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A SIMILARITY APPROACH OF HEAT TRANSFER: THE STUDY OF UNSTEADY SISKO FLUID WITH VISCOUS DISSIPATION EFFECT

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Abstract

The effect of heat transfer in unsteady Sisko fluid flow with viscous dissipation is studied using similarity technique. The main purpose of the similarity technique is to convert PDE into ODE. The converted ordinary equations are solved using MATLAB BVP4C solver by transforming into the system of a first-order ordinary differential equation. The viscous dissipation effects on heat transfer are measured by the Eckert number. The temperature enhances as an increase in Eckert number. Velocity and temperature profiles for Newtonian and non-Newtonian Sisko fluid are compared. The velocity is higher in the case of Sisko fluid than in the Newtonian fluid case and a reverse effect is observed in temperature. The velocity and temperature both are higher in case of and power-law index n zero (shear thinning) than $n = 1, 2, 3$ (shear thickening). The material parameter of Sisko fluid plays an important role on velocity and temperature profile. Impact of Prandtl number on temperature also observed in the study.

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1. Introduction

There are different non-Newtonian fluids models with various characteristics in literature, and Sisko fluid model is one of them. This model has plenty of applications in the industry because of its shear thinning and shear thickening properties. Sisko fluid model is appropriate for the flow of greases, some polymeric suspensions, drilling fluids and cement slurries without yield stress, etc.

In fluid mechanics, viscous dissipation is defined as the reduction of fluctuating velocity gradients due to viscous stresses. The kinetic energy converts into the internal energy of the fluid in the process of viscous dissipation. In the regions with large gradients, dissipation is high because of the viscosity of the fluid, which heats up the fluid. This dissipation process has applications in polymer processing flows, aerodynamic heating in the thin boundary layer around high-speed aircraft, etc.

There are different analytical and numerical methods available in the literature to solve governing nonlinear equations of non-Newtonian fluid flow models. The traveling wave and similarity solutions of nonlinear equations are desirable, as such solutions play a very important role in the study of nonlinear wave and fluid flow phenomena. The main advantage of the similarity method is to convert nonlinear partial differential equations into ordinary differential equations.

The combined influences of an applied magnetic field and viscous dissipation had numerically studied using the Shooting method, for boundary layer flow of Sisko fluid, over the stretching cylinder by Malik et al. [1]. Arif Hussain et al. [2] investigated the Sisko fluid flow characteristics over-stretching cylinder and heat transfer with viscous dissipation. Scaling group of transformations for is used to convert governing partial differential equations into a corresponding set of ordinary differential equations by applying the scaling group of transformations and further numerical technique Runge-Kutta-Fehlberg method is applied to analyze flow behaviour. They observed the opposite trends for impact of viscous dissipation through Eckert number Ec on fluid temperature and local Nusselt number. A. Megahed [3] had analyzed the non-Newtonian Sisko fluid flow due to a nonlinearly stretching sheet with viscous dissipation and heat generation effect by applying the numerical shooting technique.

Unsteady boundary layer flow of a Sisko fluid model over an axisymmetric stretching porous disk in the presence of a uniform magnetic field in cylindrical polar coordinates system is investigated numerically by applying the Shooting method with Runge-Kutta of order 5 by T. Mahmood et al. [4]. M. Khan et al. [5] analyzed the unsteady flow of a Sisko fluid in a cylindrical tube due to the translation of the tube wall parallel to the axis of the tube. They derived a similarity solution using Lie point symmetries of the partial differential equation and converted into the ordinary differential equation and determined the initial condition from a similarity solution of the partial differential equation.

The boundary layer flow of non-Newtonian Casson fluid has been numerically analysed by Ajayi et al. [6] over a horizontal melting surface embedded in a thermally stratified medium under the effect of viscous dissipation and internal space heat source. Substantial increase in temperature distribution is certain with an increase in the magnitude of Eckert number is noticed for the motion of two-dimensional Casson fluid flows with temperature dependent plastic dynamic viscosity together with thermal and solutal stratification in the presence of Lorentz force. Boubaker et al. [7] investigated the heat transfer in the presence of viscous dissipation of pseudo plastic power-law fluids aligned with a semi-infinite plate. They found numerical solutions using the discretization shooting method and the Boubaker polynomials expansion scheme (BPES). Abou-zeid [8] had studied the effects of viscous dissipation on the non-linear peristaltic mechanism with heat transfer of an incompressible micropolar non-Newtonian nanofluid in an asymmetric channel. The closed solutions of fluid velocity and Micro rotation velocity are obtained, and the solutions for temperature and nanoparticle profiles are obtained by using the homotopy perturbation method (HPM).

Viscous dissipation effects had taken into account by S. Agunbiade and M. Dada [9] on an unsteady convective rotatory Rivlin-Ericksen flow of an incompressible electrically conducting fluid under time-dependence suction. The governing equations were non-dimensionalized and reduced to ordinary differential equations using the perturbation technique. The resulting ordinary differential equations were solved using the Adomian decomposition method. M. Shojaeian et al. [10] had analyzed the flow of power-law fluids in circular channels with iso flux thermal wall boundary conditions, under the effect of viscous dissipation in convective heat transfer mode.

N. Shukla et al. [11] had analyzed unsteady MHD boundary layer stagnation point the flow of second-grade nanofluid from a horizontal stretching sheet with second-order slip velocity, entropy generation, and thermal slip effects by applying HAM. The effects of heat generation and viscous dissipation were analyzed numerically by T. Murugesan and Dinesh Kumar [12] on the MHD flow of radiative nanofluid over an exponentially stretching sheet in a porous medium by using an efficient Nachtsheim-Swigert shooting iteration scheme to satisfy asymptotic boundary conditions along with the fourth-order Runge-Kutta integration process.

