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RELATION BETWEEN RESOLVING SET AND DOMINATING SETS IN VARIOUS GRAPHS

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Abstract

In a graph $G = (V, E)$, the code of vertex v with respect to the ordered set $W = \{w_1, w_2, w_3, \ldots, w_k\} \subseteq V(G)$ is defined by $C_w(v) = (d(v, w_1), d(v, w_2), d(v, w_k))$. The set W is so-called a resolving set for G if different nodes have different codes with respect to W. A resolving set having a minimum number of nodes is a minimum resolving set or a basis for G. The (metric) dimension $G = (V, E)$. is the quantity of nodes in a basis for $G = (V, E)$. In this

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research paper, presented the idea resolving matrix in the graph and investigate relation between the dominating set and resolving set in various graphs like complete graphs, complete bipartite graphs, cycle, path, and regular graphs. Further prove some results on some results in metric of the graphs also explains the result with examples.

1. Introduction

Normally, a graph holds many dominating sets. A vertex v in a graph G is said to dominate itself and each of its neighbours. We roughly in additional words that v dominates the nodes of its closed neighbourhood $N[v]$. A set S of nodes of G is a dominating set of G if every nodes of G is dominated by at least one node of S. Consistently: a set S of nodes of G is a dominating set if all node in $V - S$ is neighbouring to at least one node in S. The minimum cardinality between the dominating sets of G is named the domination number of G and represented by $\gamma(G)$. A dominating set of cardinality $\gamma(G)$ is denoted to as a minimum dominating set. A set is independent (or stable) if no two nodes in it are adjacent. An independent dominating set of G is a set that is both dominating and independent in G. The independent domination number of G, represented by $i(G)$, is the minimum magnitude of an independent dominating set. The independence number of G, signified $\alpha(G)$, is the maximum size of an independent set in G. Several graphs hold (ordered) independent sets W such that the nodes of G are individually notable by their distances from the nodes of W. The area of this paper is to study the existence of resolving sets in graphs and, when they be existent, to examine the minimum cardinality of such a set. The distance $d(u, v)$ among two nodes u and v in a connected graph G is the length of a shortest u-vpath in G. The code of vertex v with respect to the ordered set $W = \{w_1, w_2, w_3, \ldots, w_k\} \subseteq V(G)$. In the graph $G(V, E)$ is defined by $C_w(v) = (d(v, w_1), d(v, w_2), d(v, w_k))$. The set W is termed a resolving set for G if different nodes have different codes with respect to W. A resolving set holding a minimum number of nodes is a minimum resolving set or a basis for G. The (metric) dimension $Dim(G)$ is the quantity of nodes in a basis for G.

In this research work, presented the idea of resolving matrix of the graph and investigate relation between the dominating set and resolving set in various graphs like complete graphs, complete bipartite graphs, cycle, path, and regular graphs. Further prove some results on metric of the graphs also explains the result with examples.

2. Preliminaries

A graph is an well-ordered pair $G = (V, E)$, where V is a nonempty finite set, called the set of vertices or nodes of G, and E is a set of unordered pairs (2-element subsets) of V, called the edges of G. If $xy \in E$, x and y are called adjacent and they are incident with the edge xy.

The complete graph on n vertices, denoted by K_n, is a graph on n vertices such that every pair of vertices is connected by an edge. The empty graph on n vertices, denoted by E_n, is a graph on n vertices with no edges.

A graph $G' = (V', E')$ is a sub graph of $G = (V, E)$ if and only if $V' \subseteq V$ and $E' \subseteq E$. The order of a graph $G = (V, E)$ is $|V|$ the number of its vertices. The size of G is E, the quantity of its edges. The degree of a node $|E|$, represented by $d(x)$, is the quantity of edges incident with it. A connected graph without any circuits is called a tree. A tree with n vertices has $(n-1)$ edges. A tree in which a particular node is selected as the root of the tree is called a rooted tree.

3. Main Results

In this section some relation between resolving set and dominating set of the various graphs $G(V, E)$.

