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RAYLEIGH TYPE WAVE PROPAGATION IN A ROTATING MICROPOLAR ELASTIC SOLID WITH STRETCH IN THE GRAVITY FIELD

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Abstract

In this paper, we investigate the propagation of Rayleigh type waves in a homogeneous, isotropic rotating micropolar elastic solid with stretch in its gravity field. The frequency equations are derived for symmetric and anti-symmetric modes in a rotating micropolar elastic solid with stretch and without stretch in the given gravity field. Waves in rotating stretched solid are independent of gravity field. There are two types of waves are propagate in a rotating non stretched micropolar solid, in which one is propagate in its gravity field and another is independent of gravity. Symmetric and anti-symmetric waves are inverse proportional to each other in a rotating solid without stretch and without gravity. On using MATLAB and developing theoretical illustrations, the graphical representations are presented for a particular numerical example.

1. Introduction

The study of Rayleigh wave propagation is much more important in several fields like earthquake, geophysics, seismology, acoustic, telecommunication, material sciences, oceanic paths etc. Many geological investigators are motivated by these studies in their research on ocean beds.

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Due to the constituents are dumbbell molecules, the theory of micropolar elasticity is a particular case of the micromorphic theory. By including the intrinsic rotations of the microstructure, the linear theory of micropolar elasticity was developed by Eringen [1]. This theory is model for applications in engineering studies. The surface wave propagation in an isotropic elastic solids were investigated by Rayleigh [2], Mrithyumjaya Rao and Kesava Rao [3] studied the Rayleigh waves in multilayered media. The surface wave analysis and the studies of other wave propagation problems are applicable to the investigations for the researchers like Destrade [4], Dowaikh [5], Ogden [6], Royer [7]. Wave propagation in rotating elastic solids are discussed by many others like Baljeet Singh [8], Somaiah [9] etc.

The present paper investigates the effect of angular rotation on dispersion relation of Rayleigh type waves in micropolar elastic solid with stretch in the gravity field.

2. Basic Governing Equations

With the usual notation of Eringen [10], Eringen [11] and Nowacki [12], the constitutive relation and field equations of rotating micropolar elastic solid with stretch and without body forces and body couples are given by

$$(\lambda + \mu - K)\nabla\nabla \cdot \vec{U} + (\mu + K)\nabla^2\vec{U} + 2K\nabla \times \vec{\phi} = \rho[\ddot{\vec{U}} \times \vec{\Omega} \times (\vec{\Omega} \times \vec{U})] \quad (1)$$

$$(\beta + \lambda - \alpha)\nabla\nabla \cdot \vec{\phi} + (\alpha + \lambda)\nabla^2\vec{\phi} + 2K\nabla \times \vec{U} - 4K\vec{\phi} = J\ddot{\vec{\phi}} \quad (2)$$

$$\alpha_0\nabla^2\psi - \eta_0\psi = \frac{J}{2}\ddot{\psi} \quad (3)$$

The force tensor t_{ij}, couple stress tensor m_{ij} and first moment tensor β_j are given by

$$t_{ij} = \lambda U_{k,k}\delta_{ij} + (\mu - K)(U_{i,j} + U_{j,i}) + 2K(U_{i,j} - \varepsilon_{kij}\phi_k) \quad (4)$$

$$m_{ij} = \beta_0\varepsilon_{kij}\psi_{,k} + \beta\phi_{k,k}\delta_{ij} + (\alpha + \gamma)\phi_{j,i} + (\gamma - \alpha)\phi_{i,j} \quad (5)$$

$$\beta_j = \alpha_0\psi_{,j} + \frac{\beta_0}{3}\varepsilon_{kij}\phi_{k,j} \quad (6)$$

where λ, μ, K, α, β, γ, α_0, η_0, β_0 are elastic constants, δ_{ij} is Kronecker's delta, ε_{kij} is permutation symbol, ρ is density of the solid, J is micro inertia, $\vec{U}$ is macro displacement vector, $\vec{\phi}$ is micro rotation vector, ψ is micro stretch scalar. Superposed dot is partial derivative with respect to time t. Subscripts followed by comma denote the partial derivative with respect to corresponding Cartesian co-ordinate. $\vec{\Omega} \times (\vec{\Omega} \times \vec{U})$ is Centripetal acceleration and here we neglect the Coriolis acceleration $2(\vec{\Omega} \times \dot{\vec{U}})$ of the solid.