Kotha Gangadhar et al. [13] had numerically studied free convection flow of Casson fluid over a non-linear stretching sheet under the effect of viscous dissipation using the Spectral Relaxation method. B. C. Parida et al. [14] had studied the effects of viscous dissipation on the unsteady MHD flow of an incompressible viscous fluid over a vertical permeable surface embedded in a porous medium. The Perturbation method has been applied to solve the coupled and nonlinear governing equations.

I. L. Animasaun [15] carried out study of Casson fluid flow along a vertical porous plate in the presence of viscous dissipation, nth order chemical reaction and suction. The effects of thermophoresis, Dufour, temperature dependent thermal conductivity and viscosity of an incompressible electrically conducting Casson fluid flow have been studied by utilising shooting method along with Runge-Kutta Gill and Quadratic interpolation (Muller's scheme). They found the velocity of dissipative Casson fluid flow increase with an increase in the value of temperature dependent fluid viscosity parameter.

Effects of partial slip and viscous dissipation on the dynamics of blood-gold Carreau nanofluid and dusty fluid have been investigated numerically using the classical Runge-Kutta integration scheme together with shooting techniques and Matlab bvp5c package by Olubode et al. [16]. They observed the effects of partial slip are highly significant when viscous dissipation is considerably large due to the existence of a significant difference between the impact of partial slip on the dynamics of blood-gold nanofluid and dusty fluid in the transportation of blood and GNPs mixture.

M. Patel et al. [17] studied the laminar sisko fluid flow by applying one parameter scaling Group similarity transformations and observed that the velocity profile increases more rapidly for Sisko fluid than it is in Power-law fluid. H. Parmar and M. G. Timol [18] derived proper similarity transformation for the unsteady flow of Sisko fluids past the semi-infinite flat plate and converted nonlinear partial differential equations into ordinary differential equations, using the same. R. M. Darji and M. G. Timol [19] had derived similarity variables, using the deductive group-theoretic method, for the unsteady free-convection flow of non-Newtonian Power-law fluids, over a continuous moving vertical plate systematically. The similarity variables had derived for the laminar unsteady boundary layer flow with heat conductive mass transfer by J. Surawala and M. G. Timol [20]. The Similarity variables are derived and applied to convert partial differential equations into ordinary differential equations for different steady Newtonian and non-Newtonian fluid flow problems by H. Shukla et al. [21, 22, 23, 24].

The above literature review shows that there are many papers on the steady and unsteady flow of Newtonian and non-Newtonian fluid problems under the viscous dissipation effects by directly applying the similarity variable or non-dimensional variable to convert partial differential equations into ordinary differential equations. Also, some research work was done by deriving similarity variables on steady flow problems and little work on unsteady flow problems.

M. Kabir and E. Aghdam [25] had analyzed the effect of heat transfer on the unsteady flow of a Sisko fluid model using traveling wave solutions of constant wave speed. They analysed impacts of material parameter, power-law index, and travelling wave speed in the study.

In the present investigation, we have considered viscous dissipation effects on the non-Newtonian Sisko fluid model for unsteady flow over a flat plate. Heat transfer analysis done by analysing impact of Ecrket number. Impact of various physical parameters like Prandtl number, power-law index, and material parameter on the flow are investigated in this paper. The investigation was done by deriving similarity independent and dependent variables for the purpose to reduce governing partial differential equations into ordinary differential equations. This type of analysis not done for this type of problem yet, to the author's knowledge.

2. Mathematical Formulation of Fluid Flow [25]

Here, unsteady, incompressible Sisko fluid flow, over a flat rigid plate situated at $y = 0,$ is considered for heat transfer analysis. The Sisko fluid is in the space $y > 0.$ The plate is in motion with time-dependent velocity $U_0 V_1(t).$ The Sisko fluid flow is generated because of the motion of the plate. The pressure gradient and external forces are neglected. The temperature of the plate is $\theta_0 V_2(t)$ which is supposed to be greater than the ambient temperature. The governing equations for velocity and temperature fields for Sisko fluid can be written as follows:

$$\rho \frac{\partial u}{\partial t} = \frac{\partial}{\partial y}\left\{\left[a + b\left|\frac{\partial u}{\partial y}\right|^{n-1}\right]\frac{\partial u}{\partial y}\right\} \quad (1)$$

$$\rho c_p \frac{\partial \theta}{\partial y} = k\frac{\partial^2 \theta}{\partial y^2} + \left[a + b\left|\frac{\partial u}{\partial y}\right|^{n-1}\right]\left(\frac{\partial u}{\partial y}\right)^2 \quad (2)$$

With conditions on boundary are given by

$$u(0, t) = U_0 V_1(t),\ \theta(0, t) = \theta_0 V_2(t),\ t > 0 \quad (3)$$

$$u(\infty, t) = 0,\ \theta(\infty, t) = \theta_\infty V_2(t),\ t > 0 \quad (4)$$

$$u(y, 0) = g_1(y),\ \theta(y, 0) = g_2(y),\ y > 0 \quad (5)$$

in which U_0 and θ_0 are the characteristic velocity and temperature, respectively, and $g_1(y)$ and $g_2(y)$ are the initial velocity and temperature, respectively. θ_∞ is the ambient temperature. a, b and $n(> 0)$ are the sisko fluid material parameters defined differently for different fluids. ρ the density of the fluid, c_p the specific heat, k the constant thermal conductivity.