Theorem 1. *Let $G(V, E)$ be a complete graph of n vertices $n \geq 3$. Then D^c is a resolving set, D is a minimally dominating get of $G(V, E)$ and $Dim(G) = n-1$, $\gamma_{RD}(G) = n-1$.*

Proof. Let $G(V, E)$ be a complete graph this implies there is an edge between ever pair of nodes in $G(V, E)$. Therefore distance among every pair of nodes is 1 i.e. $d(u, v) = 1$ forever $u, v \in V(G)$. D is a minimally dominating set of $G(V, E)$. Any singleton set $\{v_i\}$ for any $i = 1, 2, \ldots, n$ is a dominating set of $G(V, E)$. Now we check $V(G) - D$ is a resolving set of $G(V, E)$. Let $D = \{v_n\}$ is a dominating set of G. Therefore $R = V(G) - D$, the resolving matrix $C_R(G)$ is defined by $n \times (n-1)$ matrix

$$C_R(G) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ . \\ . \\ v_{n-1} \\ v_n \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & \ldots & v_{n-2} & v_{n-1} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & \ldots & 1 & 1 \\ 1 & 0 & 1 & \ldots & 1 & 1 \\ 1 & 1 & 0 & \ldots & 1 & 1 \\ . & . & . & \ldots & . & . \\ . & . & . & \ldots & . & . \\ 1 & 1 & 1 & \ldots & 1 & 0 \\ 1 & 1 & 1 & \ldots & 1 & 1 \end{bmatrix} \end{array}$$

Every pair of rows are not equivalent in the $C_R(G)$. This implies $R = V(G) - D$ is a resolving set of G. Further we prove $V(G) - D$ is minimal. Assume $(V(G) - D) - \{v_{n-1}\}$ is a minimal resolving set of G. Note that resolving set $R = V - \{v_n\} - \{v_{n-1}\}$, since $D = \{v_n\}$. The resolving matrix of G with respect to R is

$$C_R(G) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ . \\ v_{n-2} \\ v_{n-1} \\ v_n \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & \ldots & v_{n-3} & v_{n-1} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & \ldots & 1 & 1 \\ 1 & 0 & 1 & \ldots & 1 & 1 \\ 1 & 1 & 0 & \ldots & 1 & 1 \\ . & . & . & \ldots & . & . \\ 1 & 1 & 1 & \ldots & 1 & 0 \\ 1 & 1 & 1 & \ldots & 1 & 1 \\ 1 & 1 & 1 & \ldots & 1 & 1 \end{bmatrix} \end{array}$$

From the resolving matrix the code of $C_R(v_n)$ and $C_R(v_{n-1})$ are equivalent sets. This is contradict to your assumption $R = V(G) - \{v_n\}$

$-\{v_{n-1}\}$ is a resolving set. Therefore $V(G) - D$ is a minimal resolving set of $G(V, E)$. Clearly $V(G) - D$ is equivalent to D^c since $G(V, E)$ is a complete graph of n nodes. This implies $Dim(G) = | V - D | = | D^c | = n - 1$.

The set $V(G) - D$ is equivalent to D^c since $G(V, E)$ is a complete graph of n nodes. The set $V(G) - D$ is also a dominating set of $G(V, E)$. Therefore the $V(G) - D$ is a minimal resolving set of $G(V, E)$. This implies $\gamma_{RD}(G) = | V - D | = n - 1$. Hence proved.

Remark. The minimal resolving set of a complete graph $G(V, E)$ with n nodes is equivalent to D^c where D is dominating set of $G(V, E)$.

Theorem 2. *Let $G(V, E)$ be a K regular graph with $n \leq 2K$ nodes. The minimal resolving set of $G(V, E)$ is*

i. *If K is odd, then $R = (D_1 \cup D_2 \cup, \ldots, D_N)$ where $N = \left\lfloor \frac{K}{2} \right\rfloor$ and* $\dim(G) = k - 1$.

ii. *If K is even, then $R = (D_1 \cup D_2 \cup, \ldots, D_{N-1} \cup \{v_K\})$ where $N = \frac{K}{2}$ and* $\dim(G) = k - 1$.

