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A STUDY OF FRACTALS IN SOLID GEOMETRY

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Abstract

Fractals are distinct from the simple figures of classical, or Euclidean geometry-the square, the circle, the sphere, and so forth. A polyhedron can be ideal only when it can be represented in Euclidean geometry with all its vertices on a circumscribed sphere. Congruent triangle of Convex Polyhedral evinces the self-similarity property of Fractals. By using Euler's Characteristics can observe number C, Connected pieces of various geometric figure using $C = V - E + F$ is same for all the Solid Shapes. In a Convex polyhedral or ideal polyhedral sum of face angles is less than four right angles is manifested.

1. Introduction

Fractals 1.1. The history of fractals dates back to 1975, when fractals were discovered by Benoit Mandelbrot [14]. He explained them as being geometric shapes that when divided into parts; each part would be a smaller term Fractals which was derived from Latin word fractus [3]. The Latin word fractus means broken or fractured. A fractal is a never ending pattern. Fractals are infinitely complex patterns that are self-similar across different scales. They are created by repeating a simple process over and over in an ongoing feedback loop. Geometrically, they exist in between our familiar dimension. Fractals patterns are extremely familiar, since nature is full of

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Fractals [8]. The Study of Fractals gives us a quantitative language to describe the myriad of self-similar shapes found in the natural world. Fractals need not be natural objects; they can be human made and can also unfold in time in addition to space For instance: Trees, Rivers, Coastline, Mountains, Clouds, Seashells, the population of cities, the distribution of the number of links into web pages all can be usefully viewed as fractals [15]. Fractals objects at first blush seem intricate and complex. A few simple tools and ideas for analyzing fractals prove to be surprisingly powerful and flexible. Fractal Geometry will make you see everything differently [3]. Classical geometry provides a first approximation to the structure of physical objects; it is the language that we use to communicate the designs of technological products and, very approximately, the forms of natural creations. Fractal geometry is an extension of classical geometry. Fractal geometry is included in many disciplines like mathematics, biology, chemistry, physics, psychology, mechanical engineering, electrical engineering, aerospace engineering, computer science and geophysical engineering [9].

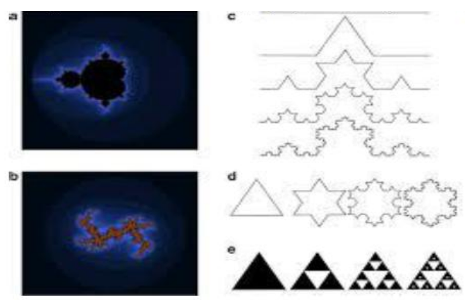

Figure 1. Fractals.

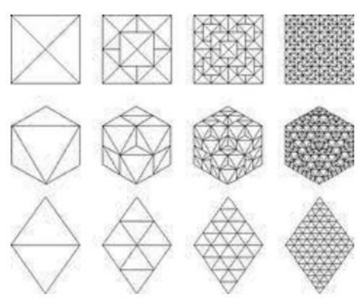

Figure 2. Fractal Geometry.

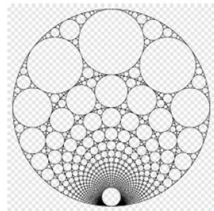

Figure 3. Fractal in Sphere.

Fractal Geometry 1.2. A Fractal is a subset of Euclidean space with a fractal dimension that strictly exceeds its topological dimension [7, 11]. Fractals appear the same at different scales, as illustrated in successive magnifications of the Mandelbrot set. Fractals often exhibit similar patterns at increasingly smaller scales, a property called self-similarity, also known as expanding symmetry or unfolding symmetry; if this replication is exactly the same at every scale, as in the menger sponge, it is called affine self-similar. Fractal geometry lies within the mathematical branch of measure theory. Euler characteristic, in mathematics, a number, C, that is a topological characteristic of various classes of geometric figures based only on a relationship between the numbers of vertices (V), edges (E), and faces (F) of a geometric figure [7]. This number, given by $C = V - E + F$, is the same for all figures whose boundaries are composed of the same number of connected pieces [4].