3. Formulation and Solution of the Problem

We consider a homogeneous isotropic and Centro symmetric micro polar elastic solid with micro stretch having thickness 2T. Assume that the Rayleigh type wave propagation along x-axis of the solid and boundary being $y = \pm T$ and y-axis is vertically downwards into the solid and medium is rotating about y-axis. So displacement vector $\vec{U}$, micro-rotational vector $\vec{\phi}$ and angular rotation vector $\vec{\Omega}$ are in the form of $\vec{U} = (u, v, 0)$, $\vec{\phi} = (0, 0, \phi_3)$ and $\vec{\Omega} = (0, \Omega, 0)$, where u and v are the displacement components along x (longitudinal) direction and y (transversal) direction respectively. Ω is the angular rotation vector component in y direction. With these assumptions, equations (1) and (2) reduce to

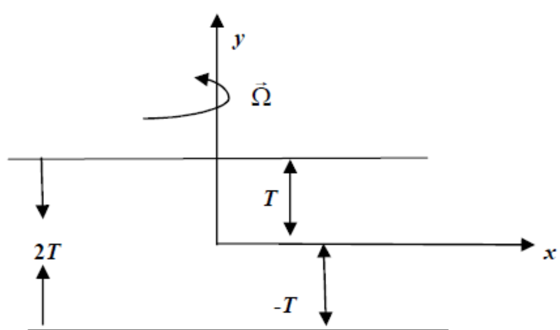

Figure 1. Geometry of the problem.

$$(\mu + K)\nabla^2 u + (\lambda + \mu - K)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y}\right) + 2K\frac{\partial \phi_3}{\partial y} = \rho(\ddot{u} - \Omega^2 u) \tag{7}$$

$$(\mu + K)\nabla^2 v + (\lambda + \mu - K)\left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2}\right) - 2K\frac{\partial \phi_3}{\partial x} = \rho\ddot{v} \tag{8}$$

$$[(\alpha + \gamma)\nabla^2 - 4K]\phi_3 + 2K\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = J\ddot{\phi}_3 \tag{9}$$

$$(\alpha_0\nabla^2 - \eta_0)\psi = \frac{J}{2}\ddot{\psi} \tag{10}$$

Introduce displacement components u and v in terms of potential functions G and H as,

$$u = \frac{\partial G}{\partial x} - \frac{\partial H}{\partial y},\ v = \frac{\partial G}{\partial y} + \frac{\partial H}{\partial x} \tag{11}$$

Substituting equations (11) into equations (7) to (10) we obtain,

$$\left[\nabla^2 + C_1^2 - \frac{1}{C_2^2}\frac{\partial^2}{\partial t^2}\right]G = 0 \tag{12}$$

$$\left[\nabla^2 + C_3^2 - \frac{1}{C_4^2}\frac{\partial^2}{\partial t^2}\right]H - P\phi_3 = 0 \tag{13}$$

$$\left[\nabla^2 - C_5^2 - \frac{1}{C_6^2}\frac{\partial^2}{\partial t^2}\right]\phi_3 + Q\nabla^2 H = 0 \tag{14}$$

and

$$\left[\nabla^2 - \frac{1}{C_7^2}\frac{\partial^2}{\partial t^2} - C_8^2\right]\psi = 0, \tag{15}$$

where

$$C_1^2 = \frac{\rho\Omega^2}{\lambda + 2\mu};\ C_2^2 = \frac{\lambda + 2\mu}{\rho};\ C_3^2 = \frac{\rho\Omega^2}{\mu + K};\ C_4^2 = \frac{\mu + K}{\rho};\ C_5^2 = \frac{4K}{\alpha + \gamma};\ C_6^2$$

$$= \frac{\alpha + \gamma}{J};\ C_7^2 = \frac{2\alpha_0}{J};\ C_8^2 = \frac{\eta_0}{\alpha_0};$$

$$P = \frac{2K}{\mu + K}; Q = \frac{2K}{\alpha + \gamma} \text{ and } \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{16}$$