Proof.

i. Let $G(V, E)$ be a K regular graph. If K is odd, write $K = 2N + 1$ where $N = \left\lfloor \frac{K}{2} \right\rfloor$. Further, one of the minimal dominating set of the K regular graph is $\{v_1, v_{K+1}\}$ since $G(V, E)$ be a K regular graph and number of nodes $n \leq 2K$. The first order dominating set is $D_1 = \{v_1, v_{K+1}\}$. In the sub graph $\langle V - D_1 \rangle$ the 2nd order dominating set is $D_2 = \{v_2, v_{K+2}\}$ since $G(V, E)$ be a K regular graph and quantity of nodes $n \leq 2K$. Similarly the dominating set $D_N = \{v_N, v_{K+N}\}$ of the sub graph $\langle V - (D_1 \cup D_1 \cup, \ldots, D_{N-1} \rangle$. Let $W = (D_1 \cup D_2 \cup, \ldots, D_N)$, the code of W is defined by $C_W(V - W) = [a_{ij}]$. a Note that the code $C_W(v_i)$ is pair wise disjoint since $v_i \in (V - W)$ and

$G(V, E)$ is a K regular graph. Hence $W = (D_1 \cup D_2 \cup, \ldots, D_N)$ is the resolving set R of K regular graph $G(V, E)$. The dimension of $G(V, E)$ is

$$\dim(G) = |R|$$

$$= |(D_1 \cup D_2 \cup, \ldots, D_N)|$$

$$= 2N \therefore |D_i| = 2 \ \forall \ i = 1, 2, \ldots, N$$

$$\dim(G) = K - 1 \therefore K = 2N + 1$$

Hence proved.

ii. Let $G(V, E)$ be a K regular graph. If K is even, write $K = 2N$ where $N = \frac{K}{2}$. Further, one of the minimal dominating set of the K regular graph is $\{v_1, v_{K+1}\}$ since $G(V, E)$ be a K regular graph and quantity of nodes $n \leq 2K$. The first order dominating set is $D_1 = \{v_1, v_{K+1}\}$. In the sub graph $\langle V - D_1 \rangle$ the 2nd order dominating set is $D_2 = \{v_2, v_{K+2}\}$ since $G(V, E)$ is a K regular graph and quantity of nodes $n \leq 2K$. Similarly the dominating set $D_1 = \{v_N, v_{K+N}\}$. of the sub graph $\langle V = (D_1 \cup D_1 \cup, \ldots, D_{N-1}) \rangle$. Let $W = (D_1 \cup D_2 \cup, \ldots, D_N \cup \{v_K\})$, the code of W is defined by $C_W(V - W) = [a_{ij}]$ a Note that the code $C_W(v_i)$ is pair wise disjoint since $v_i \in (V - W)$ and $G(V, E)$ is a K regular graph. Hence $W = (D_1 \cup D_2 \cup, \ldots, D_N \cup \{v_K\})$, is the resolving set R of K regular graph $G(V, E)$. The dimension of $G(V, E)$ is

$$\dim(G) = |R|$$

$$= |D_1 \cup D_2 \cup, \ldots, D_{N-1} \cup \{v_K\}|$$

$$= |D_1 \cup D_2 \cup, \ldots, D_{N-1}| + |v_K|$$

$$= 2(N - 1) + 1 \therefore |D_i| = 2 \ \forall \ i = 1, 2, \ldots, (N + 1)$$

$$2N - 2 + 1$$

$$\dim(G) = K - 1 \therefore K = 2N$$

Hence proved.

Theorem 3. *Let $G(V, E)$ be a cycle of n vertices C_n. Then the minimal resolving set of C_n is*

i. *If n is odd, $R = \{v_i, v_j\}$ where $v_i, v_j \in V(G)$ and $Dim(C_n) = 2$.*

ii. *If n is even, $R = \{v_i, v_j\}$ where $v_i, v_j \in V(G)$ and $j \neq i + N$ where $N = n/2$ and $Dim(C_n) = 2$.*

Proof. Let $G(V, E)$ be a cycle of n vertices C_n.

i. If n is odd the distance of the vertices u and w in to all other nodes in $V(G)$ are disjoint, i.e. $d(u, v_i) \neq d(w, v_i)$ for all $v_i \in V(G)$. Since C_n is a cycle. Therefore the resolving set of C_n is $R = \{v_{n-1}, v_n\}$. The corresponding resolving matrix is

$$C_R(G) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_{n-2} \end{array} \begin{array}{c} \begin{array}{cc} v_{n-1} & \qquad\qquad v_n \end{array} \\ \begin{bmatrix} d(v_1, v_{n-1}) & d(v_1, v_n) \\ d(v_2, v_{n-1}) & d(v_2, v_n) \\ d(v_3, v_{n-1}) & d(v_3, v_n) \\ \vdots & \vdots \\ d(v_{n-2}, v_{n-1}) & d(v_{n-2}, v_n) \end{bmatrix} \end{array}$$