Defining the non-dimensional quantities as follows

$$u' = \frac{u}{U_0},\ y' = \frac{yU_0}{\vartheta},\ t' = \frac{tU_0^2}{\vartheta},\ b' = \frac{b}{a}\left|\frac{U_0^2}{\vartheta}\right|^{n-1},\ \theta' = \frac{\theta - \theta_\infty V_2(t)}{\theta_0 - \theta_\infty},$$

$$Ec = \frac{U_0^2}{c_p(\theta_0 - \theta_\infty)},\ \Pr = \frac{ac_p}{k}$$

In the present problem we have $\frac{\partial u}{\partial y} < 0.$

Equations (1) to (5) are converted in non-dimensional form as follows:

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial y}\left\{\left[1 + b\left(-\frac{\partial u}{\partial y}\right)^{n-1}\right]\frac{\partial u}{\partial y}\right\} \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{pr}\frac{\partial^2 \theta}{\partial y^2} + Ec\left[1 + b\left(-\frac{\partial u}{\partial y}\right)^{n-1}\right]\left(\frac{\partial u}{\partial y}\right)^2 \tag{7}$$

Values at boundary are given by:

$$u(0, t) = V_1(t),\ \theta(0, t) = V_2(t),\ t > 0 \tag{8}$$

$$u(\infty, t) = 0,\ \theta(\infty, t) = 0,\ t > 0 \tag{9}$$

$$u(y, 0) = h_1(y),\ \theta(y, 0) = h_2(y),\ y > 0 \tag{10}$$

where $h_1(y) = \frac{g_1(y)}{U_0}$, $h_2(y) = \frac{g_2(y) - \theta_\infty V_2(t)}{\theta_0 - \theta_\infty}$ and des have been omitted for simplicity.

3. Method of Similarity Solution

The method used in this paper is the application of the one-parameter group transformations. By utilizing these transformations the two independent variables of governing equations will be reduced by one and the boundary value type partial differential equations which have two independent variables t and y transform into boundary value type ordinary differential equations in the only one independent variable, which are similarity equations.

We start our solution by defining the one-parameter group transformation of the form

$$G : \begin{cases} \bar{u} = A^u(a)u + B^u(a) \\ \bar{\theta} = A^\theta(a)\theta + B^\theta(a) \\ \bar{y} = A^y(a)y + B^y(a) \\ \bar{t} = A^t(a)t + B^t(a) \end{cases} \tag{11}$$

where, 'a' is the parameter of the transformation. A^S and B^S are real valued and at least differentiable in their real argument 'a'.

$$\frac{\partial\bar{u}}{\partial\bar{t}} = \frac{A^u}{A^t}\frac{\partial u}{\partial t}, \frac{\partial\bar{u}}{\partial\bar{y}} = \frac{A^u}{A^y}\frac{\partial u}{\partial y}, \frac{\partial\bar{\theta}}{\partial\bar{t}} = \frac{A^\theta}{A^t}\frac{\partial\theta}{\partial t}, \frac{\partial\bar{\theta}}{\partial\bar{y}} = \frac{A^\theta}{A^t}\frac{\partial\theta}{\partial y}, \frac{\partial^2\bar{\theta}}{\partial\bar{y}^2} = \frac{A^\theta}{(A^y)^2}\frac{\partial^2\theta}{\partial y^2},$$

$$\frac{\partial^2\bar{u}}{\partial\bar{y}^2} = \frac{A^u}{(A^y)^2}\frac{\partial^2 u}{\partial y^2} \tag{12}$$

3.1. Invariance of differential equation under one-parameter group of transformations.

$$\frac{\partial\bar{u}}{\partial\bar{t}} - \frac{\partial^2\bar{u}}{\partial\bar{y}^2} - nb\left(-\frac{\partial\bar{u}}{\partial\bar{y}}\right)^{n-1}\frac{\partial^2\bar{u}}{\partial\bar{y}^2} = \lambda(a)\left\{\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial y^2} - nb\left(-\frac{\partial u}{\partial y}\right)^{n-1}\frac{\partial^2 u}{\partial y^2}\right\} \tag{13}$$

$$\frac{\partial\bar{\theta}}{\partial\bar{t}} - \frac{1}{pr}\frac{\partial^2\bar{\theta}}{\partial\bar{y}^2} + Ec\left[1 + b\left(-\frac{\partial\bar{u}}{\partial\bar{y}}\right)^{n-1}\right]\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)^2$$

$$= H(a)\left\{\frac{\partial\theta}{\partial t} - \frac{1}{pr}\frac{\partial^2\theta}{\partial y^2} + Ec\left[1 + b\left(-\frac{\partial u}{\partial y}\right)^{n-1}\right]\left(\frac{\partial u}{\partial y}\right)^2\right\} \tag{14}$$

where $\lambda(a) = \frac{A^u}{A^t} = \frac{A^u}{(A^y)^2} = \left(\frac{A^u}{A^y}\right)^{n-1}\frac{A^u}{(A^y)^2}$ (15)

$$H(a) = \frac{A^\theta}{A^t} = \frac{A^\theta}{(A^y)^2} = \left(\frac{A^u}{A^y}\right)^2 = \left(\frac{A^u}{A^y}\right)^{n-1}\left(\frac{A^u}{A^y}\right)^2 \tag{16}$$

$A^t = (A^y)^2$ and $A^\theta = (A^u)^2$ (17)

Applying the invariance principle on auxiliary conditions we get,

$$B^u = B^\theta = B^y = B^t = 0 \tag{18}$$

Here, we derived the group G with one-parameter which transforms, the differential equations (6), (7), and the boundary conditions in equations (8) to (10) invariantly.