Eliminating H or ϕ_3 from equations (13) and (14) we get

$$\left[\left(\nabla^2 - C_3^2 + \frac{1}{C_4^2}\frac{\partial^2}{\partial t^2}\right)\left(\nabla^2 - C_5^2 - \frac{1}{C_6^2}\frac{\partial^2}{\partial t^2}\right) + PQ\nabla^2\right](H, \phi_3) = 0 \tag{17}$$

The solutions of equations (12) and (17) of the form

$$[G, H, \phi_3] = [f_1(y), f_2(y), f_3(y)]\exp[i(kx - \omega t)], \tag{18}$$

where $f_j(y)$, $j = 1, 2, 3$ are amplitudes, k is the wave number and ω is angular frequency with the phase velocity c is given by $c = \frac{\omega}{k}$.

On substituting equation (18) into equations (12) and (17) we get

$$G = [A \sinh(dy) + B\cosh(dy)]\exp[i(kx - \omega t)] \tag{19}$$

$$H = [L\sinh(n_1 y) + L^1\cosh(n_1 y) + M\sinh(n_2 y) + M^1\cosh(n_2 y)]\exp[i(kx - \omega t)] \tag{20}$$

$$\begin{aligned}\phi_3 = [&L_1 \sinh(n_1 y) + L_1^1\cosh(n_1 y) + M_1\sinh(n_2 y) \\ &+ M_1^1\cosh(n_2 y)]\exp[i(kx - \omega t)]\end{aligned} \tag{21}$$

The solution of equation (15) is of the form

$$\psi = [C\sinh(d_1 y) + D\cosh(d_1 y)]\exp[i(kx - \omega t)], \tag{22}$$

where

$$d^2 = k^2\frac{\omega^2}{C_2^2} - C_1^2,\ d_1^2 = k^2 - \frac{\omega^2}{C_7^2} + C_8^2$$

$$n_1^2, n_2^2 = k^2 + \frac{1}{2}\left[\left(C_5^2 + C_3^2 + PQ + \frac{\omega^2}{C_2^2} - \frac{\omega^2}{C_6^2}\right) \pm \left\{\left(C_5^2 + C_3^2 + PQ + \frac{\omega^2}{C_4^2} - \frac{\omega^2}{C_6^2}\right)^2\right.\right.$$

$$\left.\left.-\frac{4\omega^2}{C_4^2}\left(\frac{\omega^2}{C_6^2}-C_5^2+C_3^2\right)\right\}^{\frac{1}{2}}\right]$$

and amplitudes L_1, L_1^1, M_1 and M_1^1 are given by

$L_1 = aL; L_1^1 = aL^1; M_1 = bM; M_1^1 = bM^1$ with

$$a = \frac{k^2 - n_1^2 - \frac{\omega^2}{C_4^2}}{P}; b = \frac{k^2 - n_2^2 - \frac{\omega^2}{C_4^2}}{P} \tag{23}$$

4. Boundary Conditions

Equations (4), (5) and (6) in terms of potential functions G and H are given by

$$t_{22} = \lambda\nabla^2 G + 2\mu\left(\frac{\partial^2 G}{\partial y^2} + \frac{\partial^2 H}{\partial x \partial y}\right), \tag{24}$$

$$t_{21} = K(\nabla^2 H - 2\phi_3) + \mu\left(\frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial y^2} + 2\frac{\partial^2 G}{\partial x \partial y}\right)$$

$$m_{23} = (\alpha+\gamma)\frac{\partial\phi_3}{\partial y} + \beta_0\frac{\partial\psi}{\partial x}; \beta_2 = \alpha_0\frac{\partial\psi}{\partial y} + \frac{\beta_0}{3}\frac{\partial\phi_3}{\partial x}$$

The appropriate boundary conditions for rotating micropolar elastic solid with stretch of thickness $y = \pm T$ in the effect of gravity field with gravitational force g is given by

$$t_{22} + \rho g\vec{U} \text{ or } t_{22} + \rho gV = 0; t_{21} = 0; m_{23} = 0 \text{ and } \beta_2 = 0 \tag{25}$$

