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GENERALIZED PRE-SEMI CONTINUOUS MAPPINGS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACE

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Abstract

In this chapter, intuitionistic fuzzy generalized pre-semi continuous mappings is introduced and their relations with various other intuitionistic fuzzy continuous mappings are studied.

1. Introduction

Several authors [1, 2, 3, 4, 5, 6, 7] working in the field of intuitionistic fuzzy (IF) topology have shown more interest in studying the concept of generalizations of continuous mappings. In 2006, a weak form of continuous mappings called intuitionistic fuzzy generalized continuous mappings was introduced by Thakur and Rekha Chadurvedi [11]. Recently Thakur and J. B. Pandey [9, 10] introduced and studied other forms of generalized continuous mappings called IF w-continuous mappings, IFrw-continuous mappings, IFsg-continuous mappings and IFgpr-continuous mappings. Santhi and Jayanthi [8] introduced IF generalized semi pre continuous mappings.

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2. Intuitionistic Fuzzy Generalized Pre-Semi Continuous Mappings

Definition 1. Let $f : (X, \tau) \to (Y, \sigma)$ is called an IF generalized pre-semi continuous (IFGPS continuous) g if $f^{-1}(V)$ is an IFGPSCS in (X, τ) for every IFCS V of (Y, σ).

Example 2. Let $X = \{a, b\}, Y = \{u, v\}$ and $H_1 = \langle x, (0.5, 0.4), (0, 5, 0.6) \rangle$, $H_2 = \langle y, (0.6, 0.7), (0.4, 0.2) \rangle$. Now $\tau = \{0 \sim, H1, 1 \sim\}$, $\sigma = \{0 \sim, H1, 1 \sim\}$ are IFTs on X and Y correspondingly.

Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFGPS continuous mapping.

Theorem 3. *IF continuous mapping are IFGPS continuous mapping but the converse is not true.*

Proof of Theorem 3. $f : (X, \tau) \to (Y, \sigma)$ be an IF continuous mapping. If v is IFCS in y. Now $f^{-1}(V)$ is IFCS in X. For all IFCS is IFGPSCS, $f^{-1}(V)$ is an IFGPSCS in X.

Therefore f is IFGPS continuous mapping.

Example 4. In Example 2, $f : (X, \tau) \to (Y, \sigma)$ is an IFGPS continuous mapping but it is not an IF continuous mapping. Since $H_2 = \langle y, (0.6, 0.7), (0.4, 0.2) \rangle$. is an IFOS in Y but $f^{-1}(H_2) = \langle y, (0.6, 0.7), (0.4, 0.2) \rangle$. $H_2 = \langle y, (0.6, 0.7), (0.4, 0.2) \rangle$ is not an IFOS in X.

Theorem 5. *All IFα continuous mapping is an IFGPS continuous mapping but the converse part is not true.*

Proof of Theorem 5. $f : (X, \tau) \to (Y, \sigma)$ is IFα continuous mapping. v be an IFCS in Y. Now $f^{-1}(V)$ is an IFα CS in X. If all IFα CS is IFGPSCS, $f^{-1}(V)$ is IFGPSCS in X. Therefore f is an IFGPS continuous mapping.

Theorem 6. *All IFP continuous mapping is the IFGPS continuous mapping but the converse is not true.*

Proof of Theorem 6. $f : (X, \tau) \to (Y, \sigma)$ be an IFP continuous mapping. v be an IFCS in Y. Then $f^{-1}(V)$ is an IFPCS in X. If all IFPCS is an IFGPSCS, $f^{-1}(V)$ is an IFGPSCS in X. Therefore f is an IFGPS continuous mapping.

Example 7. Let $X = \{a, b\}$, $Y = \{u, v\}$ and $H_1 = \langle x, (0.2, 0.1), (0, 8, 0.9)\rangle$, $H_2 = \langle y, (0.5, 0.6), (0.5, 0.4)\rangle$, $H_3 = \langle y, (0.1, 0.4), (0.9, 0.6)\rangle$. Now $\tau = \{0 \sim, H1, H2, 1 \sim\}$, $\sigma = \{0 \sim, H3, 1 \sim\}$ are IFTs on X and Y correspondingly.

