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A LINEAR DIOPHANTINE EQUATION AND ITS REAL LIFE APPLICATIONS

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Abstract

In this paper, to study the suffering of Diophantine equation and also how to applied in our real life.

1. Introduction

A Diophantine equation is an indeterminate polynomial equation that allows the variables to be integers only. Diophantine problems have fewer equations that unknowns variable and involve finding integers that work correctly for all equations. They are named after the Hellenistic Mathematician Diophantine of Alexandria. The mathematical study of Diophantine problems is called Diophantine Analysis. The formulation of general theories of Diophantine equations was an achievement of the twentieth century. There are Diophantine equations which possess no solutions, finite number of solution or infinite number of solutions. The motivation to study more equations in one or more unknowns having only integral solutions lead to the origin of Diophantine equation as a polynomial equation with Integral co-efficient which is solvable in integers.

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2. Characteristics of Diophantine Equation

Stripe 1.

The general form of Diophantine Equation is a Linear Diophantine Equation in one variable $ax = b$, when $x = \frac{b}{a}$ must be integral.

To illustrate, If $\frac{3}{4^{th}}$ of a number is 4 more than $\frac{1}{3^{rd}}$ of the number than what is the integral?

$$\frac{3x}{4} - \frac{x}{3} = 4 \Rightarrow 12 \text{ (Integral)}$$

Stripe 2.

A Linear Diophantine Equation in two unknowns x and y which is of the form: $ax + by = c$. With the restriction that the solutions be only in integers, if $x = x_0$, $y = y_0$ satisfy the above equation we write the solution as (x_0, y_0).

To illustrate, $2x + 3y = 10$ is a linear Diophantine equation one it solution is $x = 2$, $y = 2$. Since, $(2 \times 2) + (3 \times 2) = 10$.

The following theorem gives a standard criterion for the solvability of the linear Diophantine equation.

Theorem 1. *The equation $ax + by = c$ is solvable in integers iff 'd' divides 'c' where d is the greatest common divisor of 'a' and 'b'.*

3. Proof of Theorem 1

Let d/c since $(a, b) = d$, there exist integers x_1 and y_1 such that

$$ax_1 + by_1 = d.$$

Multiplying both sides by c/d

$$a(c/d)x_1 + b(c/d)y_1 = c$$

$$\Rightarrow x = (c/d)x_1$$

$$y = (c/d)y_1.$$

Conversely, let (x_0, y_0) be a solution of above equation. Then $ax_0 + by_0 = c$. Now d divides a and $b \Rightarrow d/c$.

4. Real Life Application in Diophantine Equation

Word Difficulty 1.

Prakathi did her Nursery one year ago $1/5^{th}$ of her life and $1/6^{th}$ of her life from one year she doing Primary is 2 years. What is her present age?

$$\frac{P-1}{5} + \frac{P+1}{6} = 2 \Rightarrow 6 \text{ (present age).}$$

Word Difficulty 2.

Shella lasted her childhood $1/8^{th}$ of her life, her hear grew after $1/14^{th}$ more, after $1/9^{th}$ more she got married and her daughter born 7 years later. The daughter lived half her mother's age and the mother died 6 years after her daughter.

$$\frac{x}{8} + \frac{x}{14} + \frac{x}{9} + 7 + \frac{x}{2} + 6 = x \Rightarrow 68 \text{ (her age)}$$

Word Difficulty on Business.

Aravind invested a part of his investment in 0.05 Producer A and a part in 0.1 Producer B. His interest income during first year is rupee 2000. If be invests 0.5 more in 0.05 Producer A and 0.05 more in 0.1 Producer B his income during second year increases by rupee 1000. Find his initial investments.

Feature

Let his investment be rupee x and y in Producer A and B respectively. Then for first year

$$0.05x + 0.1y = 2000$$

and for second year $0.025x + 0.005y = 3000$ solve these equation as $AX = B$. Since, $|A| = -0.00225 \neq 0$ so A is invertible.

Then $X = A^{-1}B$

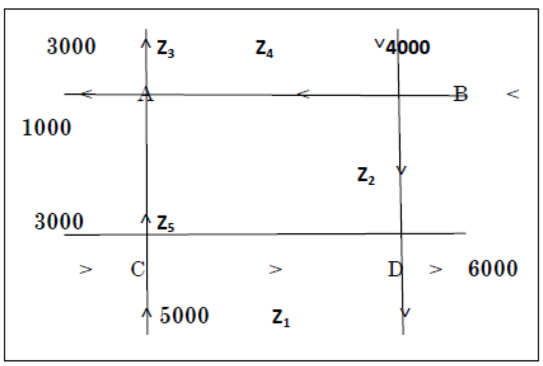
$$= \frac{1}{-0.00225} \left| \begin{matrix} 0.005 & -0.025 \\ -0.1 & 0.05 \end{matrix} \right| \left| \begin{matrix} 2000 \\ 3000 \end{matrix} \right|$$

$$= \frac{1}{-0.00225} \left| \begin{matrix} -65 \\ -50 \end{matrix} \right|$$

$$X = \left| \begin{matrix} 28{,}889 \\ 22{,}222 \end{matrix} \right|.$$

Network Flow:

In Chennai, the traffic flow in Kathipara bridge during a typical early morning in vehicles per hour, over several one way streets is given in the following diagram. Determine the general flow patterns for the network.



Inflow and outflow from the diagram is

Intersection A,

$$z_2 + z_4 = z_3 + 3000.$$

Intersection B,

$$4000 + 1000 = z_4 + z_5.$$

Intersection C,

$$3000 + 5000 = z_2 + z_1$$

Intersection D,

$$z_1 + z_5 = 6000.$$

Now going to equate the sum of total inflow as well as outflow. Then

$$z_2 + z_4 + 4000 + 1000 + 3000 + 5000 + z_1 + z_5 = z_3 + 3000$$

$$+z_4 + z_5 + z_2 + z_1 + 6000$$

$$z_3 = 7000.$$

For a simultaneous solution express the above conditions

$$z_2 + z_4 - z_3 = 3000$$

$$z_4 + z_5 = 5000$$

$$z_1 + z_2 = 8000$$

$$z_1 + z_5 = 6000$$

$$z_3 = 7000.$$

Here z_1, z_2, z_3, z_4, z_5 are represents vehicles. Therefore, these values must be whole numbers. A solution is obtained as Linear Diophantine Equation in five variables as follows.

$$z_1 = 6000 - z_5$$

$$z_2 = 10000 - z_4$$

$$z_3 = 7000$$

$$z_4 = 5000 - z_5$$

$$z_5 = \text{free var iables.}$$

From above equation clear that positive flow shows that one direction and negative flow means that opposite direction flow. Since, the bridges in the problem are one way, so none of the variable can be negative. It leads to certain limitations on the possible variable.

5. Conclusion

In finite or infinite number of variables are solvable in many linear as well as non-linear Diophantine equation also used in real life, they can be really helpful and have wide application.

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