5. Symmetric Waves

For Rayleigh type symmetric waves with respect to the plane $y = 0$, we consider from equations (19) to (22)

$$G = B\cosh(dy)\exp[i(kx - \omega t)] \tag{26}$$

$$H = [L \sinh (n_1 y) + M \sinh (n_2 y)] \exp [i(kx - \omega t)] \tag{27}$$

$$\phi_3 = [L_1 \sinh (n_1 y) + M_1 \sinh (n_2 y)] \exp [i(kx - \omega t)] \tag{28}$$

$$\psi = D \cosh (d_1 y) \exp [i(kx - \omega t)] \tag{29}$$

Substituting equations (26) to (29) in boundary conditions (25) and with the help of equation (24) we get the system of four homogeneous equations in B, L, M and D and this system has a non-trivial solution if and only if,

$$\det (a_{ij}) = 0; i, \; j = 1, 2, 3, 4 \tag{30}$$

where

$$a_{11} = [(\lambda + 2\mu) d^2 - \lambda k^2] \cosh (dT) + \rho g d \sinh(dT);$$

$$a_{12} = 2i\mu k n_1 \cosh (n_1 T) + ik\rho g \sinh (n_1 T);$$

$$a_{13} = 2i\mu k n_2 \cosh (n_2 T) + ik\rho g \sinh (n_2 T); a_{14} = 0 \; a_{21} = 2\mu i k d \sinh (dT);$$

$$a_{22} = [K(n_1^2 - k^2 - 2a) - \mu(k^2 + n_1^2)] \sinh (n_1 T);$$

$$a_{23} = [K(n_2^2 - k^2 - 2b) - \mu(k^2 + n_2^2)] \sinh (n_2 T;)$$

$$a_{24} = 0, \; a_{31} = 0; a_{32} = a n_1 (\alpha + \gamma) \cosh (n_1 T);$$

$$a_{33} = b n_2 (\alpha + \gamma) \cosh (n_2 T); \tag{31}$$

$$a_{34} = i\beta_0 k \cosh (d_1 T); a_{41} = 0; a_{42} = \frac{iak\beta_0}{3} \sinh (n_1 T);$$

$$a_{43} = \frac{ibk\beta_0}{3} \sinh (n_2 T); a_{44} = \alpha_0 d_1 \sinh (d_1 T)$$

Solving the determinant (30), we get the following two frequency equations

$$\tanh (n_2 T) = \frac{-3\alpha_0 d_1 n_2 (\alpha + \gamma)}{\beta_0^2 k^2} \tanh (d_1 T) \tag{32}$$

and

$$\rho g + [(\lambda + 2\mu)d - \frac{\lambda}{d}k^2]\coth(dT) = \frac{2\mu k^2[\rho g + 2\mu n_1 \coth(n_1 T)]}{\mu(k^2 + n_1^2) - K(n_1^2 - k^2 - 2a)} \tag{33}$$

Equation (32) is symmetric dispersion relation of Rayleigh type waves in rotating micropolar elastic solid with stretch and this relation is not depends on gravity field. Equation (33) is symmetric dispersion relation of Rayleigh type waves in rotating micro polar elastic solid in the gravity field and this relation is not affected by micro stretch.

5.1 Particular case

If we neglect stretch parameter α_0 in equation (32), we get the dispersion relation as

$$n_2^2 \approx \frac{3}{T^2} \tag{34}$$

From equation (34), we say that the frequencies of rotating micropolar elastic solid without stretch and gravity field are inverse proportional to the thickness of solid.

If we neglect the gravity field in equation (33) we obtain the following frequency equation.

$$\frac{\tanh(n_1 T)}{\tanh(dT)} = \frac{4\mu^2 k^2 n_1}{[\mu(k^2 + n_1^2) - K(n_1^2 - k^2 - 2a)][(\lambda + 2\mu)d - \frac{\lambda k^2}{d}]} \tag{35}$$

6. Anti-symmetric Waves

For Rayleigh type anti-symmetric waves with respect to the plane $y = 0$, we consider from equations (19) to (22)

$$G = A\sinh(dy)\exp[i(kx - \omega t)] \tag{36}$$

$$H = [L^1 \cosh(n_1 y) + M^1 \cosh(n_2 y)]\exp[i(kx - \omega t)] \tag{37}$$

$$\phi_3 = [L_1^1 \cosh(n_1 y) + M_1^1 \cosh(n_2 y)]\exp[i(kx - \omega t)] \tag{38}$$