The group G is in the form,

$$G:\begin{cases}\overline{u} = A^{u}(a)u\\ \overline{\theta} = (A^{u})^{2}(a)\theta\\ \overline{y} = A^{y}(a)y\\ \overline{t} = (A^{y})^{2}(a)t\end{cases} \tag{19}$$

3.2 Derivation of Complete set of absolute invariant:

To convert the boundary value problem in the form of similarity equations in a single independent variable we will find the complete set of absolute invariants.

If $\eta = \eta(y, t)$ is the absolute invariants of the independent variables, then

$$g_i(y, t; u, \theta) = F_j(\eta(y, t)) \quad j = 1, 2 \tag{20}$$

which are the two absolute invariants corresponding to u and θ. The application of a basic theorem in group theory; states that: A function $g_i(y, t; u, \theta)$ is an absolute invariant of a one-parameter group if it satisfies the following first-order linear differential equation: (see Moran and Gaggioli [27])

$$\sum_{i=1}^{4}(a_i S_i + \beta_i)\frac{\partial g}{\partial S_i} = 0 \text{ where, } S_i = y, t; u, \theta$$

$$\alpha_i = \frac{\partial A^{S_i}}{\partial a}(a_0) \quad \beta_i = \frac{\partial B^{S_i}}{\partial a}(a_0) \quad i = 1, 2, 3 \tag{21}$$

where, a_0 denotes the value of which yield the identity element of the group.

3.2.1 Deduction of similarity independent variable

$\eta(x, y)$ is an absolute invariant if it satisfies the first-order linear partial differential equation:

$$(\alpha_1 y + \beta_1)\frac{\partial \eta}{\partial y} + (\alpha_2 t + \beta_2)\frac{\partial \eta}{\partial t} = 0 \tag{22}$$

Since $\beta_1 = 0, \beta_2 = 0, \alpha_2 = 2\alpha_1$

$$(\alpha_1 y)\frac{\partial \eta}{\partial y} + (2\alpha_1 t)\frac{\partial \eta}{\partial t} = 0 \tag{23}$$

the solution of (23) is given by,

$$\eta = \frac{y}{\sqrt{t}} \tag{24}$$

3.2.2 Deduction of similarity dependent variables u, θ

By applying, the same concept which we applied for derivation of similarity independent variable, we get

$$(\alpha_1 y)\frac{\partial f}{\partial y} + (2\alpha_1 t)\frac{\partial f}{\partial t} + \alpha_3 u \frac{\partial f}{\partial u} = 0 \tag{25}$$

$$f(\eta) = \frac{u}{t^{\frac{m}{2}}} \text{ where } m = \frac{\alpha_3}{\alpha_1} \tag{26}$$

$$u = f(\eta)\, t^{\frac{m}{2}} \tag{27}$$

$$(\alpha_1 y)\frac{\partial g}{\partial y} + (2\alpha_1 t)\frac{\partial g}{\partial t} + 2\alpha_3 \theta \frac{\partial g}{\partial \theta} = 0 \tag{28}$$

$$g(\eta) = \frac{\theta}{t^m} \tag{29}$$

$$\theta = g(\eta)t^m \tag{30}$$

3.3 The reduction to similarity equations.

Applying derived transformations in equations (24), (27), (30), equations (6)-(7) are converted into the following form.

$$\rho \frac{m}{2} f(\eta) - \frac{\eta}{2} f'(\eta) = f''(\eta) + nb[-t^{\frac{m-1}{2}} f'(\eta)]^{n-1} f''(\eta) \tag{31}$$

$$mg(\eta) - \frac{\eta}{2} g'(\eta) = \frac{1}{pr} g''(\eta) + Ec\left[1 + b\left(-f'(\eta)t^{\frac{m-1}{2}}\right)^{n-1}\right](f'(n))^2 \tag{32}$$

With boundary conditions

$\eta = 0 \Rightarrow f(\eta) = 1,\ g(\eta) = 1$

$\eta = \infty \Rightarrow f(\eta) = 0,\ g(\eta) = 0$

Here we observed equations (31) and (32) which have two independent variables η and t. So, we required coefficients either constants or functions of η only.

So, for the same purpose, we choose $m = 1$. So, the above equations are reduced into the following ODEs.

$$\frac{1}{2} f(\eta) - \frac{\eta}{2} f'(\eta) = f''(\eta) + nb[-f'(\eta)]^{n-1} f''(\eta) \tag{33}$$

$$g(\eta) - \frac{\eta}{2} g'(\eta) = \frac{1}{pr} g''(\eta) + Ec[1 + b(-f'(n))^{n-1}](f'(\eta))^2 \tag{34}$$

With boundary conditions

$\eta = 0 \Rightarrow f(\eta) = 1,\ g(\eta) = 1$

$\eta = \infty \Rightarrow f(\eta) = 0,\ g(\eta) = 0$

4. Numerical Solution

It is a very difficult task to solve these differential equations analytically. So, here, we used BVP4C MATLAB coding to obtain a numerical solution to the problem. To apply MATLAB Bvp4c coding we have to reduce the above system of equations in a system of first-order differential equations as follows:

Substitute y_i, for $i = 1, 2, 3, 4$ for functions

f, f', g, g' and $x = \eta$

$$y_1' = y_2 \tag{35}$$

$$y_2' = \frac{y_1 - xy_2}{2[1 + nb(-y_2)^{n-1}]} \tag{36}$$

$$y_3' = y_4 \tag{37}$$

$$y_4' = pr[y_3 - \frac{x}{2} y_4 - Ec(y_2)^2(1 + b(-y_2)^{n-1})] \tag{38}$$

with boundary conditions

$$x = 0 \Rightarrow y_1 = 1,\ y_3 = 1$$

$$x = \infty \Rightarrow y_1 = 1,\ y_3 = 1 \qquad (39)$$

5. Results and Discussion

Equations (35) to (39) are solved by BVP4C coding.