The $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Now f is an IFGPS continuous mapping but not an IFP continuous mapping.

Theorem 8. *All IFW continuous mapping is an IFGPS continuous mapping but the converse part is not true.*

Proof of Theorem 8. $f : (X, \tau) \to (Y, \sigma)$ be an IFW continuous mapping. v be an IFCS in Y. Then $f^{-1}(V)$ is an IFWCS in X. All IFWCS is an IFGPSCS, $f^{-1}(V)$ is an IFGPSCS in X. Therefore f is an IFGPS continuous mapping.

Theorem 9. *All IFGPS continuous mapping is an IFGPR continuous mapping but the converse part is not true.*

Proof of Theorem 9. $f : (X, \tau) \to (Y, \sigma)$ be IFGPS continuous mapping. v be an IFCS in Y. Then $f^{-1}(V)$ is an IFGPSCS in X. All IFGPSCS is an IFGPRCS, $f^{-1}(V)$ is an IFGPRCS in X. Therefore f is an IFGPR continuous mapping.

Example 10. Let $X = \{a, b\}$, $Y = \{u, v\}$ and $H_1 = \langle x, (0.3, 0.2), (0.7, 0.8)\rangle$, $H_2 = \langle y, (0.5, 0.6), (0.5, 0.4)\rangle$. Now $\tau = \{0 \sim, H1, 1 \sim\}$, $\sigma = \{0 \sim, H1, 1 \sim\}$ are IFTs on X and Y respectively.

The $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFGPR continuous mapping but not an IFGPS continuous mapping.

Theorem 11. *All IFGPS continuous mapping is an IFGSP continuous mapping but the converse part is not true.*

Proof of Theorem 11. $f:(X, \tau) \to (Y, \sigma)$ be an IFGPS continuous mapping. v be an IFCS in Y. Now $f^{-1}(V)$ is an IFGPSCS in X. All IFGPSCS is an IFGSPCS, $f^{-1}(V)$ is an IFGSPCS in X. Therefore f is an IFGSP continuous mapping.

Example 12. In Example 10, $f:(X, \tau) \to (Y, \sigma)$ is an IFGSP continuous mapping but it is not an IFGPS continuous mapping.

Theorem 13. *All IFGPS continuous mapping is an IFGSPR continuous mapping but the converse is not true.*

Proof of Theorem 13. $f:(X, \tau) \to (Y, \sigma)$ be an IFGPS continuous mapping. v be an IFCS in Y. Now $f^{-1}(V)$ is an IFGPSCS in X. All IFGPSCS is an IFGSPRCS, $f^{-1}(V)$ is an IFGSPRCS in X.

Therefore f is an IFGSPR continuous mapping.

Example 14. In Example 10, $f:(X, \tau) \to (Y, \sigma)$ is an IFGSPR continuous mapping but it is not an IFGPS continuous mapping.

Theorem 15. *Let $f:(X, \tau) \to (Y, \sigma)$ be a mapping where $f^{-1}(V)$ is an IFRCS in X for every IFCS in Y. Hence f is an IFGPS continuous mapping but not conversely.*

Proof of Theorem 15. A be an IFCS in Y. Now $f^{-1}(V)$ is an IFRCS in X. All IFRCS is an IFGPSCS, $f^{-1}(V)$ is an IFGPSCS in X.

Therefore f is an IFGPS continuous mapping.

Example 16. In Example 2, $f:(X, \tau) \to (Y, \sigma)$ is an IFGPS continuous mapping but inverse image of a IFCS in Y is not IFRCS in X.