Each and every rows are distinct in the resolving matrix $C_R(G)$ since $d(v_i, v_{n-1}) \neq d(v_i, v_{n-1})$ for $i = 1, 2, 3, n-2$. This implies $R = \{v_{n-1}, v_n\}$ is a resolving set of C_n. Clearly $R = \{v_{n-1}, v_n\}$ is a minimal resolving set of C_n. Since $d(v_i, v_{n-1}) \neq d(v_i, v_{n-1})$ for $i = 1, 2, 3, n-2$. The $Dim(C_n) = |R| = |\{v_{n-1}, v_n\}| = 2$.

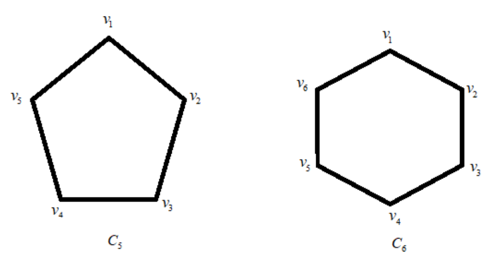
ii. If n is even the distance of the nodes u and w in to all other nodes in $V(G)$ are disjoint, i.e. $d(u, v_i) \neq d(w, v_i)$ for all $v_i \in V(G)$. Since C_n is a cycle. Note that $d(u, v_i) \neq d(w, v_j)$ if $j = i + N$, where $N = n/2$.

Therefore the resolving set of C_n is $R = \{v_i, v_j\}, j \neq i + N$. The corresponding resolving matrix is

$$C_R(G) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_m \end{array} \begin{array}{c} \begin{array}{cc} \quad v_i \quad & \quad v_j \quad \end{array} \\ \begin{bmatrix} d(v_1, v_i) & d(v_1, v_j) \\ d(v_2, v_i) & d(v_2, v_j) \\ d(v_3, v_i) & d(v_3, v_j) \\ \vdots & \vdots \\ d(v_m, v_i) & d(v_m, v_j) \end{bmatrix} \end{array} \text{ where } m \neq i, j.$$

Each and every rows are distinct in the resolving matrix $C_R(G)$. This implies $R = \{v_{n-1}, v_n\}$, $j \neq i + N$ is a resolving set of C_n. Clearly $R = \{v_{n-1}, v_n\}$, $j \neq i + N$ is a minimal resolving set of C_n. Since $d(v_i, v_m) \neq d(v_j, v_m)$ for $v_m \in V(G)$. The $Dim(C_n) = |R| = |\{v_i, v_j\}| = 2$. Hence proved.

Illustration.



The first graph $G(V, E)$ is a cycle C_5 with 5 vertices. The any pair of vertices $R = \{v_i, v_j\}$ is a resolving set of cycle C_5. Assume $R = \{v_1, v_2\}$ and the resolving matrix $C_R(G)$ is

$$C_R(G) = \begin{array}{c} \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{cc} v_1 & v_2 \end{array} \\ \begin{bmatrix} 2 & 1 \\ 2 & 2 \\ 1 & 2 \end{bmatrix} \end{array}$$

The 2nd graph $G(V, E)$ is a cycle C_6 with vertices. The any pair of vertices $R = \{v_i, v_j\}$, $j \neq i + N$. is a resolving set of cycle C_6. Assume $R = \{v_1, v_2\}$ and the resolving matrix $C_R(G)$ is

$$C_R(G) = \begin{array}{c} \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array}\begin{array}{c} \begin{array}{cc} v_1 & v_2 \end{array} \\ \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 2 & 3 \\ 1 & 2 \end{bmatrix} \end{array}$$

Conclusion

In this paper, presented the idea of resolving matrix of the graph and investigate relation between the dominating set and resolving set in the various graphs like complete graphs, complete bipartite graphs cycle, path, and regular graphs. Further prove some results on metric of the graphs also explains the result with examples. In future investigate the metric of different types of the products.

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