$$\psi = C\sinh(d_1 y)\exp[i(kx - \omega t)] \tag{39}$$

On substituting equations (36) to (39) in equation (24) with the help of eq. (23) we obtain the system of four homogeneous equations in A, L^1, M^1, C and the system has a non-trivial solution if and only if

$$\det(c_{ij}) = 0; i, \ j = 1, 2, 3, 4 \tag{40}$$

where

$$c_{11} = [(\lambda + 2\mu)d^2 - \lambda k^2] \sinh(dT) + \rho g d \cosh(dT);$$

$$c_{12} = 2i\mu k n_1 \sin(n_1 T) + ik\rho g \cosh(n_1 T);$$

$$c_{13} = 2i\mu k n_2 \sinh(n_2 T) + ik\rho g \cosh(n_2 T); c_{14} = 0; c_{21} = 2\mu i k d \cosh(dT);$$

$$c_{22} = [K(n_1^2 - k^2 - 2a) - \mu(k^2 + n_1^2)] \cosh(n_1 T); c_{24} = 0$$

$$c_{23} = [K(n_2^2 - k^2 - 2b) - \mu(k^2 + n_2^2)] \cosh(n_2 T); c_{31} = 0$$

$$c_{32} = a n_1 (\alpha + \gamma) \sinh(n_1 T) : c_{33} = b n_2 (\alpha + \gamma) \sinh(n_2 T) \tag{41}$$

$$c_{34} = i\beta_0 k \sinh(d_1 T); c_{41} = 0; c_{42} = \frac{iak\beta_0}{3} \cosh(n_1 T);$$

$$c_{43} = \frac{ibk\beta_0}{3} \cosh(n_2 T); c_{44} = \alpha_0 d_1 \cosh(d_1 T)$$

After solving equation (40) one can obtain the following two frequency equations

$$\tanh(d_1 T) = \frac{-3\alpha_0 d_1 n_2 (\alpha + \gamma)}{\beta_0^2 k^2} \tanh(n_2 T) \tag{42}$$

$$\rho g + [(\lambda + 2\mu)d - \frac{\lambda}{d} k^2] \tanh(dT) = \frac{2\mu k^2 [\rho g + 2\mu n_1 \tanh(n_1 T)]}{\mu(k^2 + n_1^2) - K(n_1^2 - k^2 - 2a)} \tag{42}$$

Equation (42) is anti-symmetric dispersion relation of Rayleigh type waves in rotating micropolar elastic solid with stretch and it is independent of gravity field, while equation (43) is antisymmetric dispersion relation of Rayleigh type waves in a rotating micro polar elastic solid without stretch in the gravity field.

6.1 Particular case.

If we neglect stretch parameter α_0 in equations (42), we get the thickness $T \to 0$ as $\alpha_0 \to 0$

If we neglect gravity field g in equation (43) we obtain,

$$\frac{\tanh(n_1 T)}{\tanh(dT)} = \frac{[\mu(k^2 + n_1^2) - K(n_1^2 - k^2 - 2a)][(\lambda + 2\mu)d - \frac{\lambda k^2}{d}]}{4\mu^2 k^2 n_1} \tag{44}$$

From equations (35) and (44) we observed that the symmetric and anti-symmetric Rayleigh type waves are inverse proportional to each other in a rotating micropolar elastic solid without stretch and gravity field.

7. Numerical Computation and Graphical Representation

To study the effect of rotation and gravity field on Rayleigh type waves, we adopt the following relevant parameters from [13] as:

$\lambda = 7.583 GPa$; $\mu = 6.334 GPa$; $K = 14.905 MPa$; $\rho = 1189 kg/\ m^3$;
$J = 6.25 \times 10^{-7} m^2$; $\gamma = 2.89 GN$; $\alpha = 3.688 GPam^2$.

Micro stretch parameters Kumar et al. [14] as;

$\alpha_0 = 0.779 \times 10^{-9} N$; $\beta_0 = 0.5 \times 10^{-9} N$; $\eta_0 = 0.5 \times 10^{10} N/m^2$. Thickness of the solid T as $T = 0.5m$; natural frequency $\omega = 10$ (non-dimensional); gravity $g = 9.87 m/\sec^2$; the speed of angular rotation components Ω taken as, $\Omega = 0, 0.5, 1.5$ (non-dimensional). The variation of frequency and wave number k is shown in the range of k with $10 \le k \le 25$.