Theorem 17. *If $f:(X, \tau) \to (Y, \sigma)$ is an IFGPS continuous mapping, then for each IFP $p(\alpha, \beta)$ of X and each $A \epsilon \sigma$ with $f(p(\alpha, \beta)) \epsilon A$, there exists an IFGPSOSB of X containing $p(\alpha, \beta)$ such that $f(B) \subseteq A$.*

Proof of Theorem 17. Let $p(\alpha, \beta)$ be an IFP of X and $A€\sigma$ with

$f(p(\alpha, \beta)) \in A$. Put $B = f^{-1}(A)$. Then by hypothesis B is an IFGPSOS in X such that $p(\alpha, \beta) \in B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

Theorem 18. *Let $f : (X, \tau) \to (Y, \sigma)$ be an IFGPS continuous mapping. Then f is an IFP continuous mapping if X is an IFPST$_{1/2}$ space.*

Proof of Theorem 18. Let v be an IFCS in Y.

Now $f^{-1}(V)$ is an IFGPSCS in X, by hypothesis.

Hence X is an IFPST$_{1/2}$ space, $f^{-1}(V)$ is an IFPCS in X.

Therefore f is an IFP continuous mapping.

Theorem 19. *Let $f : (X, \tau) \to (Y, \sigma)$ be an IFGPS continuous mapping and $g : (Y, \sigma) \to (Z, \eta)$ be an IF continuous mapping, then $g \circ f : (X, \tau) \to (Z, \eta)$ is an IFGPS continuous mapping.*

Proof of Theorem 19. v be an IFCS in Z.

Then $g^{-1}(V)$ is an IFCS in Y, by hypothesis.

Since f is an IFGPS continuous mapping, $f^{-1}(g^{-1}(v)) = (g \circ f)^{-1}(v)$ is an IFGPSCS in X. Therefore $g \circ f$ is an IFGPS continuous mapping.

Theorem 20. *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is a IFPST$_{1/2}$ space*:

(i) *f is an IFGPS continuous mapping*,

(ii) $f^{-1}(B)$ *is an IFGPSOS in X for all IFOS B in Y*,

(iii) *All IFP $p(\alpha, \beta)$ in X and for every IFOS B in Y so that $f(p(\alpha, \beta)) \in B$, there will be an IFGPSOS A in X such that $p(\alpha, \beta) \in A$ and $f(A) \subseteq B$.*

Proof of Theorem 20.

(i) $\Leftrightarrow$ (ii) is true, $f^{-1}(A^c) = (f^{-1}(A))^c$.

(ii) $\Leftrightarrow$ (iii) Let $p(\alpha, \beta) \in X$. Let B be any IFOS in Y containing $f(p(\alpha, \beta))$. By hypothesis $f^{-1}(B)$ is an IFGPSOS in X. Take $A = f^{-1}(B)$. Now $p(\alpha, \beta) \in f^{-1}(f(p(\alpha, \beta)))$. Therefore $f^{-1}(f(p(\alpha, \beta))) \in f^{-1}(B) = A$. This implies $p(\alpha, \beta) \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.

(iii) $\Leftrightarrow$ (i) Let A be an IFCS in Y. Then its complement, say $B = A^c$, is an IFOS in Y. Let $p(\alpha, \beta) \in f^{-1}(B)$

$\Rightarrow f(p(\alpha, \beta)) \in B$

$\Rightarrow$ There will be an IFGPSOS in X so that $p(\alpha, \beta) \in C$ and $f(C) \subseteq B$ for every $p(\alpha, \beta) \in f^{-1}(B)$

$\Rightarrow$ There exists an IFGPSOS C in X with $p(\alpha, \beta) \in C \in f^{-1}(B)$

Therefore $f^{-1}(B)$ is an IFGPSOS in X by Theorem 3.4.3.

Hence $f^{-1}(A^c) = (f^{-1}(A))^c$ is an IFGPSOS in X and hence $f^{-1}(A)$ is an IFGPSCS in X.

Therefore f is an IFGPS continuous mapping.

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