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SUPER EDGE-MAGIC LABELING OF OCTOPUS GRAPH AND VERTEX SWITCHING OF JEWEL GRAPH

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Abstract

A graph G with v vertices and e edges is said to be edge-magic if there exists a bijection $f: V(G) \cup E(G) \to \{1, 2, 3, \ldots, |V(G)| + |E(G)|\}$ such that for all the edges uv of G, $f(u) + f(v) + f(uv)$ is a constant k. An edge-magic labeling f of G becomes a super edge-magic labeling if the bijection is said to have an additional property that the vertex set $f(V(G)) = \{1, 2, 3, \ldots, |V(G)|\}$. In this paper we obtain a super edge-magic labeling of Octopus graph O_n and vertex switching of jewel graph J_n.

1. Introduction

Graph labeling is an assignment of integers to the vertices or edges or both subject to certain limitations. It was first introduced by Rosa in 1967 [4]. In the intervening years dozens of graph labeling techniques have been studied. One such graph labeling technique is the magic labeling introduced by Sedlacek [6] when he was motivated by the notion of magic squares in number theory. Ringel and Llado [5] rediscovered this notion and called it as edge-magic labeling. Enomoto et al. [2] called a graph $G(V, E)$ with an edge-magic labeling f of G such that $f(V(G)) = \{1, 2, 3, \ldots, |V(G)|\}$ as a super

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edge-magic labeling. Enomoto et al. [2] also proved that C_n is super edge-magic iff n is odd; $K_{m,n}$ is super edge-magic iff $m = 1$ or $n = 1$, W_n of order n is not super edge-magic. Kotzig and Rosa [4] proved that every caterpillar is super edge-magic. Amara Jothi et al. [1] have proved super edge-magic labeling for duplication graphs. For more results on super edge-magic labeling refer to survey by Gallian [3]. One of the real life application of super edge-magic labeling is that, we could assign workstations or computers to each department in a company under certain constraints. In this paper, we prove that octopus graph O_n and vertex switching of jewel graph J_n admits super edge-magic labeling.

2. Main Section

In this section, we state few definitions and prove two theorems.

Definition 1. A graph G with distinct v vertices and distinct e edges is called edge-magic if there exists a bijection $f : V(G) \cup \{1, 2, 3, \ldots, |V(G)| + |E(G)|\}$ such that there exists a constant k for any (u, v) in E satisfying $f(u) + f(u, v) + f(v) = k$.

Definition 2. An edge-magic graph G is called a super edge-magic graph if $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ and $f : E(G) \to \{|V(G)| + 1, |V(G)| + 2, \ldots, |V(G)| + |E(G)|\}$.

Definition 3. An Octopus graph $O_n, (n \geq 2)$ can be constructed by joining a Fan graph $F_n(n \geq 2)$ to a Star graph $K_{1,n}$ by sharing a common vertex, where n is any positive integer. Octopus graph O_n is shown in Figure 1.

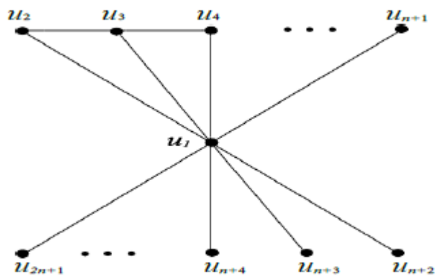

Figure 1. Octopus graph O_n.

Definition 4. The Jewel graph J_n is the graph with the vertex set $V(J_n) = \{v_x, v_y, u_x, u_y, u_i : 1 \leq i \leq n\}$ and the edge set $E(J_n) = \{v_x, u_x, v_x v_y, v_y u_x, u_x u_y, v_y u_y, v_x u_i, u_y u_i : 1 \leq i \leq n\}$. Jewel graph J_n is shown in Figure 2.

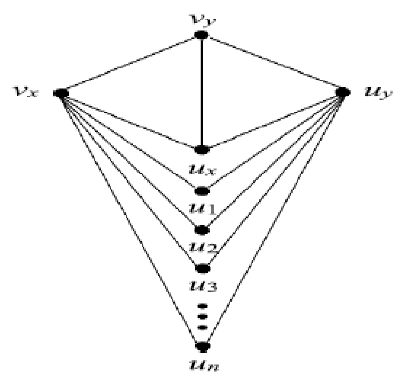

Figure 2. Jewel graph J_n.

Definition 5. In a graph $G(V, E)$ vertex switching at $v \in V$, denoted by G_v is obtained by removing all edges $u_v \in E$ incident to v and adding edges v_w, whenever $v_w \notin E$, $w \in V$.