2. Solid Geometry

In mathematics Solid geometry or Stereometry is the traditional name for the geometry of three dimensional, Euclidean spaces (i.e., geometry) [2]. Stereometry deals with the measurements of volumes of various solid figures (or 3D figures), including pyramids, prisms and other polyhedrons; cylinders, cones, truncated cones, and balls bounded by spheres.

Definition 2.1 Solid Shapes. Solid shapes are three-dimensional shapes that have length, breadth, and height as the three dimensions. We will now learn about each solid shape in detail. Solid shapes are classified into several categories. Some of them have curved surfaces; some are in the shape of pyramids or prisms. Let’s explore types of solid shapes-Sphere, Cylinder, Cone, Pyramid, and Prism [12].

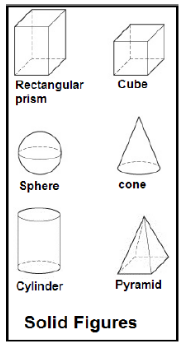

Figure 4. Solid Shape.

Table 1. Solid Shapes its Properties, Area and Volume.

SOLID SHAPES	PROPERTIES	AREA	VOLUME
SPHERE	IT HAS NO EDGES OR VERTICES AND HAS ONLY ONE SURFACE. ALL POINTS ON THE SURFACE ARE SAME DISTANCE (R) FROM THE CENTER.	$4\pi r^2$	$\frac{4}{3}\pi r^3$
CYLINDER	IT IS A 3D OBJECT WITH TWO IDENTICAL ENDS THAT ARE EITHER CIRCULAR OR OVAL. THE BASES ARE ALWAYS CONGRUENT AND PARALLEL.	$2\pi(r+h)$	$\pi r^2 h$
CONE	IT HAS A CIRCULAR OR OVAL BASE WITH A VERTEX. A CONE IS ROTATED TRIANGLE.	$\pi r(r+s)$	$\frac{1}{3}\pi r^2 h$
PYRAMID	A PYRAMID IS A POLYHEDRON WITH A POLYGON BASE AND ALL LATERAL FACES ARE TRIANGLE. BASED ON THEIR APEX ALIGNMENT WITH THE CENTER AS BASE THEY CAN BE CLASSIFIED INTO REGULAR AND OBLIQUE	$BA + \frac{1}{2} \times P(SH)$	$\frac{1}{3} BA^2$

	PYRAMIDS.		
PRISMS	IT HAS A IDENTICAL ENDS AND FLAT FACES AND IT HAS THE SAME CROSS SECTION ALL ALONG ITS LENGTH.	$2 \times BA + P \times h$	$BA \times h$

Table 2. Number of Faces, Edges, and Vertices of All Solid Shapes Solid Shapes.

SOLID SHAPES	FACES	EDGES	VERTICES
SPHERE	1	0	0
CYLINDER	2	2	0
CONE	1	1	1
CUBE	6	12	8
RECTANGULAR PRISM	6	12	8
TRIANGULAR PRISM	5	9	6
PENTAGONAL PRISM	7	15	10
HEXAGONAL PRISM	8	18	12
SQUARE PYRAMID	5	8	5
TRIANGULAR PYRAMID	4	6	6
PENTAGONAL PYRAMID	6	10	6
HEXAGONAL PYRAMID	7	12	7

By Euler characteristic $C = V - E + F$, the connected pieces are 2 for all the solid shapes (Figure 4) except for sphere and cone [10, 13].

3. Polyhedral

In Euclidean geometry, a three-dimensional object is composed of a finite number of polygonal surfaces (faces) [5]. A polyhedral is a three-dimensional shape with flat polygonal faces, straight edges, and sharp corners or vertices. A Polyhedral angle is defined a portion of space partly enclosed by three or more planes whose intersections meet in a vertex. A polyhedral is convex if any two points on its surface can be connected by a segment that lies entirely inside or on the polyhedron. If this segment goes outside the polyhedron is not convex or concave [6].

Theorem 3.1. *The sum of the face angles of any convex polyhedral angle is less than four right angles.*

Given: *A Convex Polyhedral angle V, all of its edges being cut by a plane making the section ABCDE* [1].