We had taken $\eta_\infty = 6$. Flow Velocity and temperature values are obtained for fixed parameter values $b = 0.2,\ pr = 1,\ Ec = 0.8$ and for different flow index value $n = 1, 2, 3$.

Table 1. Comparison of velocity and temperature for different $b = 0.2,\ pr = 1,\ Ec = 0.8$.

η	Velocity ($n = 1$)	Velocity ($n = 2$)	Velocity ($n = 3$)	Temperature ($n = 1$)	Temperature ($n = 2$)	Temperature ($n = 3$)
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.5	0.5887	0.5891	0.5930	0.6360	0.6280	0.6232
1	0.3150	0.3232	0.3313	0.3700	0.3482	0.3444
1.5	0.1520	0.1645	0.1721	0.1927	0.1724	0.1706
2	0.0656	0.0772	0.0821	0.0791	0.0766	0.0760
2.5	0.0252	0.0332	0.0357	0.0316	0.0307	0.0305
3	0.0086	0.0130	0.0141	0.0114	0.0111	0.0110

Table 2. Velocity and temperature for $n = 0,\ b = 0.2,\ pr = 1,\ Ec = 0.8$.

η	Velocity $(n = 0)$	Temp $(n = 0)$
0	1.0000	1.0000
0.5	0.6187	0.6404
1	0.3538	0.3700
1.5	0.1858	0.1927
2	0.0891	0.0906

2.5	0.0388	0.0385
3	0.0153	0.0148

We observed from Table 1 that velocity is higher in the case of $n = 2$ and $n = 3$ for non-Newtonian fluid than in the case of $n = 1$ for the Newtonian fluid. We observed that temperature are higher in the case of $n = 1$ for Newtonian fluid than in the case of $n = 2$ and $n = 3$ for non-Newtonian fluid. From Table 1 and Table 2, we observed that velocity and temperature for $n = 0$ is higher than $n = 1,\ 2$ and 3. 6. Graphical presentation:

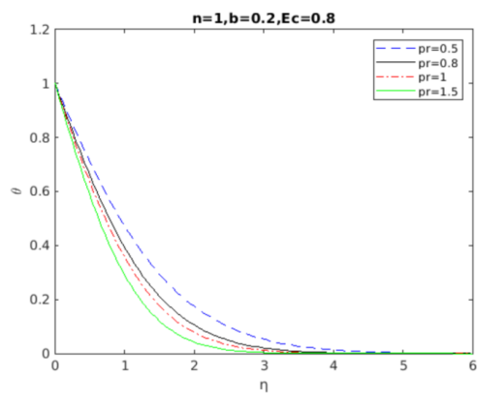

Figure 1. Effect of pr on temperature for Newtonian fluid flow.

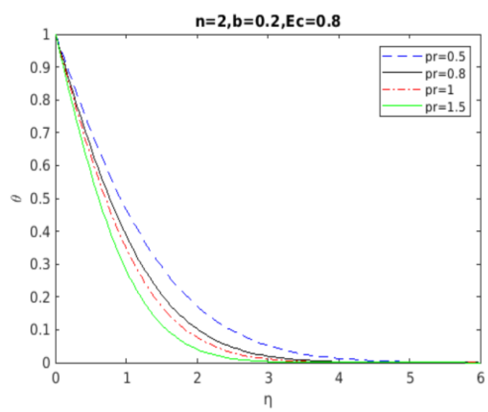

Figure 2. Effect of pr on temperature for non-Newtonian fluid flow.

Figures 1 and 2 describe the impact of Prandtl number Pr on the temperature profile. The thermal conductivity of the fluid declines by enhancing the Prandtl number Pr. Thus the transfer of the heat slows which falls down the temperature of flow distribution. This figure validates the above result i.e. the temperature of the flow distribution falls when Prandtl number Pr increases.

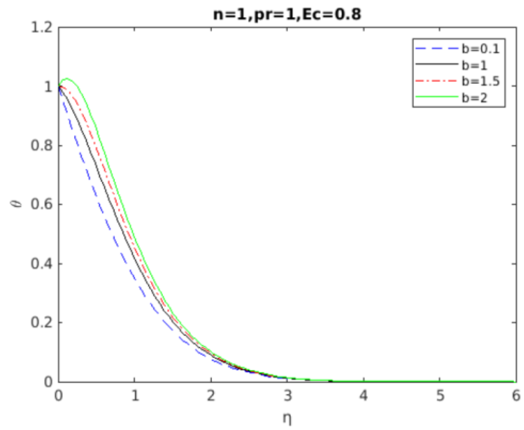

Figure 3. Effect of fluid parameter b on temperature for Newtonian fluid flow $(n = 1)$.

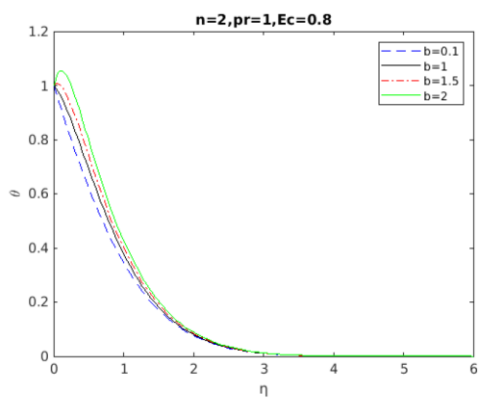

Figure 4. Effect of fluid parameter b on temperature for non-Newtonian fluid flow $(n = 2)$.

