \le i \le m$

$$\operatorname{supp}(y) = \sum_{v \in N(y)} \deg v = \deg x + \deg v_1$$

$$= m+2$$

Hence $\operatorname{supp} S_2^+(G) = m(m+1)+(m+2)$

Therefore $\operatorname{supp} \alpha^+(G) = \max\{\operatorname{supp} S_1^+(G), \operatorname{supp} S_2^+(G)\}$

$$= \max\{m(m+1)+2,\ m(m+1)+(m+2)\}$$

$$= m(m+1)+(m+2)$$

Similarly $\operatorname{supp} S_1^\times(G) = \prod_{v \in S_2} mult\,(v)$

$$= (m+1)^m(m+2)$$

Therefore $\operatorname{supp} \alpha^\times(G) = \max\{mult\, S_1^\times(G),\ mult\, S_2^\times(G)\}$

$$= \max\{2(m+1)^m,\ (m+1)^m(m+2)\}$$

$$= (m+1)^m + (m+2).$$

III. Conclusion

In this paper, the open support independence number of some standard graphs under addition and multiplication are studied. Further studies can be observed for some special graphs.

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