The symmetric frequency curves are shown in figures (2) to (5) and anti-symmetric frequency curves are shown in figure (6) to figure (9), while their comparative frequency curves for non-stretched rotating solids in their gravity are shown in figure (10).

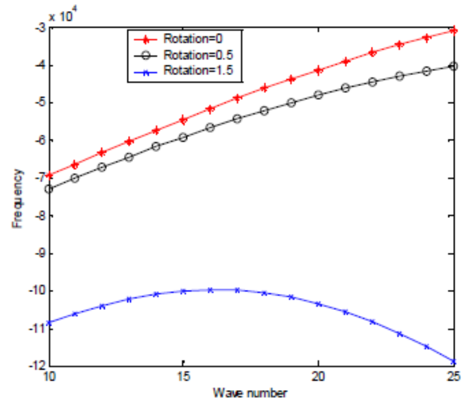

Figure 2. Symmetric frequency vs. wave number in rotating solid with gravity [without stretch].

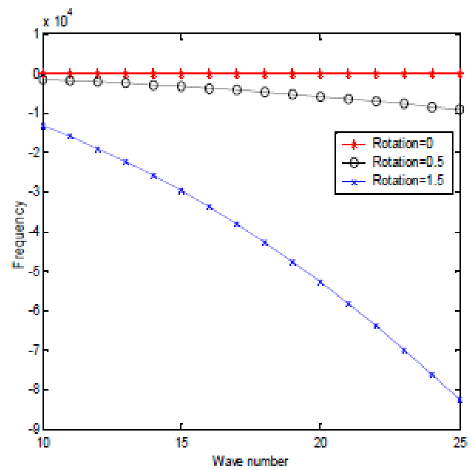

Figure 3. Symmetric frequency vs. wave number in rotating solid [without stretch and gravity].

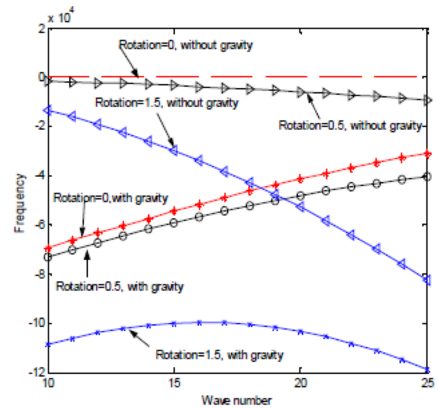

Figure 4. Symmetric frequency vs. wave number in rotating solid with and [without stretch].

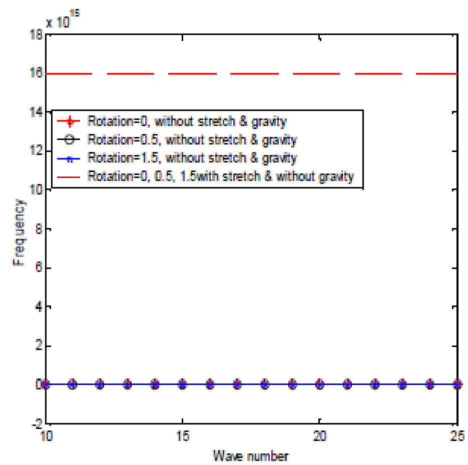

Figure 5. Symmetric frequency vs. wave number in rotating solid without gravity [stretch and without stretch].

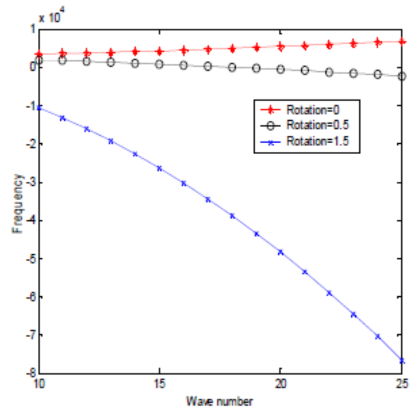

Figure 6. Anti-symmetric frequency vs. wave number in rotating solid with gravity [without stretch].