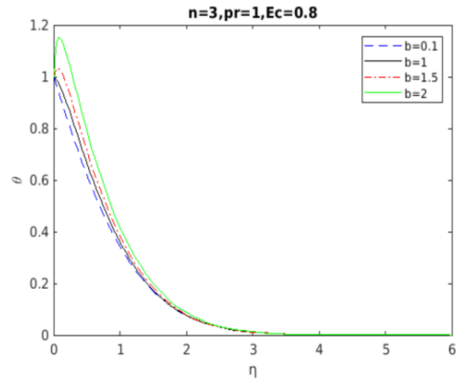

Figure 5. Effect of fluid parameter b on temperature for non-Newtonian fluid flow $(n = 3)$

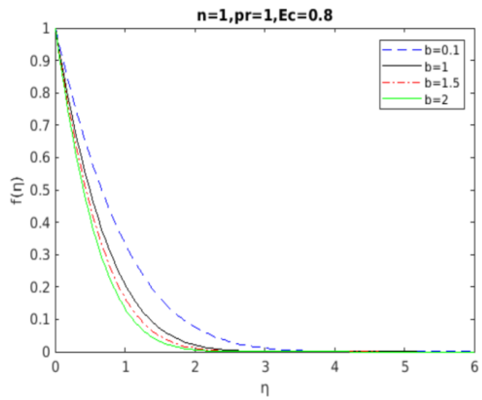

Figure 7. Effect of fluid parameter b on velocity for non-Newtonian fluid flow $(n = 2)$.

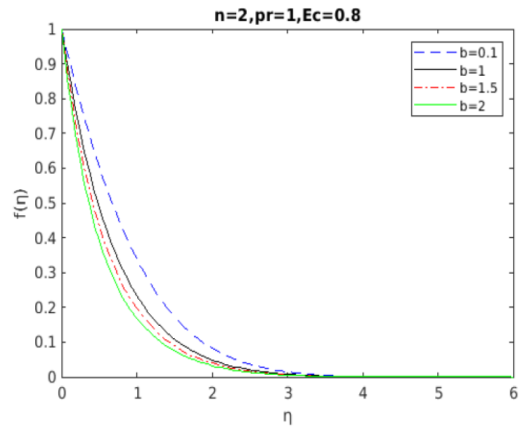

Figure 6. Effect of fluid parameter b on velocity for Newtonian fluid flow $(n = 1)$.

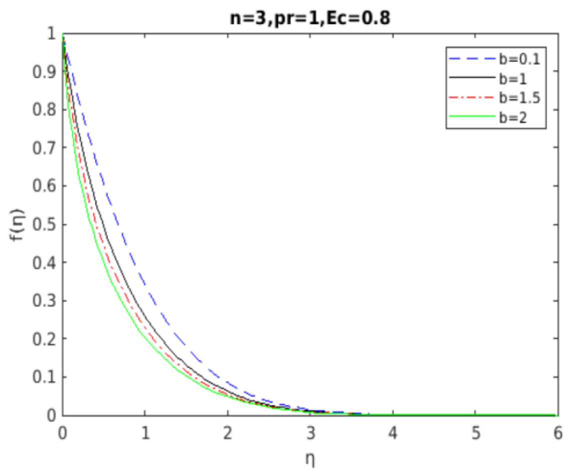

Figure 8. Effect of fluid parameter b on velocity for non-Newtonian fluid flow $(n = 3)$.

Figure 3 shows the characteristics of the fluid parameter b on a temperature for Newtonian fluid and Figures 4 and 5 show the impact of fluid parameter b on a temperature profile for non-Newtonian Sisko fluid. As increasing fluid parameter b temperature enhances for all three power-law indexes. Figure 6 describes the characteristics of the fluid parameter b on velocity for Newtonian fluid and Figures 7 and 8 show the impact of fluid parameter b on velocity profile for non-Newtonian Sisko fluid. Here opposite trends for velocity than temperature. As increasing fluid parameter b, velocity decreases for all three power-law index. So, by controlling fluid parameter b and power-law index we can control the temperature and velocity of fluid flow.

The impact of Eckert number Ec on temperature is demonstrated in Figure 9 for power law index $(n = 1)$ Newtonian fluid. Figures 10 and 11 show temperature profile for different Eckert number Ec of power-law index $n = 2$ and 3 for non-Newtonian fluid. As Eckert number Ec is the relation between flow kinetic energy to heat enthalpy difference. So increase in Eckert number causes an enhancement in the kinetic energy. We knew that temperature is defined as average kinetic energy. Thus, the temperature of the fluid rises. It is depicted from these graphs that fluid temperature increases when Eckert number Ec increases.

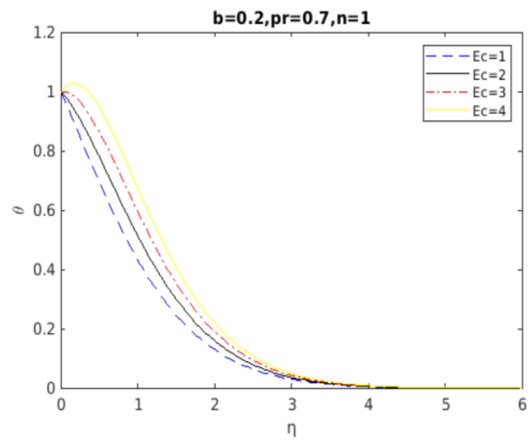

Figure 10. Impact of Eckert number on temperature for non-Newtonian fluid flow $(n = 2)$.