To prove that $\angle AVB + \angle BVC$, *etc., are less than four right angles*

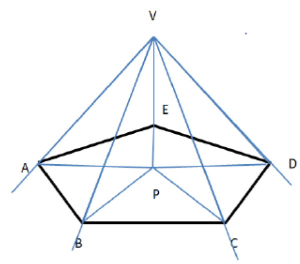

Figure 5. Polyhedral angle.

Proof. From any point P within the polygon draw PA, PB, PC, PD, PE. The number of the triangles having the common vertex P is the same as the number having common vertex V.

The sum of any two face angles of a trihedral angle is greater than the third face angle.

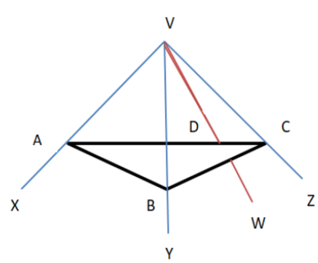

Figure 6. Trihedral angle.

Given the trihedral angle $V - XYZ$ with the face angle XVZ greater than either of the angles XVY or YVZ.

To prove that $\angle XVY + \angle YVZ$ is greater than $\angle XVZ$.

Proof. In the $\angle XVZ$ draw VW making $\angle XVW = \angle XVY$. Through any point D to VW draw ADC in the plane XVZ. On VY take VB equal to VD. Pass a plane through the line AC and the point B. Then since $AV = AV$, $VD = VB$, and $\angle AVD = \angle AVB$,

Therefore ΔAVD is congruent to ΔAVB. Therefore $AD = AB$. In the ΔABC, $AB + BC > AC$. Since $AB = AD$,

Therefore $BC > DC$.

In the ΔBVC and ΔDVC, $VC = VC$ and $VB = VD$ but $BC > DC$.

Therefore $\angle BVC$ is greater than $\angle DVC$.

Therefore $\angle AVB + \angle BVC$ are greater than $\angle AVD + \angle DVC$.

But $\angle AVD + \angle DVC = \angle AVC$.

$\angle AVB + \angle BVC$ are greater than $\angle AVC$.

That is $\angle XVY + \angle YVZ$ is greater than $\angle XVZ$.

Therefore the sum of the angles of all the triangles having the common vertex V is equal to the sum angles of all the triangles having the common vertex P.

But in the trihedral angles formed at A, B, C, etc.,

$\angle EAV + \angle BAV$ is greater than $\angle BAE$,

$\angle VBA + \angle CBV$ are greater than $\angle VBA$, etc.,

If un equals are added to un equals in the same order the sums are unequal in the same order; if un equals are subtracted from equals the reminders are unequal in the reverse order. Hence the sum of the angles at the base of the triangles whose common vertex is V is greater than the sum of the angles at the bases of the triangles whose common vertex is P.

Therefore the sum of the angles at the vertex V is less than the sum of the triangles at the vertex P. The whole angular space in a plane about a point is called a perigon. But sum of the angles at P is equal to 4 right angles. Therefore sum of angles at V is less than 4 right angles.

Two triangles are Congruent if their corresponding sides are equal in length and corresponding angles are equal. Consider the triangle ΔVB, ΔCVD with common vertex V.

Vertices. A and C, B and D are same

Sides. $AV = CV$, $AB = VD$ and $CV = DV$

Angles. $\angle A = \angle C$, $\angle B = \angle D$. From this all sides and angles are equal therefore AVB, CVD is congruent. Hence all the triangles with common vertex V are congruent and also all the triangles with common vertex P are also congruent. From Figure 4 it shows polyhedral angle satisfies self-similarity property of fractals.

Consider vertex V in figure 4 therefore ΔAVB, ΔBVC, ΔCVD, ΔDVE, ΔEVA all the triangles are self-similar to each other. In same way if consider the vertex P, ΔAPB, ΔBPC, ΔCPD, ΔDPE, ΔEPA also satisfy the self-similar property. From this concluded that polyhedral angles satisfy property of Fractals.

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