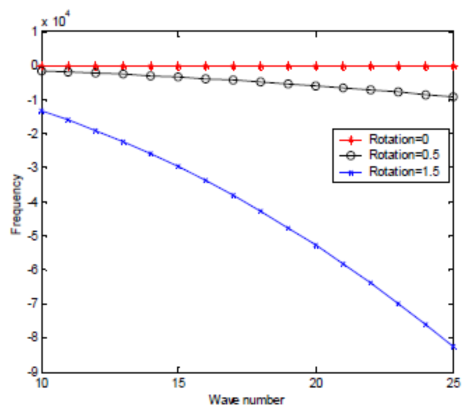

Figure 7. Anti-symmetric frequency vs. wave number in rotating solid [without stretch & gravity].

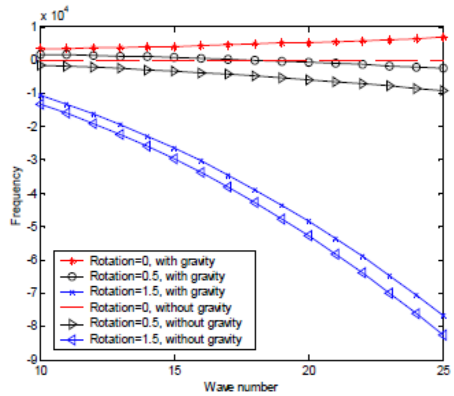

Figure 8. Anti-symmetric frequency vs. wave number in rotating solid with without gravity [without stretch].

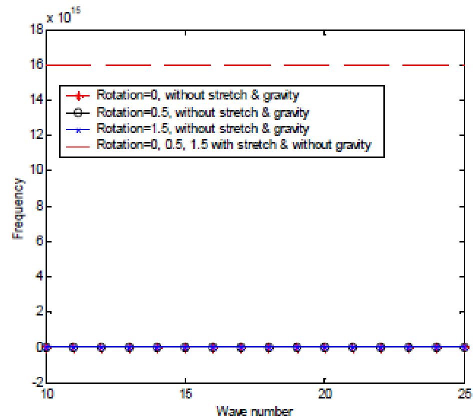

Figure 9. Anti- symmetric frequency vs. wave number in rotating solid without gravity [stretch and without stretch].

From figures (2), (3), (6) and (7) we observed that the frequencies in symmetric and anti-symmetric modes are inverse proportional to the angular rotation in non-stretched solid of its gravity and without gravity field. From figure (4) we observed that symmetric lower frequencies are obtained in rotating, non-stretched solids in the gravity field, while from fig (8), we observed that antisymmetric higher frequencies are obtained in rotating, non-stretched solids in their gravity field.

Constant frequencies are shown in figures (5) and (9) for rotating stretched and non-stretched solids without their gravity.

8. Overall Observations

In this paper, basic equations for a rotating micropolar elastic solid with stretch has been considered and solved by the method of plane harmonic solution and dispersion relation has been derived for Rayleigh type waves in symmetric and anti symmetric modes in the gravity field. From presented theoretical illustrations and numerical example, we observed that;

(i) Waves in a rotating solid with stretch are independent of gravity.

(ii) Two types of waves exist in a rotating non stretched micropolar solid, in which one is depends on gravity and other is independent of gravity.

(iii) Symmetric and anti-symmetric waves are inverse proportional to each other in a rotating solid without stretch and without gravity.

(iv) Rayleigh type waves in a non-stretched solid with gravity and without gravity are inverse proportional to its angular rotation.

(v) Anti symmetric waves with gravity are faster than without gravity in rotating non-stretched solids.

(vi) Symmetric waves without gravity are faster than with gravity in rotating non stretched solids.

(vii) Constants waves are propagate in rotating stretched and non stretched solids in their gravity field.

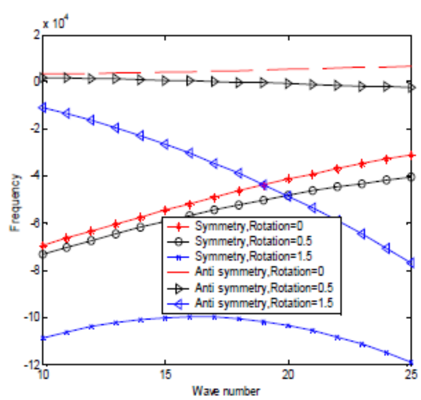

Figure 10. Symmetric and Anti-symmetry frequency vs. wave number in rotating solid with gravity [without stretch].

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