Theorem 1. *Octopus graph O_n admits super edge-magic labeling.*

Proof. Consider an Octopus graph O_n. Let $V(O_n) = \{u_1, u_2, u_3, \ldots, u_{2n+1}\}$ denote the set of vertices of an octopus graph. Let $E(O_n) = \{E_1 \cup E_2 \cup E_3 \cup E_4\}$ be the set of edges an octopus graph, where $E_1 = \{u_2 u_3, u_3 u_4, \ldots,$

$u_i u_{i+1} : 2 \le i \le n\}, E_2 = \{u_1 u_{2n+1} : n \ge 3\}, E_3 = \{u_1 u_3, u_1 u_5, \ldots, u_1 u_{2i+1} : 1 \le i \le n-1\}$ and $E_4 = \{u_1 u_2, u_1 u_4, \ldots, u_1 u_{2i} : 1 \le i \le n\}$. Thus we have $|V(O_n)| = 2n + 1$ and $|E(O_n)| = 3n - 1$ to be the number of vertices and edges of an octopus graph respectively.

Case (i). When n is odd.

We define the vertex labeling $f : V \to \{1, 2, 3, \ldots, 2n + 1\}$ as follows: $f(u_1) = 1$, $f(u_{2i}) = 2(n + 1) - i$, $1 \le i \le n$, $f(u_{2i+1}) = n + 1 - i$, $1 \le i \le n - 1$ and $f(u_{2n+1}) = n + 1$, $n \ge 3$ where all the vertex labelings are distinct. Now, We define the edge labeling $f : E \to \{2n + 2, 2n + 3, \ldots, 5n\}$ by $f(u_i u_{i+1})$ $= 2n + i, 2 \le i \le n, f(u_1 u_{2i}) = 3n + i, 1 \le i \le n, f(u_1 u_{2i+1}) = 4n + 1 + i, 1 \le i \le n - 1$ and $f(u_1 u_{2n+1}) = 4n + 1$, $n \ge 3$ where all the edge labelings are distinct.

We observe that, $f(u_1) + f(u_{2i}) + f(u_1 u_{2i}) = 5n + 3, 1 \le i \le n, f(u_1) + f(u_{2i+1})$ $+ f(u_1 u_{2i+1}) = 5n + 3$, $1 \le i \le n - 1$, $f(u_1) + f(u_{2n+1}) + +f(u_1 u_{2n+1}) = 5n + 3$, $n \ge 3$ and $f(u_i) + f(u_{i+1}) + f(u_1 u_{i+1}) = 5n + 3$, $2 \le i \le n$.

Case (ii). When n is even.

We define the vertex labeling $f : V \to \{1, 2, 3, \ldots, 2n + 1\}$ by $f(u_1) = 1$, $f(u_{2i}) = n + 1 - i, 1 \le i \le n - 1, f(u_{2i+1}) = 2(n + 1) - i, 1 \le i \le n$ and $f(u_{2n}) = n + 1$, where $n \ge 2$ all the vertex labelings are distinct. Now we define the edge labeling $f : E \to \{2n + 2, 2n + 3, \ldots, 5n\}$ by $f(u_i u_{i+1}) = 2n + i, 2 \le i \le i \le n$, $f(u_1 u_{2i}) = 4n + 1, i, 1 \le i \le n - 1$, $f(u_1 u_{2i+1}) = 3n + i$, $1 \le i \le n$ and $f(u_1 u_{2n})$ $= 4n + 1$, $n \ge 2$ where all the edge labelings are distinct.

We observe that, $f(u_1) + f(u_{2i}) + f(u_1 u_{2i}) = 5n + 3$, $1 \le i \le n - 1$, $f(u_1)$ $+ f(u_{2i+1}) + f(u_1 u_{2i+1}) = 5n + 3$, $1 \le i \le n$, $f(u_1) + f(u_{2n}) + f(u_1 u_{2n}) = 5n + 3$, $n \ge 2$ and $f(u_i) + f(u_{i+1}) + f(u_1 u_{i+1}) = 5n + 3$, $2 \le i \le n$. In both cases, we have, $f(u) + f(v) + f(uv) = 5n + 3 = k$, a magic constant.

Thus the Octopus graph O_n admits super edge-magic labeling.

Illustration 1. Octopus graph O_5 admits super edge-magic labeling with a magic constant $k = 28$. Refer figure 3.

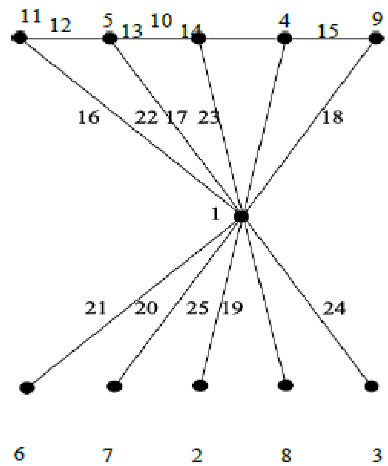

Figure 3. Super edge-magic labeling of Octopus graph O_5.