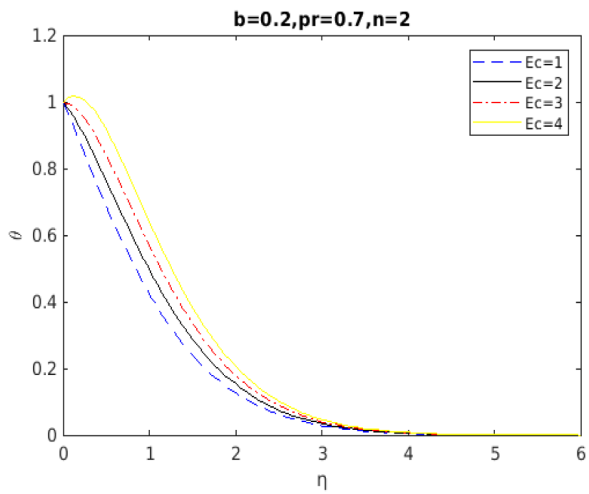

Figure 9. Impact of Eckert number on temperature for Newtonian fluid flow $(n = 1)$.

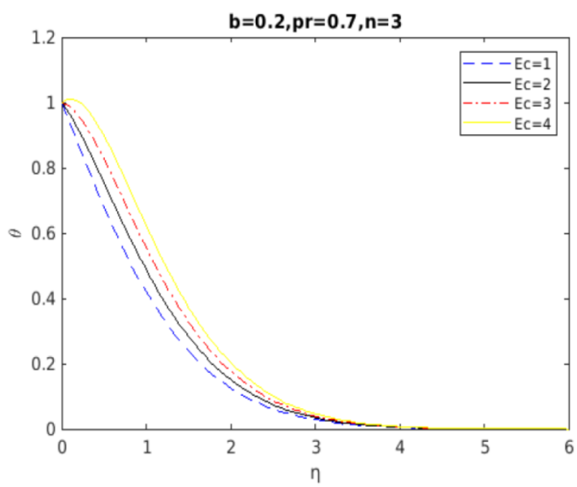

Figure 11. Impact of Eckert number on temperature for non-Newtonian fluid flow $(n = 3)$.

7. Conclusions

In this research work, one parameter group-theoretic similarity method is applied. The similarity solution is derived by converting a nonlinear system of partial differential equations to a nonlinear system of ordinary differential equations using similarity variables. MATLAB BVP4C coding was employed to find numerical solutions to the reduced problem. The results are presented graphically for the velocity and temperature profiles to show the influence of the pertinent parameters.

- The temperature is lower in the Newtonian fluid $(n = 1)$ than the Sisko fluid $(n = 2, 3)$ and the reverse trend is observed for velocity. The different trend observed for case $n = 0$. The temperature and velocity are higher in the case of $n = 0$ than $n = 1, 2, 3$.

- The velocity and temperature of the fluid can be controlled at a required level by adjusting the Sisko fluid material parameter b and the power index n.

- The viscous dissipation effects are observed through the Eckert number. Temperature increases as increasing Eckert number in all cases of power-law index $n = 1, 2, 3$.

- The temperature profile decreases as increasing Prandtl number.

References

[1] M. Y. Malik, A. Hussain, T. Salahuddin and M. Awais, Effects of viscous dissipation on MHD boundary layer flow of Sisko fluid over a stretching cylinder, AIP Advances 6 (2016), https://doi.org/10.1063/1.4944347

[2] Hussain, M. Malik, S. Bilal, M. Awai and T. Salahuddin, A note on numerical study of Sisko fluid flow over stretching cylinder and heat transfer with viscous dissipation, Mathematical Sciences Letters 2 (2017), 199-205. http://dx.doi.org/10.18576/msl/060213

[3] A. M. Megahed, Flow and heat transfer of non-Newtonian Sisko fluid past a nonlinearly stretching sheet with heat generation and viscous dissipation, J. Braz. Soc. Mech. Sci. Eng. 40 (2018), 492, https://doi.org/10.1007/s40430-018-1410-3.

[4] T. Mahmood, Z. Iqbal and A. Shahzad, A numerical investigation of time-dependent MHD axisymmetric transport of Sisko fluid towards elongating porous disk, Pramana J. Phys 92(14) (2019), https://doi.org/10.1007/s12043-018-1689-8.

[5] M. Khan, S. Abelman, D. P. Mason and F. M. Mahomed, Self-Similar unsteady flow of a Sisko fluid in a cylindrical tube undergoing translation, Mathematical Problems in Engineering, Article ID 135424, 14 (2015), http://dx.doi.org/10.1155/2015/135424.

[6] T. M. Ajayi, A. J. Omowaye and I. L. Animasaun, Viscous dissipation effects on the motion of casson fluid over an upper horizontal thermally stratified melting surface of a

paraboloid of revolution: boundary layer analysis, Journal of Applied Mathematics (2017) 13, Article ID 1697135, https://doi.org/10.1155/2017/1697135

[7] Karem Boubaker, Botong Li, Liancun Zheng, Ali H Bhrawy and Xinxin Zhang, Effects of Viscous Dissipation on the Thermal Boundary Layer of Pseudoplastic Power-Law Non-Newtonian Fluids Using Discretization Method and the Boubaker Polynomials Expansion Scheme, International Scholarly Research Notices 6, 2012 (2012), https://doi.org /10.5402/2012/181286.