Illustration 2. Octopus graph O_4 admits super edge-magic labeling with a magic constant $k = 23$. Refer figure 4.

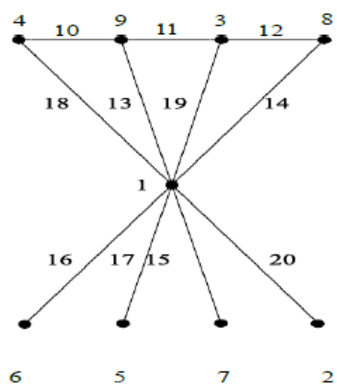

Figure 4. Super edge-magic labeling of Octopus graph O_4.

Theorem 2. *Vertex switching of Jewel graph* J_n *admits super edge-magic labeling.*

Proof. Consider a Jewel graph J_n. Let $V(J_n) = \{v_x, v_y, u_x, u_y, u_i : 1 \le i \le n\}$ be the vertices of the graph. Also denote by $E(J_n) = \{v_x u_x, v_x v_y, v_y u_x, u_x u_y, v_x u_i, u_y u_i : 1 \le i \le n\}$ the edges of the graph. Thus $|V(J_n)| = n + 4$ and $|E(J_n)| = 2n + 5$ denotes the number of vertices and edges of the jewel graph respectively. Now, Let us switch the vertex u_y of the Jewel graph J_n. We note that, the vertex set remains the same whereas the edge set becomes, $E^*(J_n) = \{v_x u_x, v_x v_y, v_y u_x, v_x u_y, v_x u_i : 1 \leq i \leq n\}$. Thus, we have $|V(J_n)| = n + 4$ and $|E^*(J_n)| = n + 4$ to be the number of vertices and edges of the jewel graph with switched vertex respectively. See Figure 5.

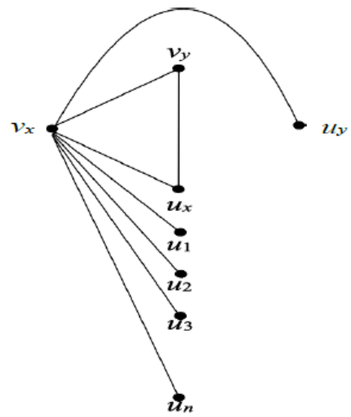

Figure 5. Vertex Switching of Jewel graph J_n.

Now, we define the vertex labeling of the jewel graph with switched vertex u_y as $f : V \to \{1, 2, 3, \ldots, n + 4\}$ by $f(v_x) = 1$, $f(v_y) = 2$, $f(u_x) = 4 + n$, $f(u_y) = 3$ and $f(u_i) = 3 + i, 1 \leq i \leq n$ where all the vertex labelings are distinct. Now we define the edge labeling of the jewel graph with switched vertex u_y as $f : E^* \to \{n + 5, \ldots, 2(n + 4)\}$ by $(v_x v_y) = 2(n + 4)$, $(v_x u_x) = 6 + n$, $(v_y u_x) = 5 + n$, $(v_x u_y) = 7 + 2n$ and $(v_x u_i) = 7 + 2n - i, 1 \leq i \leq n$ where all the edge labelings are distinct.

We observe that, $f(v_x) + f(v_y) + f(v_x v_y) = 2n + 11$; $f(v_x) + f(u_x) + f(v_x u_x) = 2n + 11$; $f(v_y) + f(u_x) + f(v_y u_x) = 2n + 11$; $f(v_x) + f(u_y) + f(v_x u_y) = 2n + 11$ and $f(v_x) + f(u_i) + f(v_x u_i) = 2n + 11, 1 \leq i \leq n$.

We have, $f(u) + f(v) + f(uv) = 2n + 11 = k$, a magic constant.

Thus the Vertex switching of Jewel graph J_n admits super edge-magic labeling.

Illustration 3. Vertex Switching of Jewel graph J_3 admits super edge-magic labeling with a magic constant $k = 17$. Refer Figure 6.

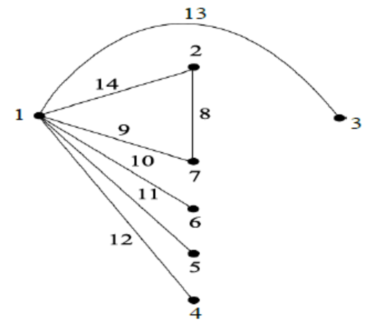

Figure 6. Super edge-magic labeling of Vertex Switching of J_3.

3. Conclusion

In this paper we have proved that Octopus graph O_n and Vertex switching of Jewel graph J_n admits super edge-magic labeling. Further we intend to prove that Path union of Octopus graph O_n admits super edge-magic labeling.

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