[8] Mohamed Abou-Zeid, Effects of thermal-diffusion and viscous dissipation on peristaltic flow of micropolar non-Newtonian nanofluid, Application of homotopy perturbation method, Results in Physics 6 (2016), 481-495, https://doi.org/10.1016/j.rinp.2016.08.006

[9] S. A. Agunbiade and M. S. Dada, Effects of viscous dissipation on convective rotatory chemically reacting Rivlin-Ericksen flow past a porous vertical plate, Journal of Taibah University for Science 13(1) (2019), 402-413, https://doi.org/10.1080/16583655.2019.1582149.

[10] Mostafa Shojaeian, Mehmet Yildiz and Ali Koşar, Convective heat transfer and second law analysis of non-Newtonian fluid flows with variable thermophysical properties in circular channels, International Communications in Heat and Mass Transfer 60 (2015), 21-31, http://dx.doi.org/10.1016/j.icheatmasstransfer.

[11] Nisha Shukla, Punit Rana and O. Anwar Beg, Unsteady MHD Non-Newtonian Heat Transfer Nanofluids with Entropy Generation Analysis, Nonlinear Engineering 8(1) (2019), 630-644. https://doi.org/10.1515/nleng-2017-0177

[12] T. Mrugusen and M. Dineshkumar, Effects of thermal radiation and heat generation on hydromagnetic flow of nanofluid over an exponentially stretching sheet in a porous medium with viscous dissipation, World Scientific News, 128(2) (2019), 130-147.

[13] Kotha Gangadhar, D. N. Bhargavi and V. S. Munagala, Steady Boundary Layer Flow of Casson Fluid over a Nonlinear Stretched Sheet in Presence of Viscous Dissipation Using the Spectral Relaxation Method, Mathematical Modelling of Engineering Problems, 7(3) (2020), 351-358, https://doi.org/10.18280/mmep.70304.

[14] B. C. Parida, B. K. Swain, N. Senapati and S. Sahoo, Viscous Dissipation Effect on MHD Free Convective Flow in the Presence of Thermal Radiation and Chemical Reaction, Mathematical Modelling of Engineering Problems 7(3) (2020), 387-394, https://doi.org/10.18280/mmep.70308.

[15] I. L. Animasaun, Effects of thermophoresis, variable viscosity and thermal conductivity on free convective heat and mass transfer of non-Darcian MHD dissipative Casson fluid flow with suction and nth order of chemical reaction, Journal of the Nigerian Mathematical Society 34 (2015), 11-31, http://dx.doi.org/10.1016/j.jnnms.2014.10.008

[16] O. K. Koriko, K. S. Adegbie, N. A. Shah, I. L. Animasaun and M. A. Olotu, Numerical solutions of the partial differential equations for investigating the significance of partial slip due to lateral velocity and viscous dissipation: The case of blood-gold Carreau nanofluid and dusty fluid, Numerical Methods of Partial Differential Equations (2021), 1-29, https://doi.org/10.1002/num.22754

[17] M. Patel, J. Patel and M. G. Timol, Laminar Boundary Layer Flow of Sisko Fluid, Applications and Applied Mathematics: An International Journal (AAM) 10(2) (2015), 909-918.

[18] Hiral Parmar and M. G. Timol, Group theoretic analysis of Rayleigh problem for a non-Newtonian MHD Sisko fluid past semi infinite plate, Int. J. Adv. Appl. Math. and Mech. 1(3) (2014), 96-108.

[19] R. M. Darji and M. G. Timol, Similarity analysis for unsteady natural convective boundary layer flow of Sisko fluid, Int. J. Adv. Appl. Math. and Mech. 1(3) (2014), 22-36.

[20] J. Surawala and M. G. Timol, Analysis of thermal boundary layer flow of viscous fluids by new similarity method, Int. J. Adv. Appl. Math. and Mech. 6(1) (2018), 69-77.

[21] H. C. Surati, H. S. Shukla and M. G. Timol, On the Transformation of Non-Newtonian Boundary value problem to Initial value problem, International Journal of Engineering Science Invention Research and development 3(3) (2016), 180-189.

[22] H. S. Shukla, H. C. Surati and M. G. Timol, Similarity Analysis of MHD Three Dimensional Nanofluid Flow for Non-Newtonian Power-law Model over Linearly Stretching Sheet with Convective Boundary Conditions, International Journal of Heat and Technology 38(1) (2020), 203-212, https://doi.org/10.18280/ijht.380122.

[23] H. S. Shukla, J. Patel, H. C. Surati, M. Patel and M. G. Timol, Similarity Solution of Forced Convection Flow of Powell-Eyring and Prandtl-Eyring Fluids by Group-Theoretic Method, Mathematical Journal of Interdisciplinary Sciences 5(2) (2017), 151-165. https://doi.org/10.15415/mjis.2017.52012

[24] H. S. Shukla, H. C. Surati and M. G. Timol, Similarity analysis of three dimensional nanofluid flow by deductive group theoretic method, Applications and Applied Mathematics 13(2) (2018), 1260-1272.

[25] M. M. Kabir and E. A. Aghdam, Unsteady flow and heat transfer analysis on a sisko fluid, Thermal Science (2012). https://doi.org/10.2298/TSCI120223013K.

[26] M. J. Moran and R. A. Gaggioli, A new systematic formalism for similarity analysis, with application to boundary layer flow, U.S. Army Mathematics Research Center, Technical Summary Report No. 918, (1968).